CMB lensing overview

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Berkeley, 21 April 2011

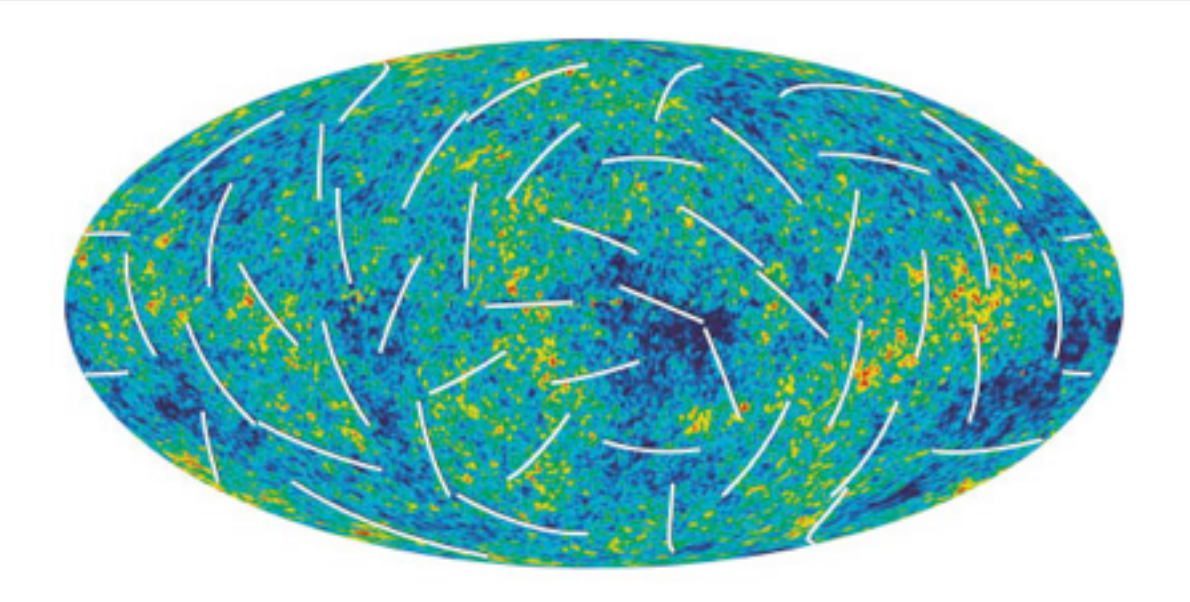
1. CMB lensing: general picture

2. Non-Gaussian statistics

3. B-modes

4. Cosmological information from lensing

CMB fields



Temperature field $\Delta T(\mathbf{n})$

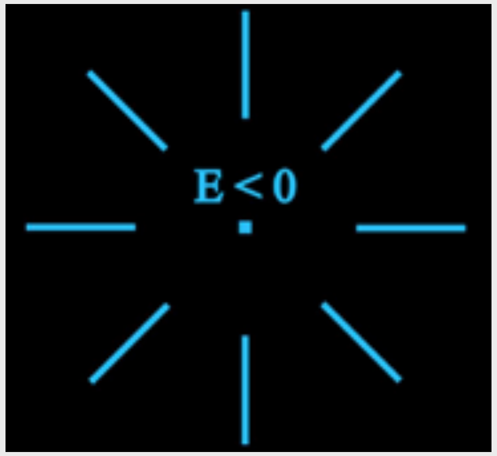
In Fourier space:

$$T(\mathbf{l}) = \int d^2\mathbf{n}\, T(\mathbf{n}) e^{i\mathbf{l}\cdot\mathbf{n}}$$

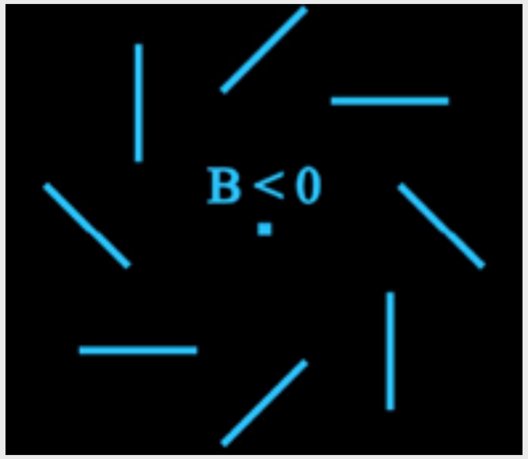
Polarization fields $Q(\mathbf{n}), U(\mathbf{n})$

E-B decomposition:

$$\begin{pmatrix} E(\mathbf{l}) \\ B(\mathbf{l}) \end{pmatrix} = \begin{pmatrix} \cos(2\varphi_l) & \sin(2\varphi_l) \\ -\sin(2\varphi_l) & \cos(2\varphi_l) \end{pmatrix} \begin{pmatrix} Q(\mathbf{l}) \\ U(\mathbf{l}) \end{pmatrix}$$

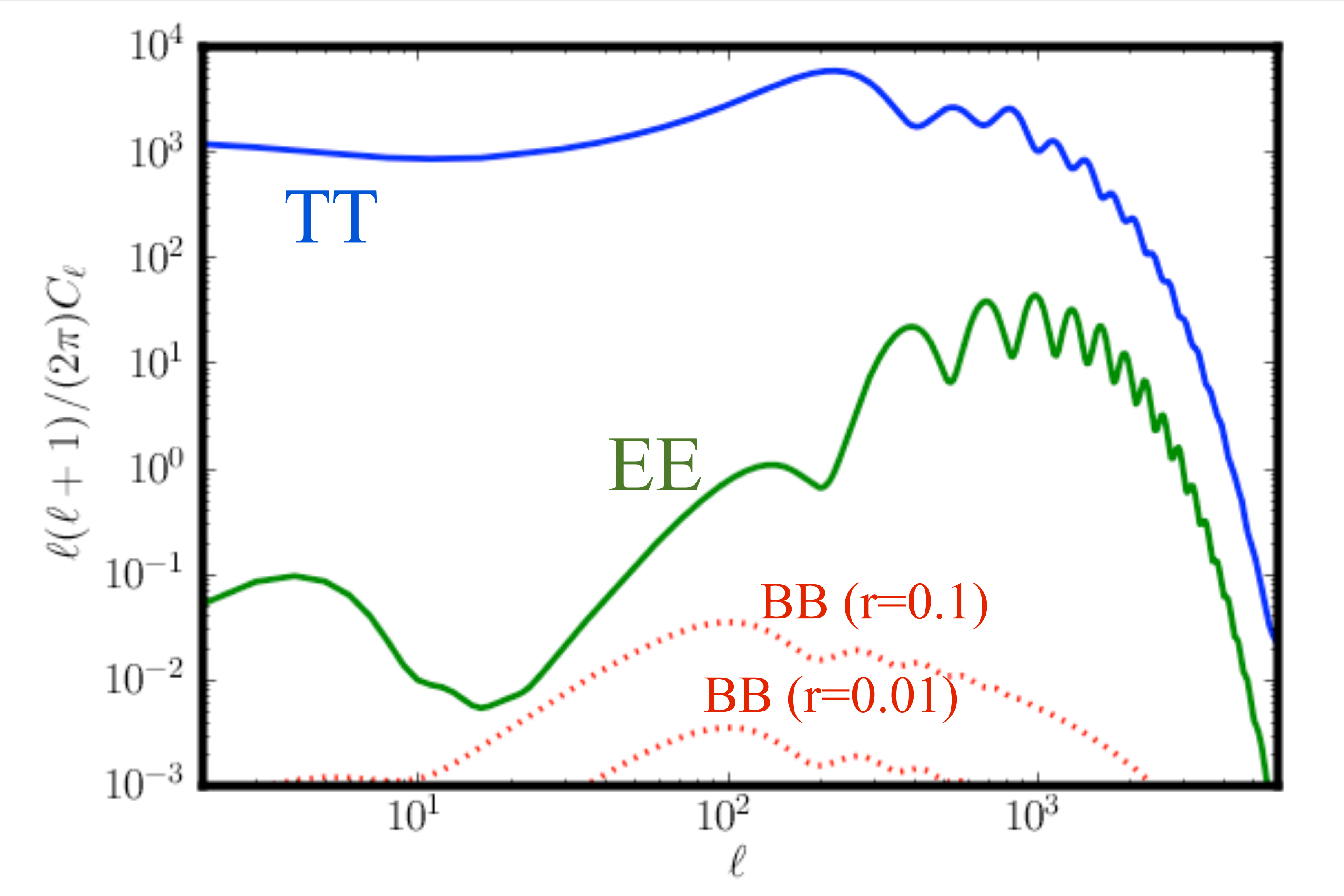


E-mode
("gradient-like")



B-mode
("curl-like")

Unlensed CMB: power spectra

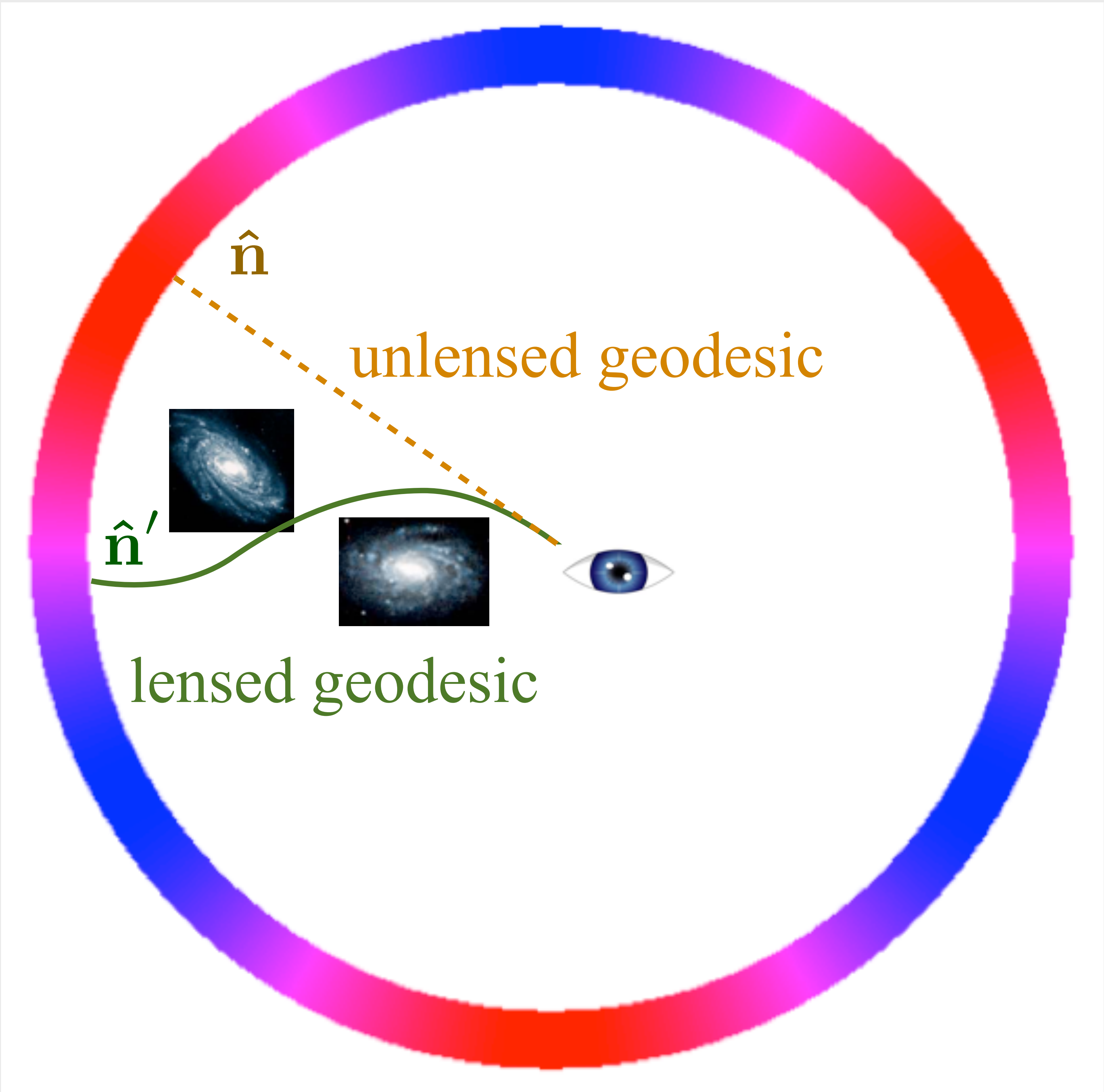


Two-point function in Fourier space:

$$\langle T(\mathbf{l})T(\mathbf{l}')^*\rangle = C_l^{TT}\delta^2(\mathbf{l}-\mathbf{l}')$$

If the CMB is a Gaussian field, then power spectrum contains all the information in the original map ("sufficient statistic")

Lensed CMB: general picture



Apparent anisotropy in direction $\hat{\mathbf{n}}$
= unlensed anisotropy in direction $\hat{\mathbf{n}}'$

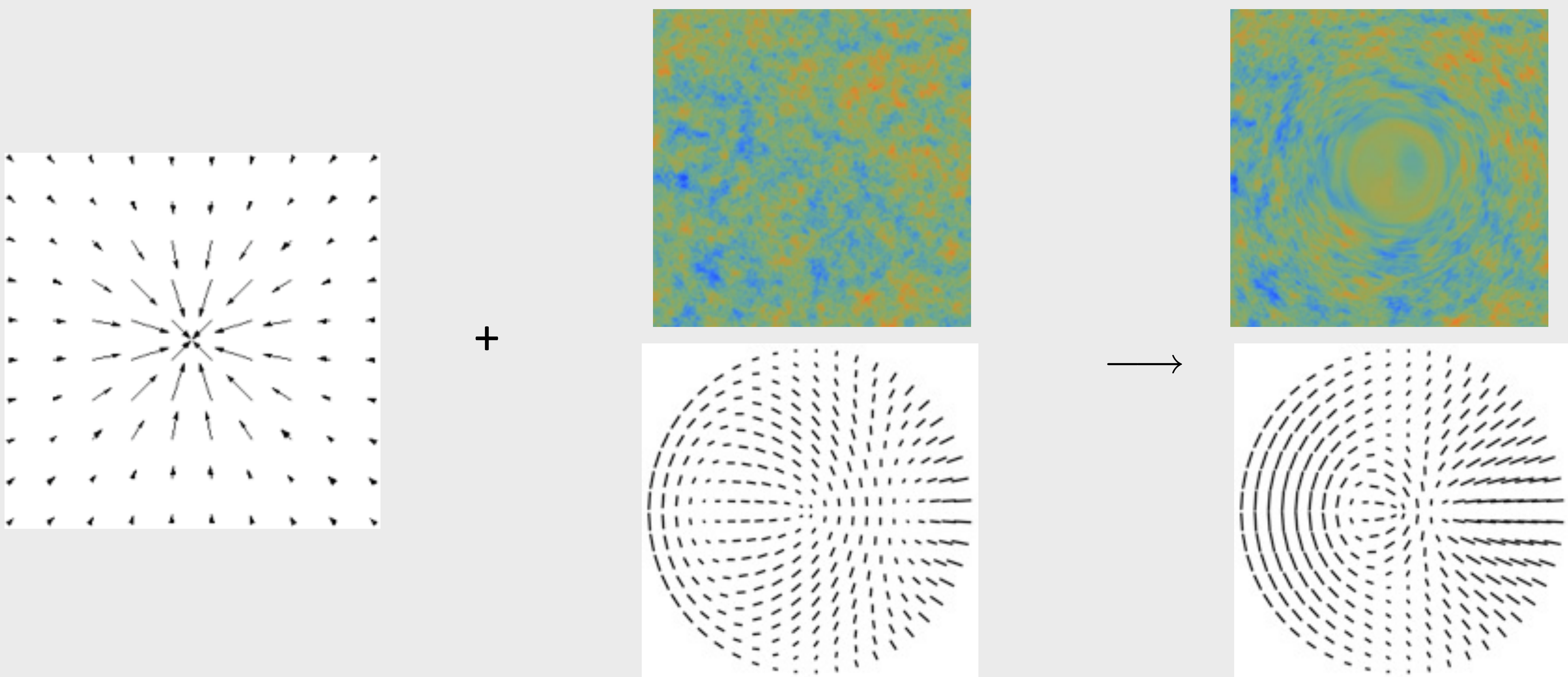
Moves existing temperature fluctuations around; does not generate new anisotropy (conserves surface brightness)

Lensed CMB: general picture

More formally: can define vector field $\mathbf{d}(\hat{\mathbf{n}})$; lensed CMB is given by:

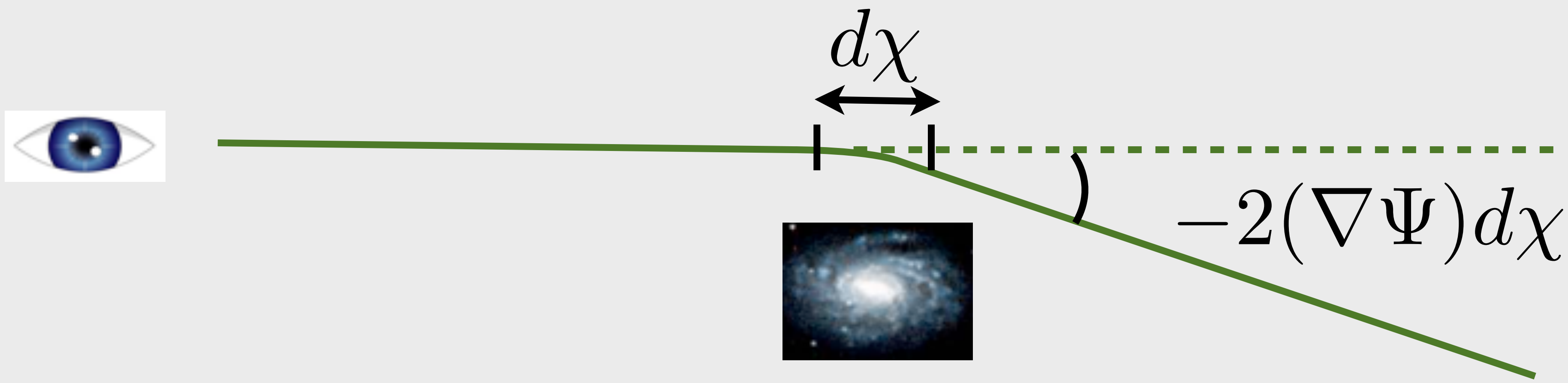
$$\Delta T(\mathbf{n})_{\text{lensed}} = \Delta T(\mathbf{n}+\mathbf{d}(\mathbf{n}))_{\text{unlensed}}$$

$$(Q \pm iU)(\mathbf{n})_{\text{lensed}} = (Q \pm iU)(\mathbf{n}+\mathbf{d}(\mathbf{n}))_{\text{unlensed}}$$

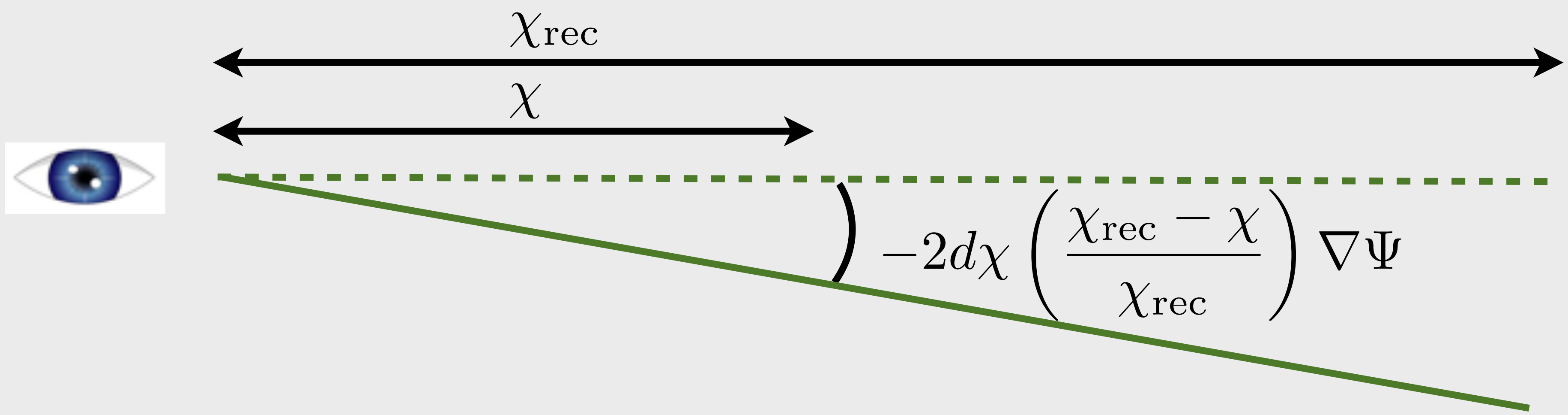


deflection field $\mathbf{d}(\hat{\mathbf{n}})$ unlensed CMB lensed (observed) CMB

Lensed CMB: line-of-sight integral



Lensed CMB: line-of-sight integral

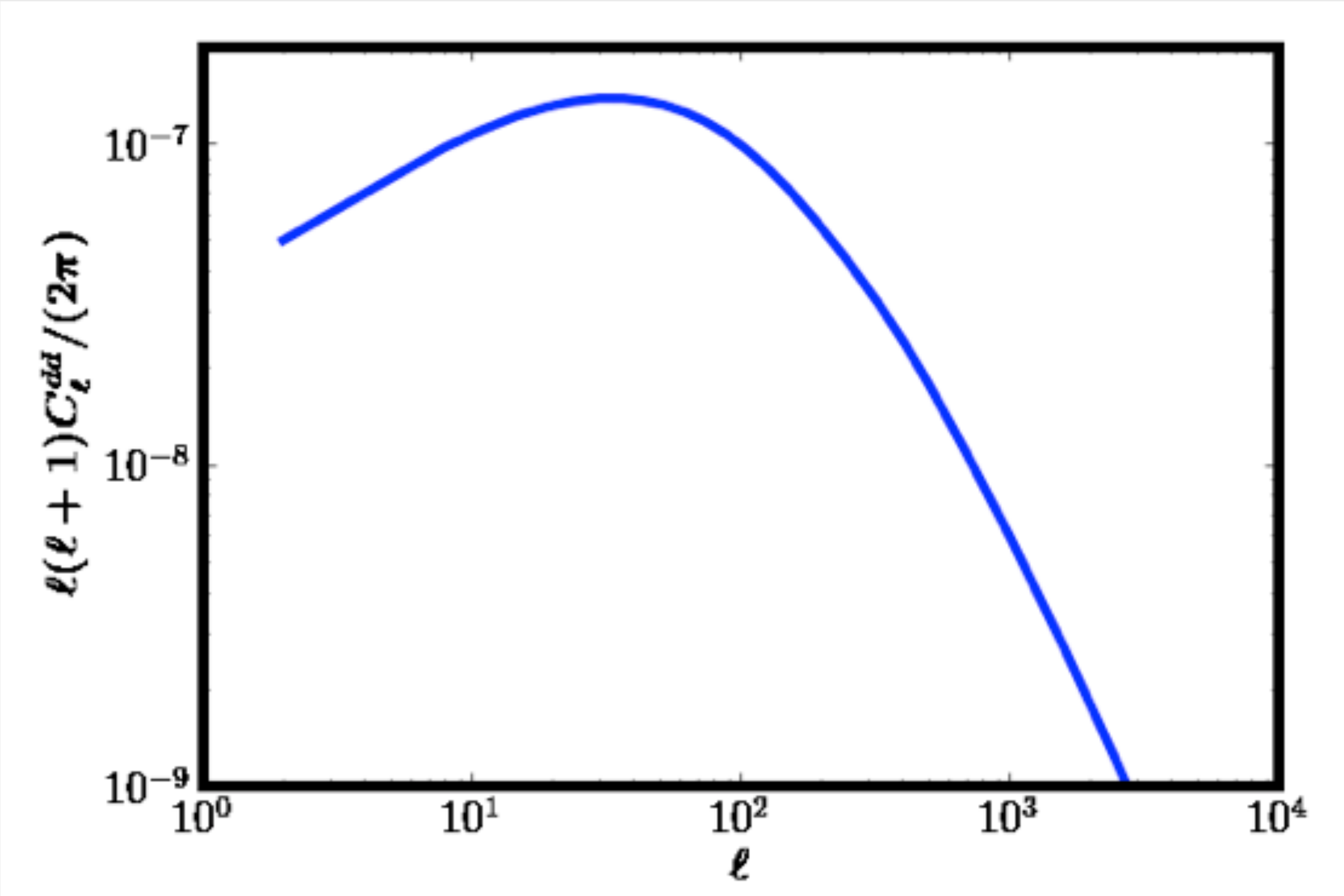


$$
\begin{aligned}
\mathbf{d}(\hat{\mathbf{n}}) &= -2\int d\chi\left(\frac{\chi_{\rm rec}-\chi}{\chi_{\rm rec}}\right)\nabla_{\perp}\Psi(\chi\hat{\mathbf{n}},\chi) \\
&= \nabla\underbrace{\left[-2\int d\chi\left(\frac{\chi_{\rm rec}-\chi}{\chi\chi_{\rm rec}}\right)\Psi(\chi\hat{\mathbf{n}},\chi)\right]}_{\phi(\hat{\mathbf{n}})}
\end{aligned}
$$

Deflection field is a pure gradient (no curl component)

Deflection field

$$\mathbf{d}(\hat{\mathbf{n}}) = -2 \int d\chi \left(\frac{\chi_{\mathrm{rec}} - \chi}{\chi_{\mathrm{rec}}} \right) \nabla_\perp \Psi(\chi \hat{\mathbf{n}}, \chi)$$



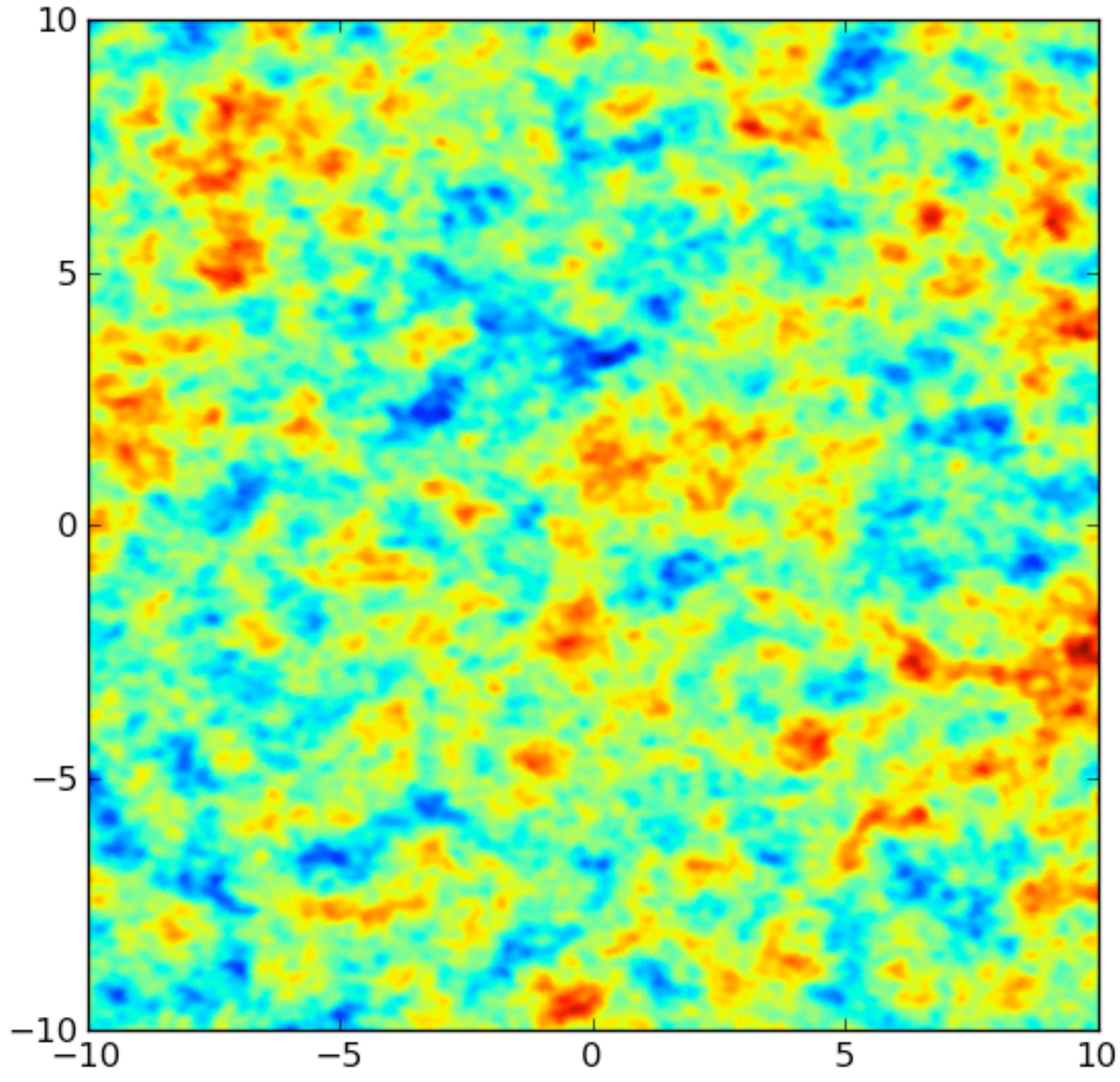
Line-of-sight integral is broadly peaked at $z \approx 2$.

Angular power spectrum of deflection field $\mathbf{d}(\hat{\mathbf{n}})$ is broadly peaked at $\ell \approx 40$.

RMS deflection: 2.5 arcmin

"Few-arcminute deflections, coherent on ~2 deg scales, sourced by large-scale structure at redshift ~2"

Unlensed vs lensed CMB



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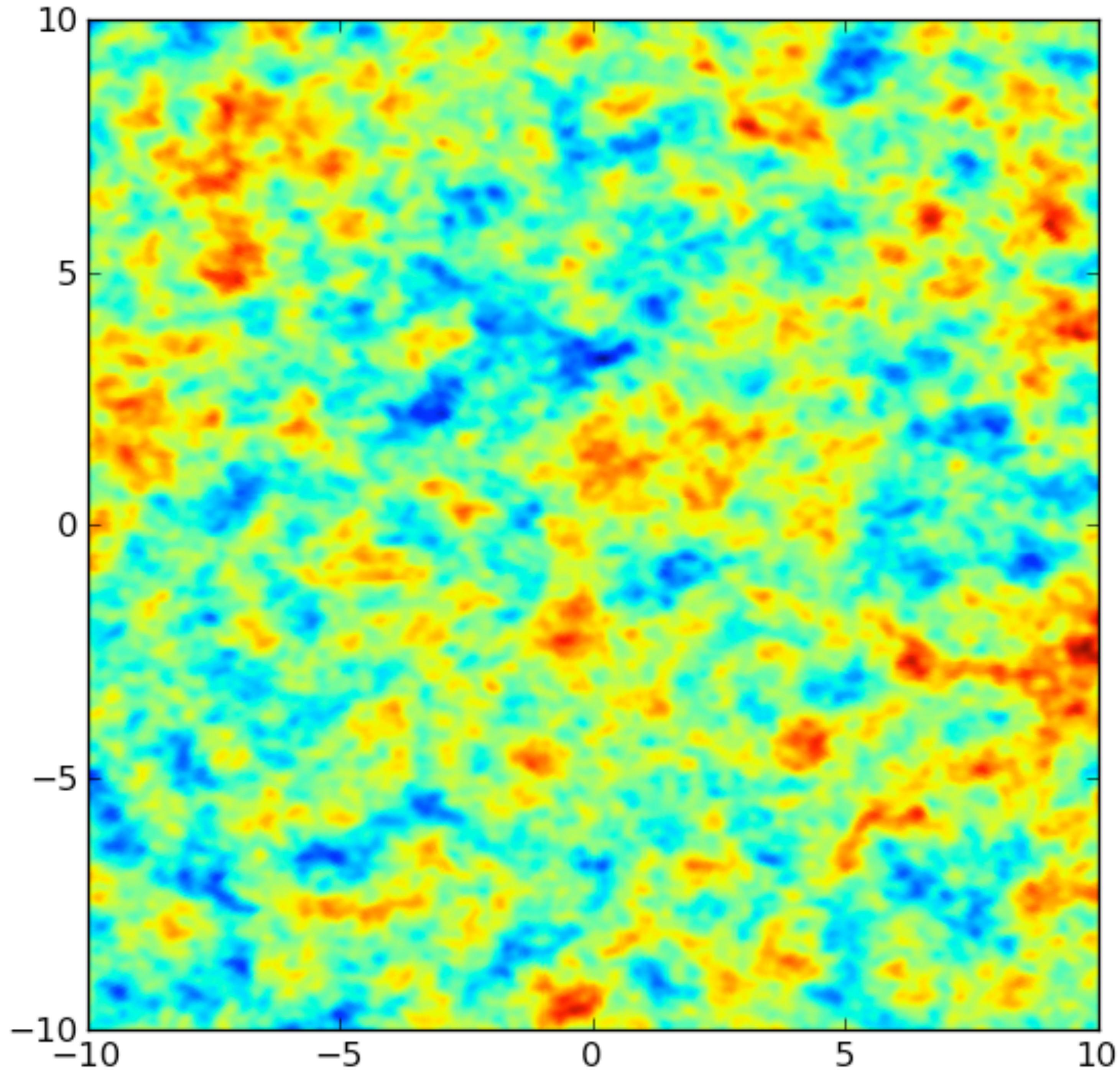
Unlensed vs lensed CMB



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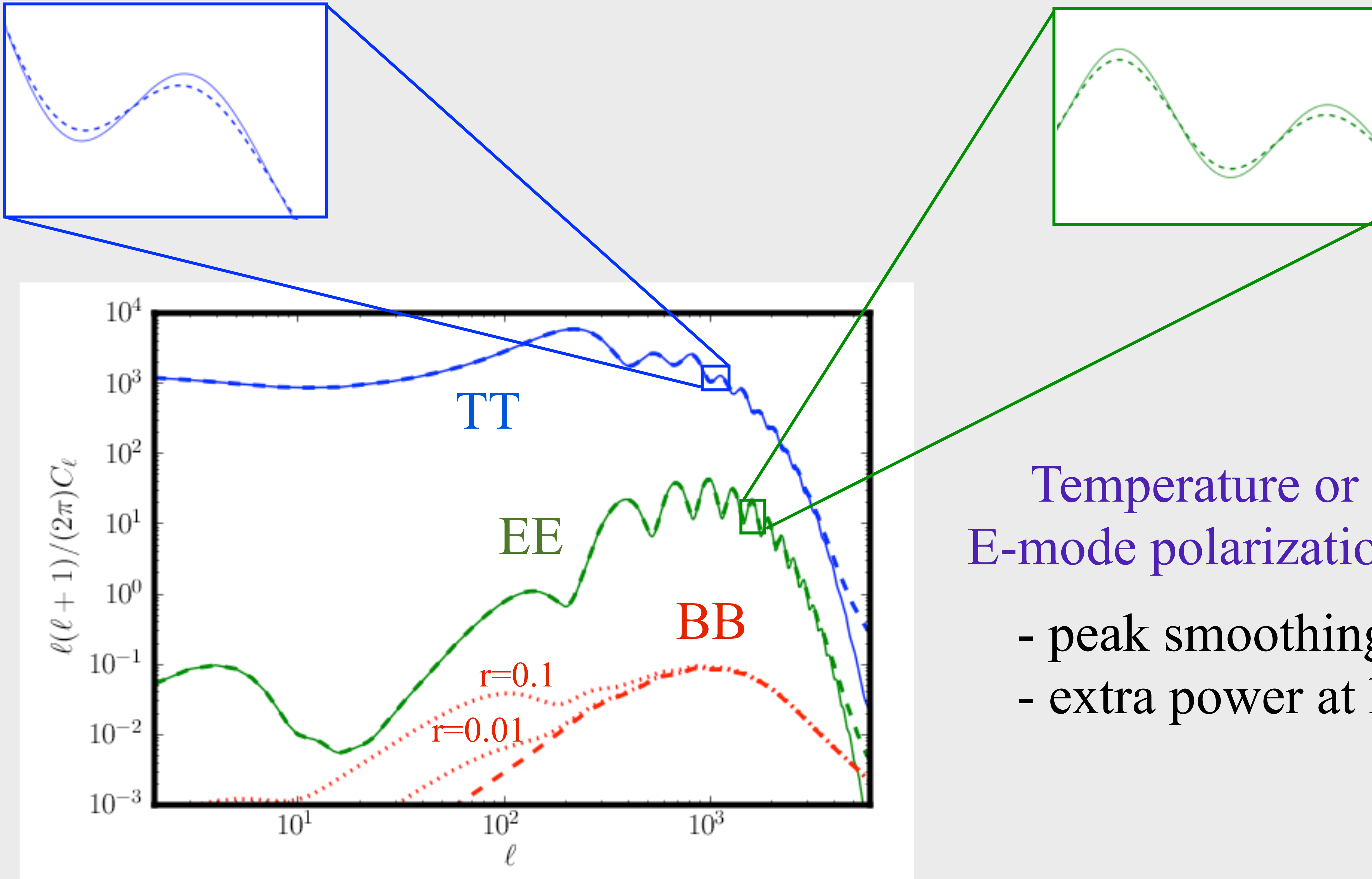
Lensed CMB: power spectra



Temperature or
E-mode polarization:

- peak smoothing
- extra power at high ℓ

Lensing converts primary E-mode to mixture of E and B
=> largest guaranteed source of B-mode polarization

Lensed TT spectrum: observations

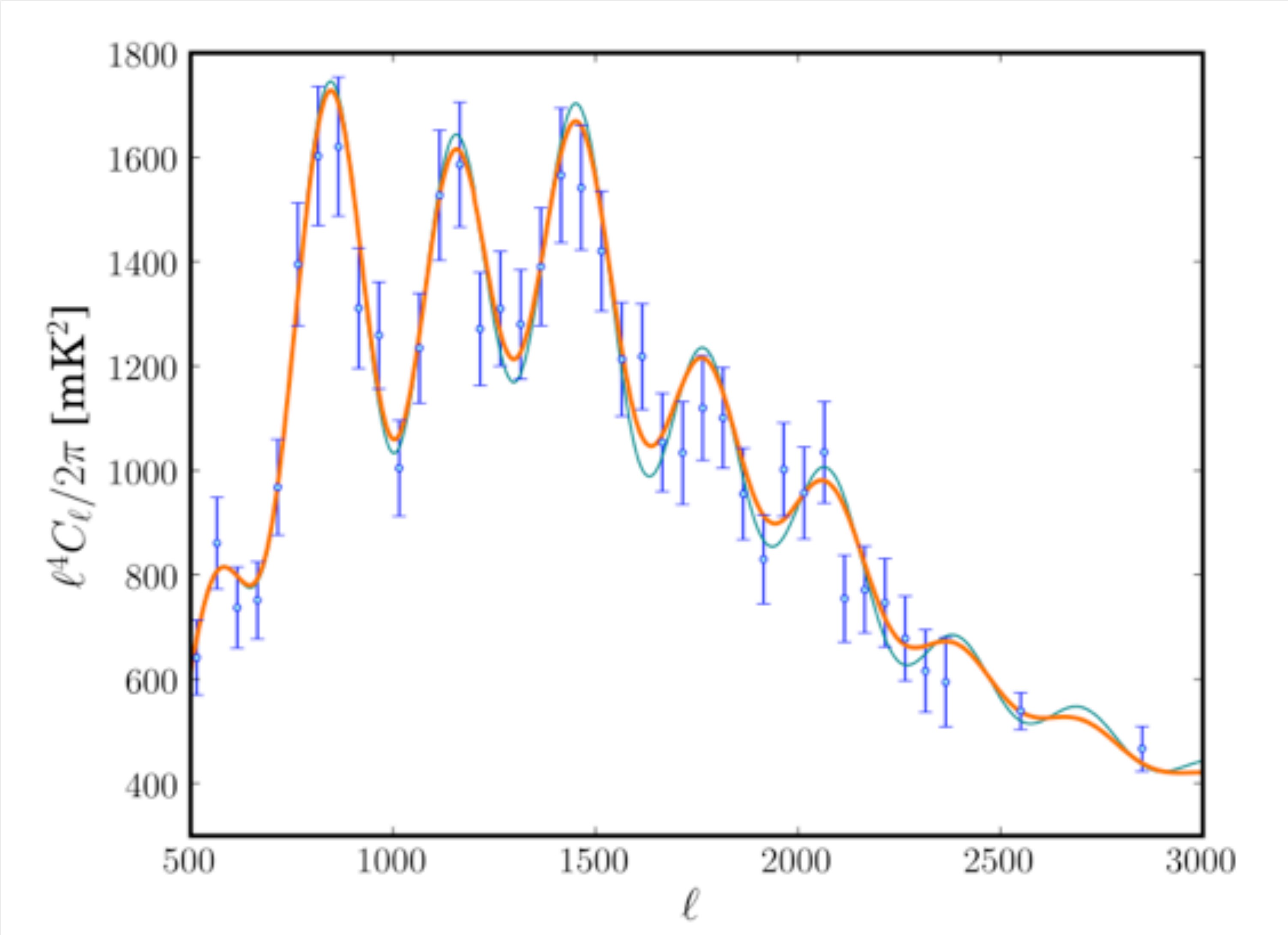
Current TT power spectrum measurements prefer a lensed spectrum to an unlensed spectrum at few-sigma level

WMAP+ACT: 2.8σ

(Das et al 2010)

WMAP+ACBAR +QUAD+SPT: 3.4σ

(Shirokoff et al 2010)



Das et al 2010

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Lensed CMB: non-Gaussian statistics

Taylor-expand lensed CMB in powers of deflection field:

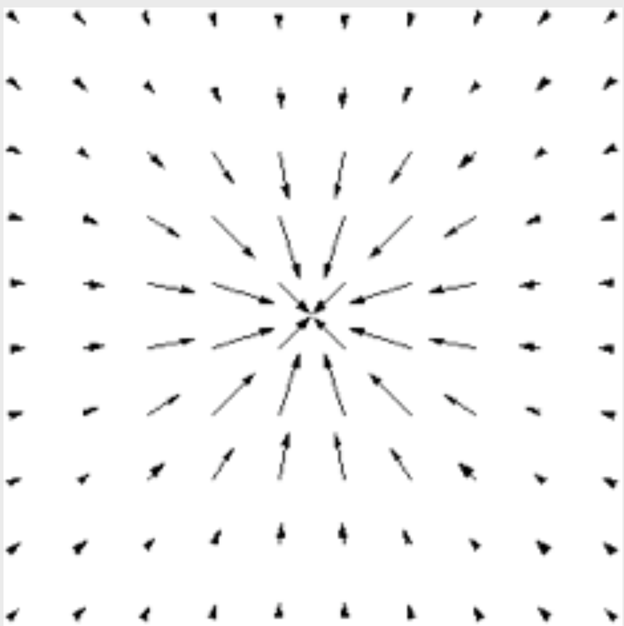
$$\begin{aligned} T(\mathbf{n})_{\rm lensed} &= T(\mathbf{n}+\mathbf{d}(\mathbf{n}))_{\rm unl} \\ &= T(\mathbf{n})_{\rm unl} + d_i(\mathbf{n})\nabla_i T(\mathbf{n})_{\rm unl} \\ &\quad + \frac{1}{2} d_i(\mathbf{n}) d_j(\mathbf{n}) \nabla_i \nabla_j T(\mathbf{n})_{\rm unl} + \cdots \end{aligned}$$

All terms beyond the first are non-Gaussian

=> statistics not fully determined by power spectrum

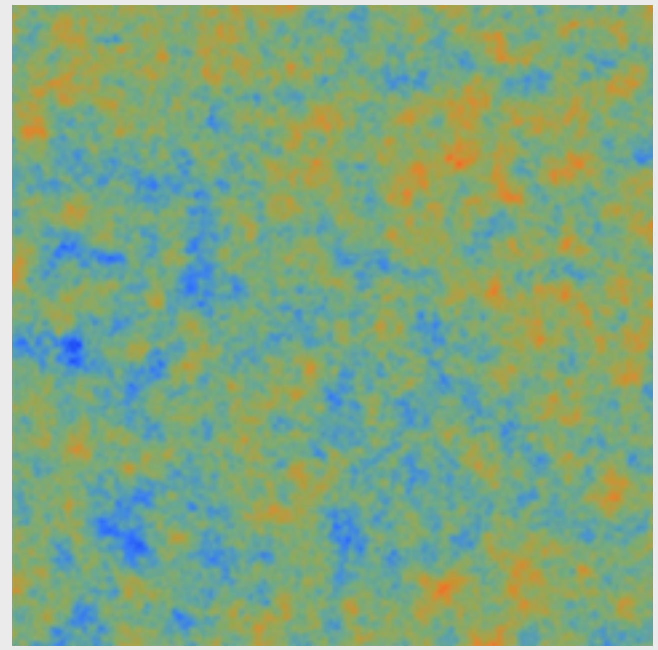
Next part of talk: describe higher-point statistics which complement the power spectrum and extract characteristic non-Gaussianity generated by gravitational lensing

Lens reconstruction: idea



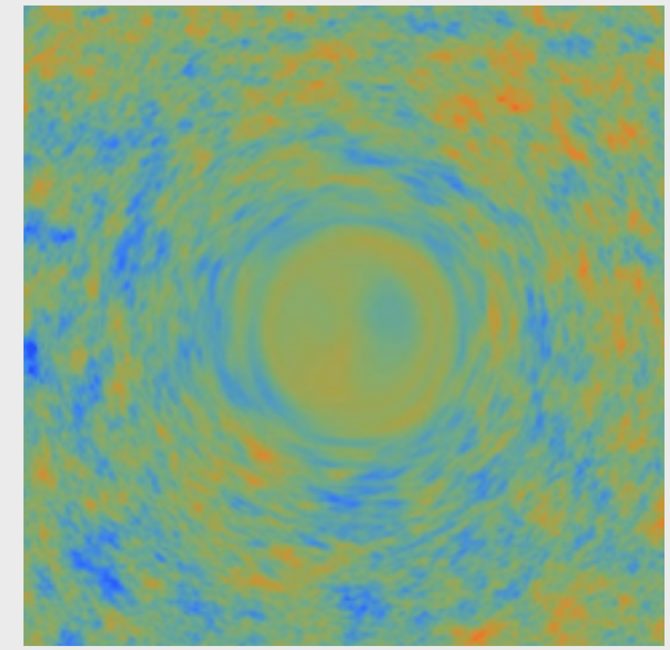
Deflection angles

+



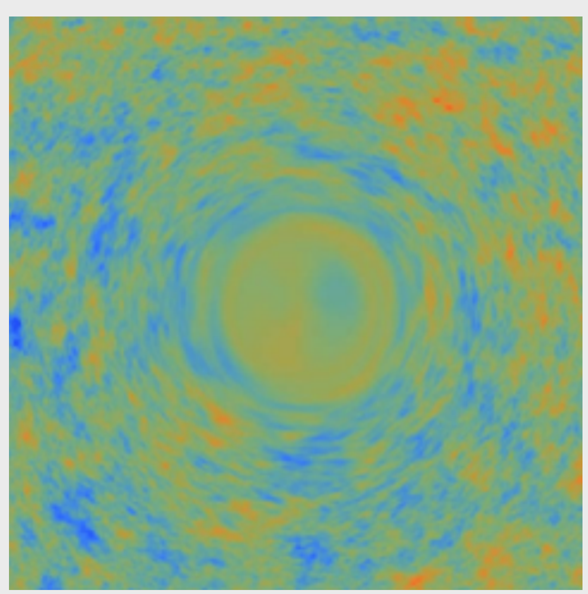
Unlensed CMB

→



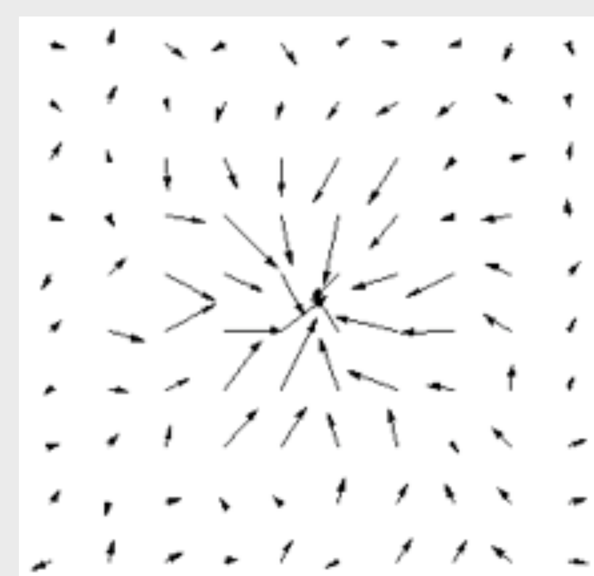
Lensed CMB

Idea: from observed CMB, reconstruct deflection angles (Hu 2001)



Lensed CMB

→



Reconstruction + noise

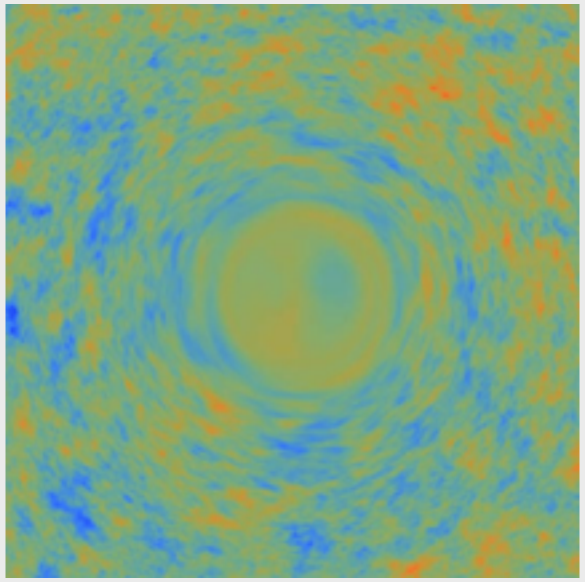
Quadratic estimator

In a fixed lens, Fourier modes with $\mathbf{l} \neq \mathbf{l}'$ are weakly correlated:

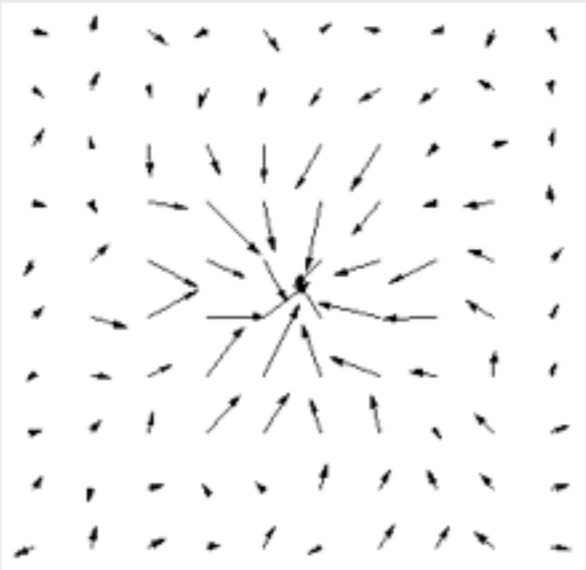
$$\langle T(\mathbf{l})T(\mathbf{l}')^* \rangle = iC_l[\mathbf{l} \cdot \mathbf{d}(\mathbf{l} - \mathbf{l}')] + [\mathbf{l} \leftrightarrow \mathbf{l}']^*$$

Formally: can define estimator $\widehat{\mathbf{d}}(\mathbf{l})$ which is quadratic in temperature

$$\widehat{\mathbf{d}}(\mathbf{l}) \propto \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} [\mathbf{l}_1 C_{\ell_1} + \mathbf{l}_2 C_{\ell_2}] \frac{T(\mathbf{l}_1)T(\mathbf{l}_2)}{(C_{\ell_1} + N_{\ell_1})(C_{\ell_2} + N_{\ell_2})}$$



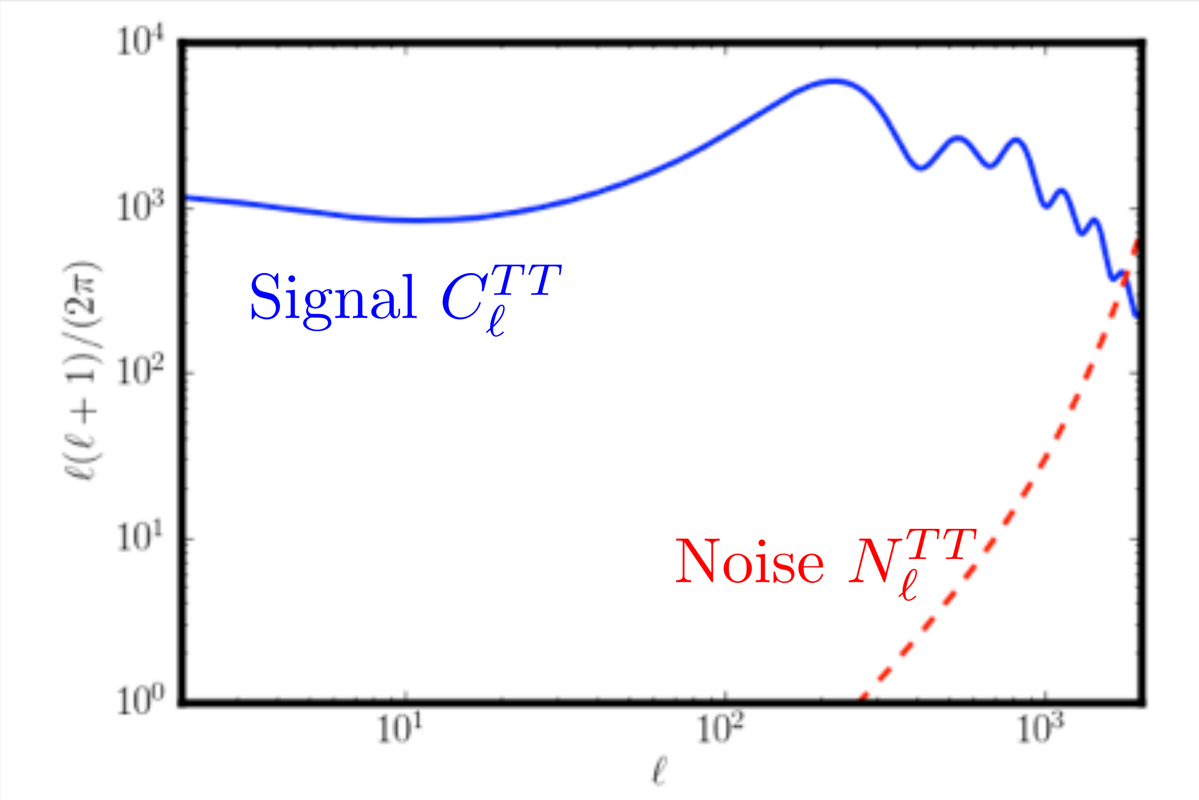
$\longrightarrow$



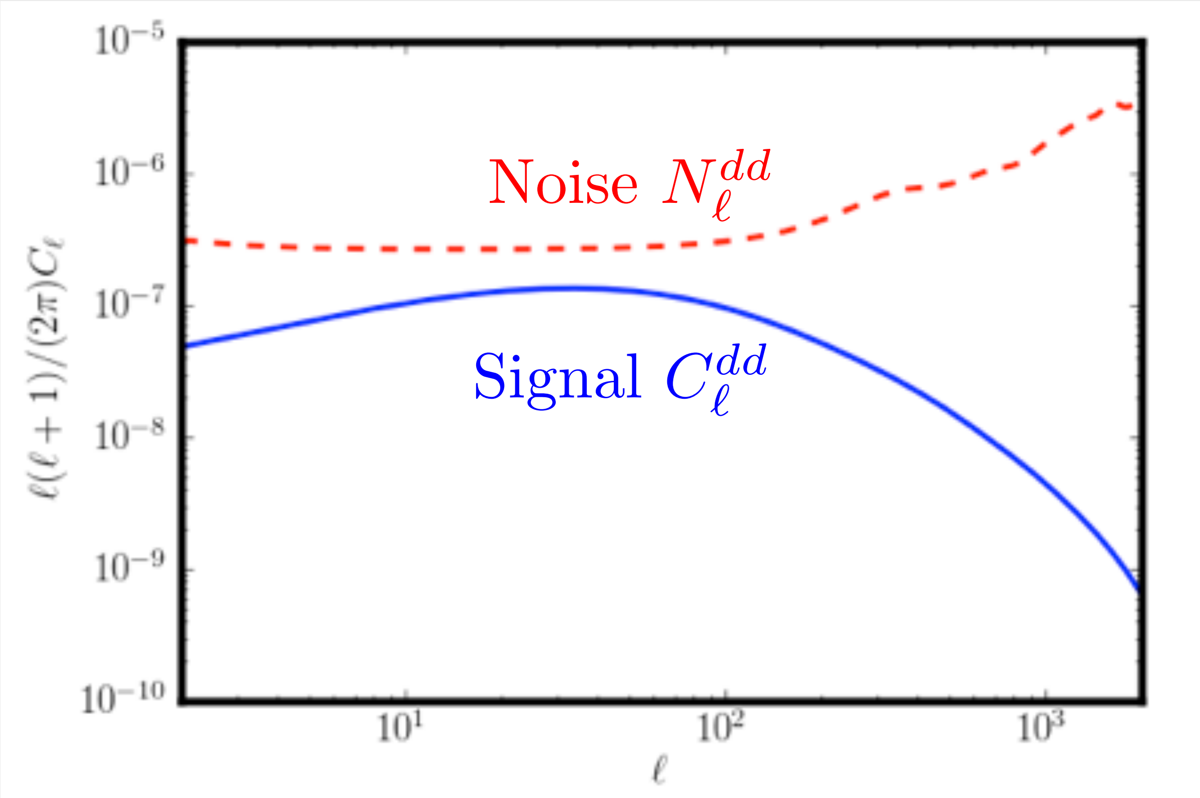
Lensed CMB

Reconstruction + noise

Example: Planck forecasts



Signal/noise power spectra (temperature)

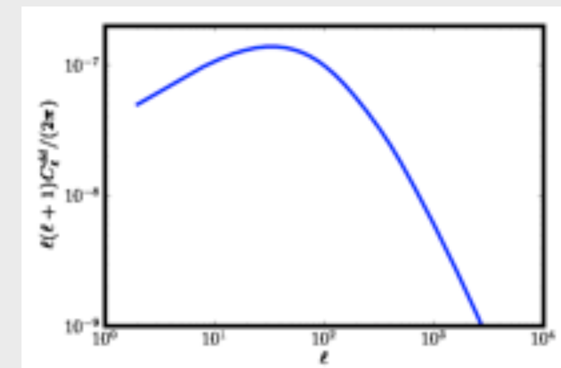


Signal/noise power spectra (lens reconstruction)

Higher-point statistics

Lens reconstruction naturally leads to higher-point statistics:

e.g. start with observed CMB temperature $T(\mathbf{n})$



=> apply quadratic estimator $\widehat{\mathbf{d}}(\mathbf{l})$

=> estimate deflection power spectrum C_l^{dd}

Estimator for C_l^{dd} is a 4-point estimator in the CMB

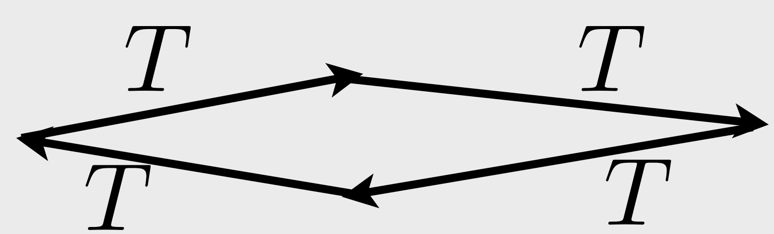
or: start with CMB temperature $T(\mathbf{n})$ and galaxy counts $g(\mathbf{n})$

=> apply quadratic estimator $\widehat{\mathbf{d}}(\mathbf{l})$

=> estimate deflection power spectrum C_l^{dg}

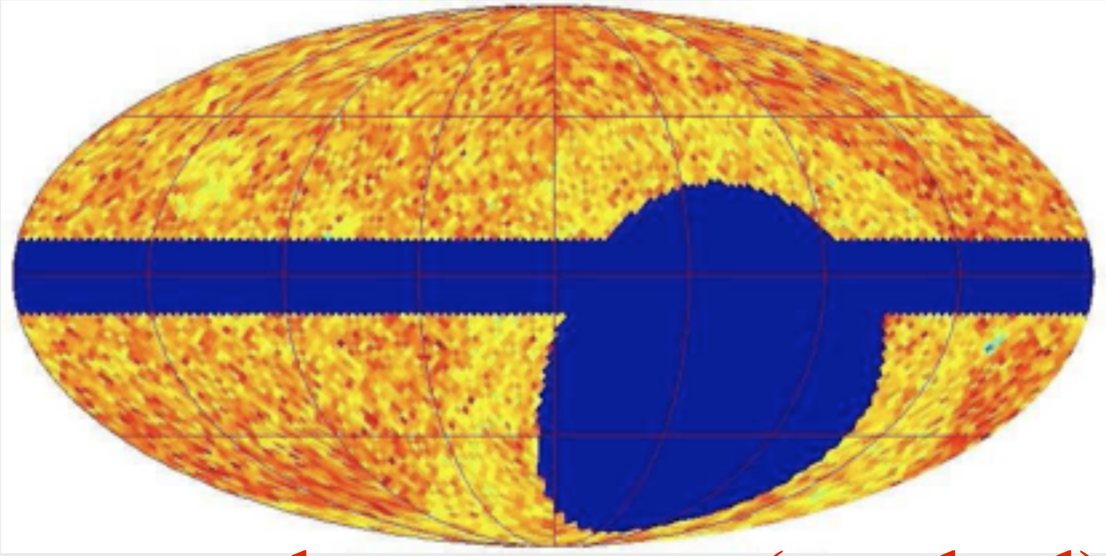
Estimator for C_l^{dg} is a (2+1)-point estimator in (T, g)

Can think of the lensing signal formally as a contribution to the 3-point or 4-point function, but lens reconstruction is more intuitive



NVSS: NRAO VLA Sky Survey

1.4 GHz source catalog,
50% complete at 2.5 mJy



galaxy counts (masked)

Mostly extragalactic sources:

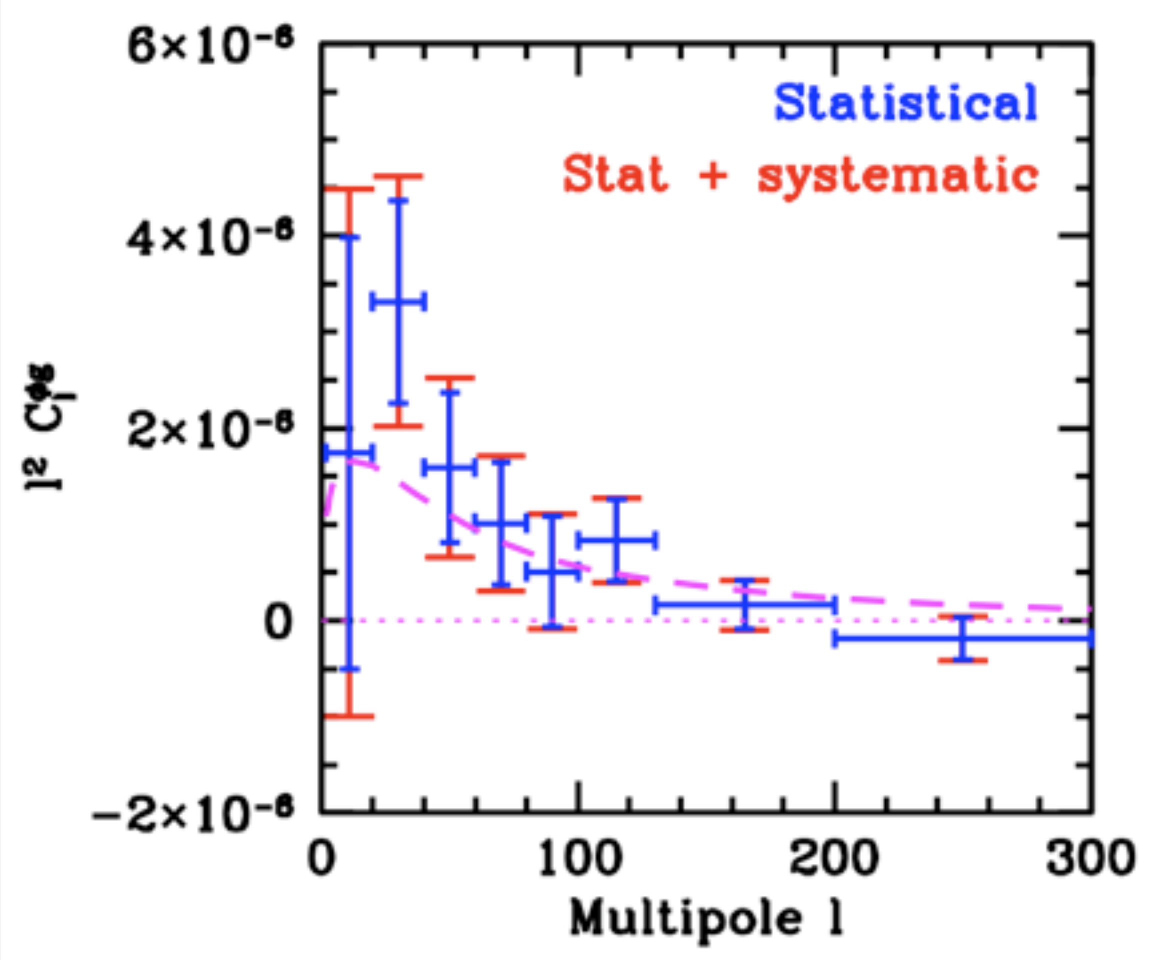
AGN-powered radio galaxies
Quasars
Star-forming galaxies

Well-suited for cross-correlating to WMAP lens reconstruction:

Nearly full sky coverage $(f_{\rm sky} = 0.8)$
Low shot noise $(b_g = 2,\ N_{\rm gal} = 1.8 \times 10^6)$
High redshift $(z_{\rm median} = 2)$

WMAP-NVSS analysis

First detection (3.4σ) of CMB lensing, via 3-point signal (C_l^{dg})

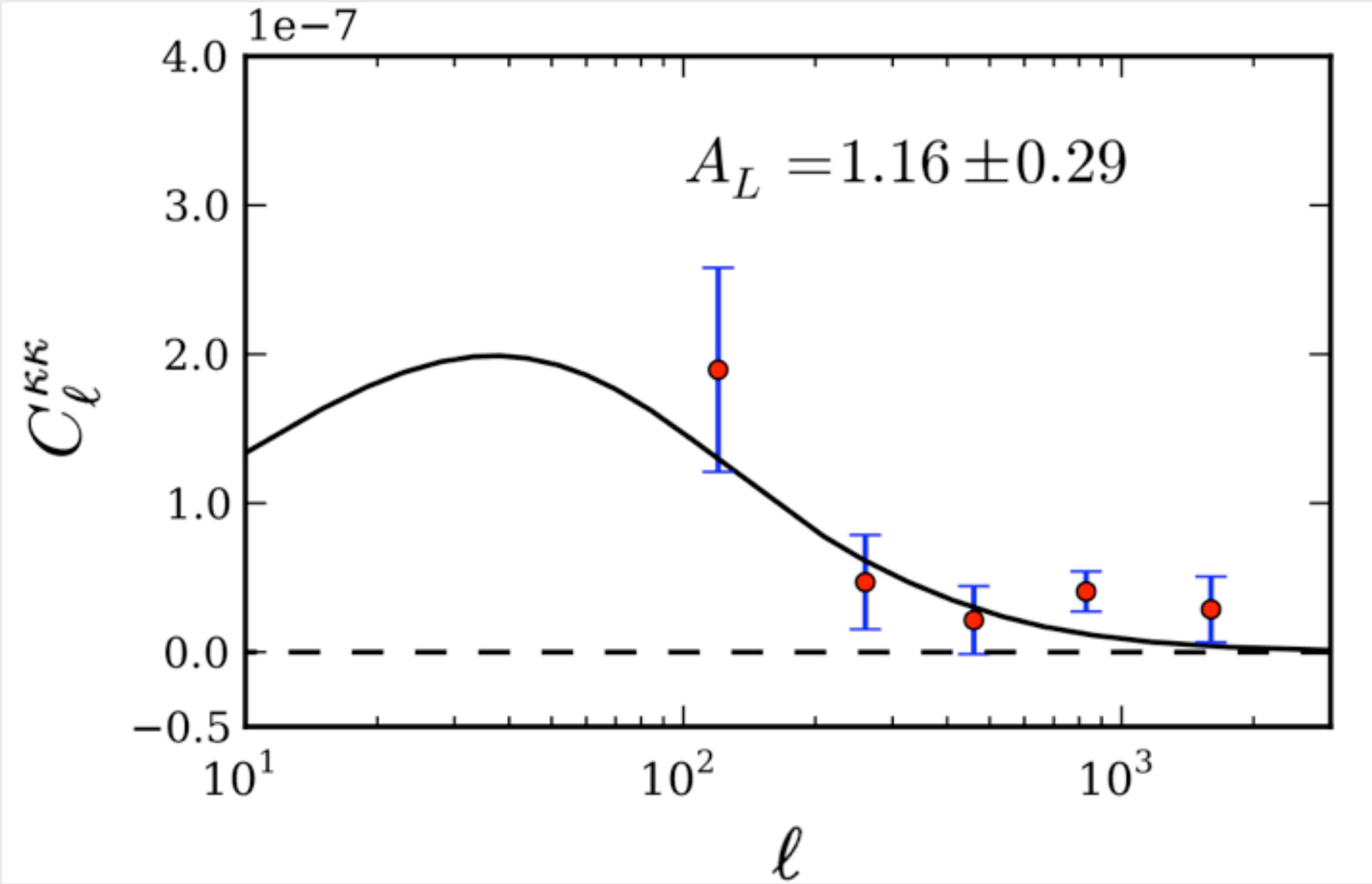


Smith, Zahn, Dore & Nolta 2007
(see also Hirata et al 2008)

		Beam			Galactic			Point source + SZ			
$(\ell_{\min}, \ell_{\max})$	Statistical	Asymmetry	Uncertainty	Total	Dust	Free-free	Total	Unresolved	Resolved	Total	Stat + systematic
(2, 20)	17.4 ± 22.4	± 0.9	± 0.3	± 1.2	± 0.4	± 1.4	± 3.6	± 10.9	± 0.5	± 11.4	17.4 ± 27.4
(20, 40)	33.2 ± 10.5	± 0.2	± 0.1	± 0.3	± 0.2	± 0.5	± 1.4	± 4.9	± 1.0	± 5.9	33.2 ± 13.0
(40, 60)	15.9 ± 7.8	± 0.1	± 0.1	± 0.2	± 0.2	± 0.3	± 1.0	± 2.8	± 1.5	± 4.3	15.9 ± 9.3
(60, 80)	10.1 ± 6.3	± 0.1	± 0.1	± 0.2	± 0.1	± 0.3	± 0.8	± 2.0	± 0.3	± 2.3	10.1 ± 7.0
(80, 100)	5.1 ± 5.8	± 0.1	± 0.1	± 0.2	± 0.1	± 0.3	± 0.8	± 1.1	± 0.2	± 1.3	5.1 ± 6.0
(100, 130)	8.3 ± 4.3	± 0.1	< 0.1	± 0.2	± 0.1	± 0.2	± 0.6	± 0.6	± 0.2	± 0.8	8.3 ± 4.4
(130, 200)	1.6 ± 2.5	< 0.1	< 0.1	± 0.1	± 0.1	± 0.1	± 0.4	± 0.3	± 0.1	± 0.4	1.6 ± 2.6
(200, 300)	-1.9 ± 2.2	< 0.1	< 0.1	± 0.1	± 0.1	± 0.1	± 0.4	± 0.3	± 0.1	± 0.4	-1.9 ± 2.3

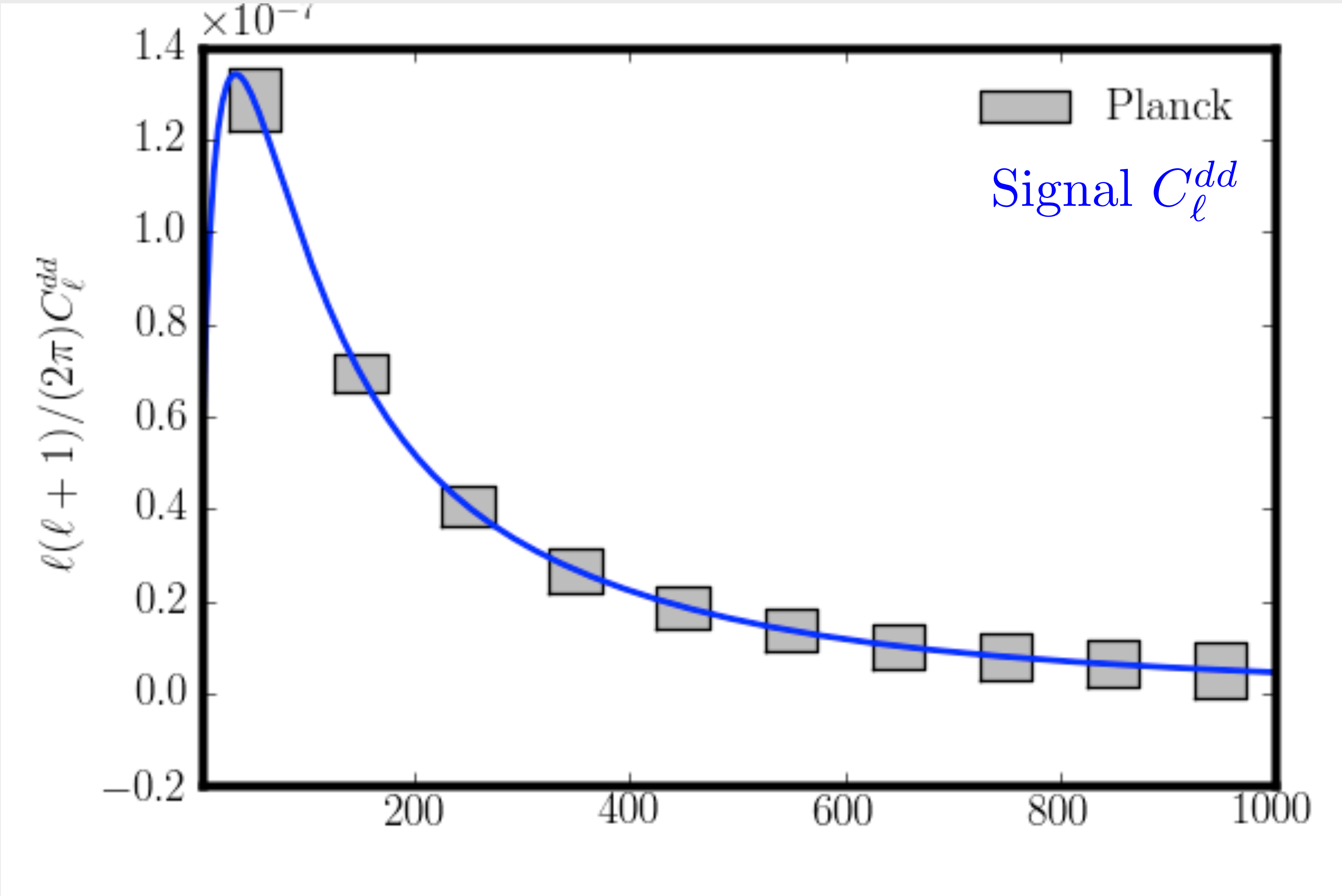
ACT deflection power spectrum

First clear detection (4σ) of 4-point lensing signal (C_ℓ^{dd})



Das et al 2011

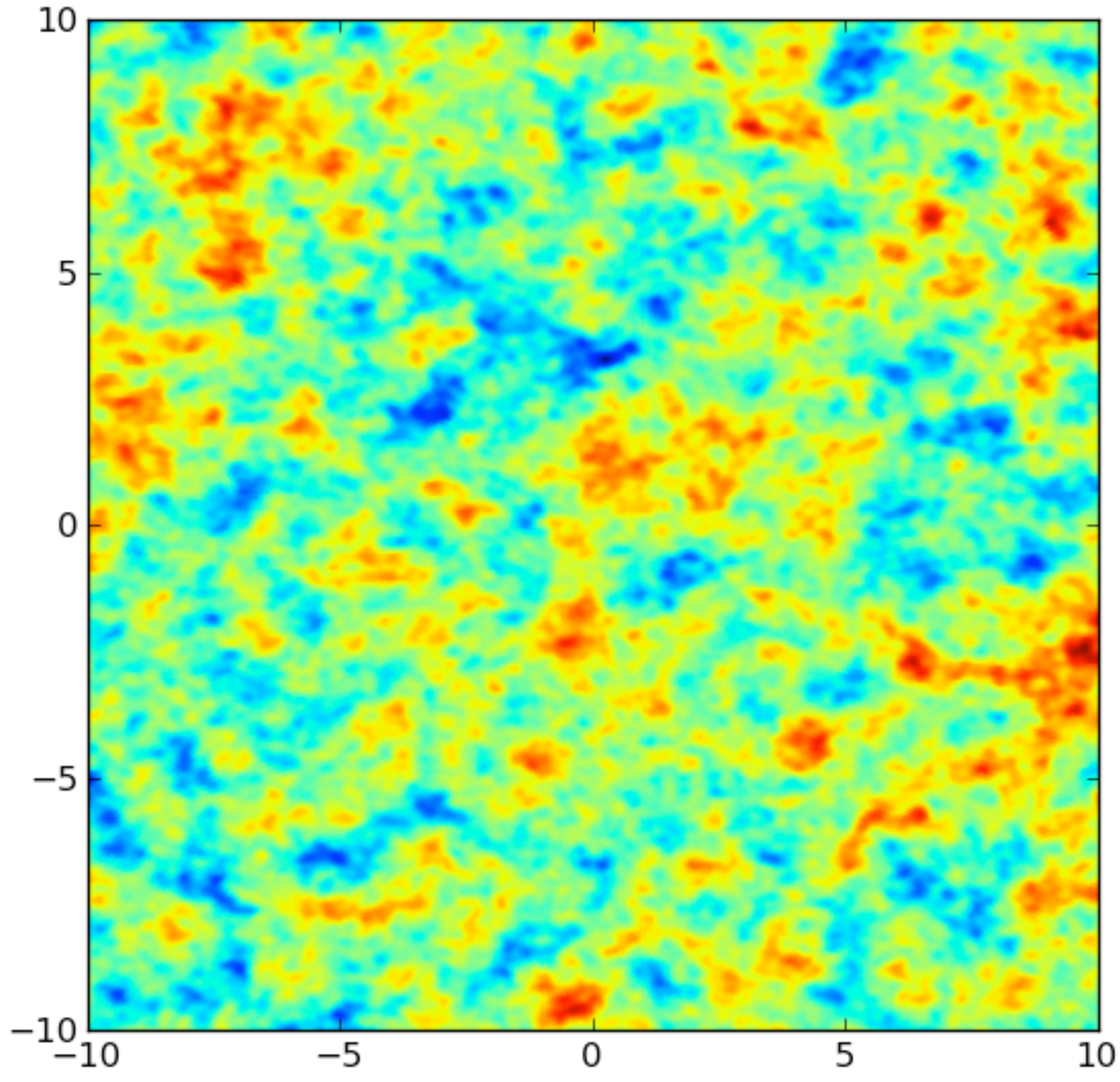
Planck forecast



Cumulative detection significance = 27 sigma!

We are entering the era of precision measurements of CMB lensing
High-resolution CMB experiments "contain" lensing experiments

Unlensed vs lensed CMB



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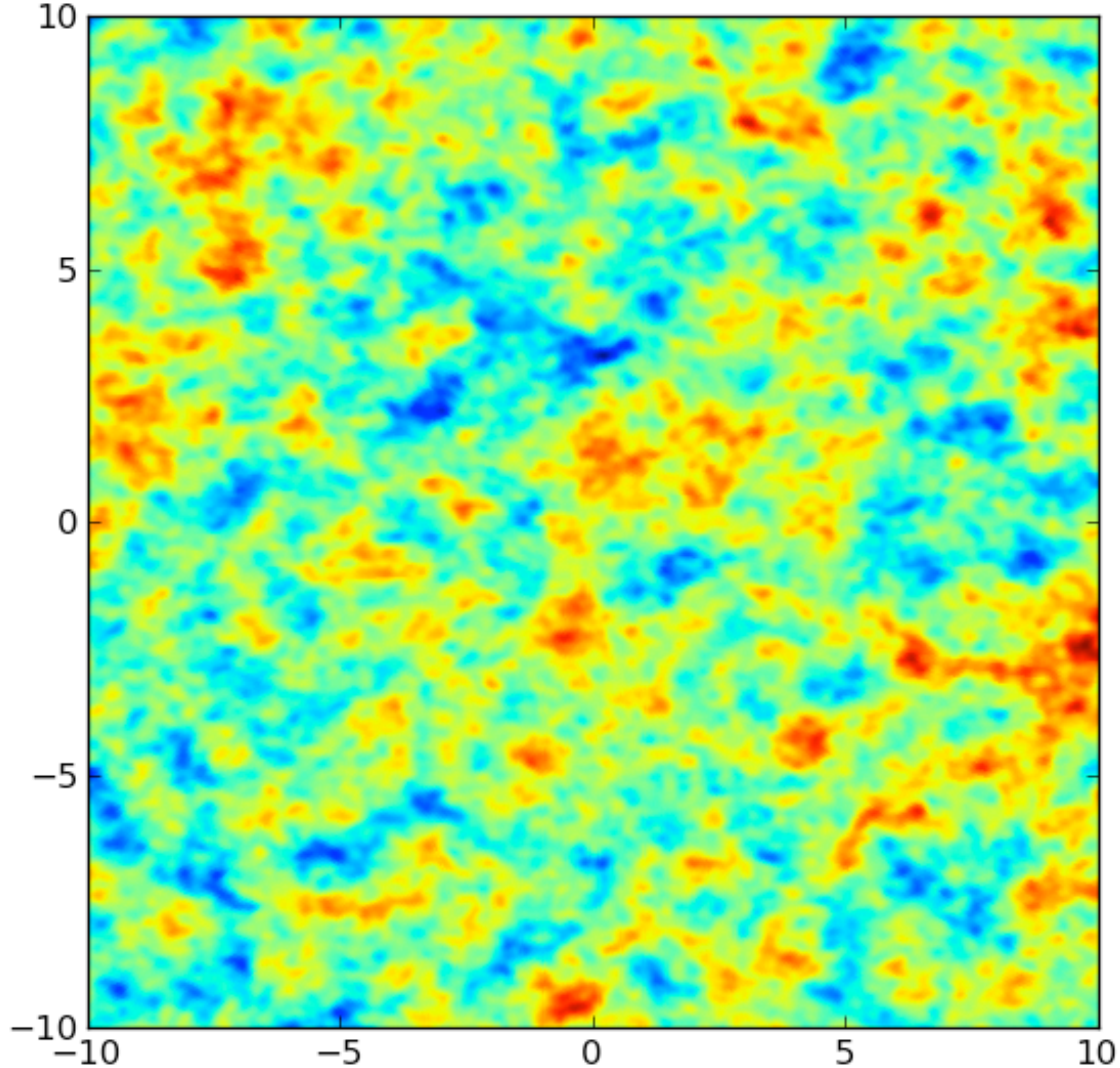
Unlensed vs lensed CMB



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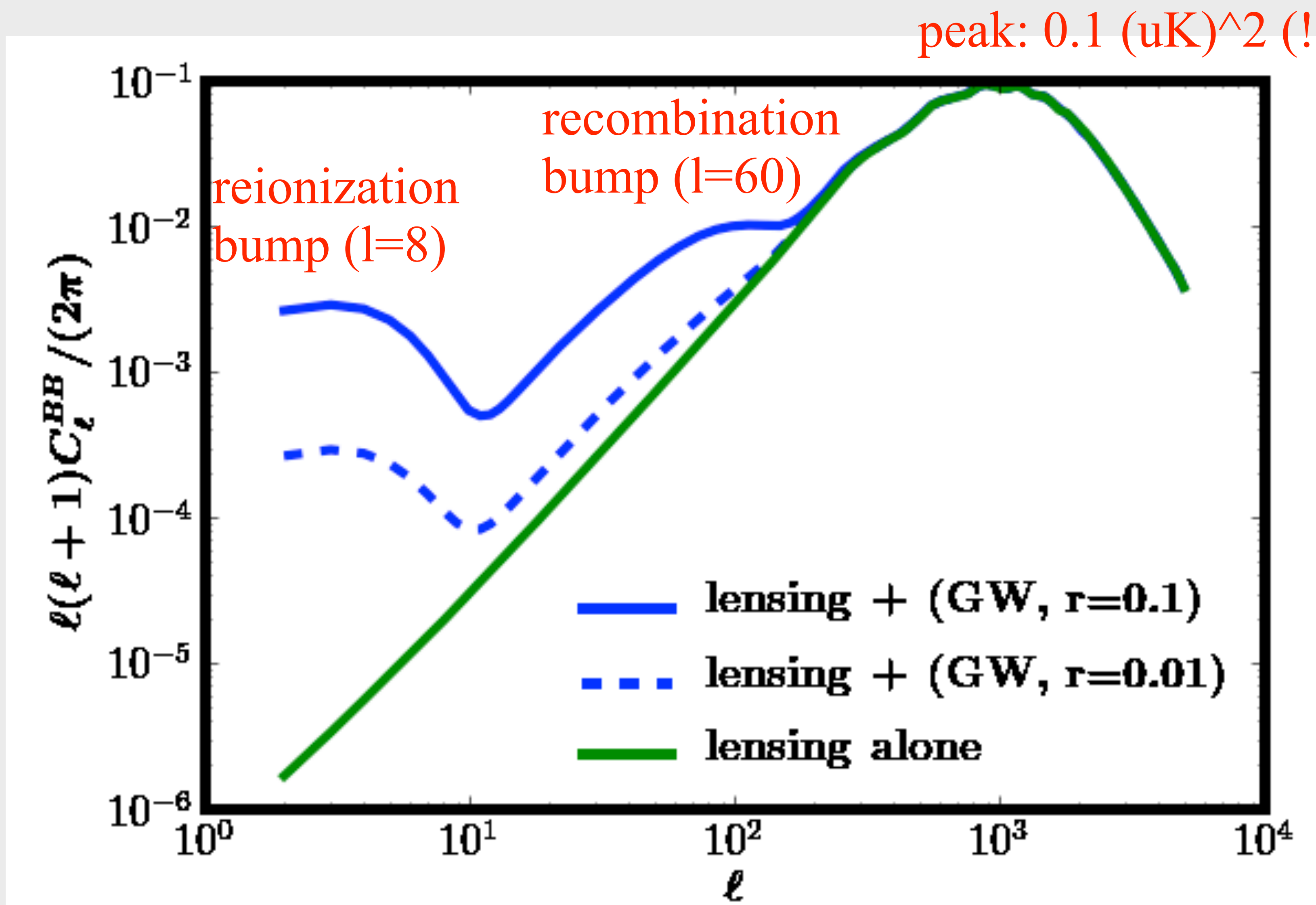
B-mode power spectrum

General symmetry argument implies: B-modes are only generated by non-scalar sources (e.g. GW background from inflation) OR nonlinear evolution

Gravitational lensing (nonlinear effect) is largest guaranteed B-mode
Lensing converts primary E-mode to mixture of E and B



B-modes as probe of inflation

Qualitative distinction between models with detectable r and undetectably small r.

E.g. in single-field inflation with standard kinetic term

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right)$$

models with detectable gravity waves are models in which:

Energy scale of inflation is GUT-scale:

$$\rho^{1/4} = (3.35 \times 10^{16} \text{ GeV})\, r^{1/4}$$

Change in inflaton field per e-folding is Planck scale:

$$d\phi/d(\log a) = (0.354\, M_{\rm Pl})\, r^{1/2}$$

B-mode power spectrum: low l

Lensing looks like white noise with $(\Delta_P)_{\rm lensing} = 5\ \mu$K-arcmin

Combines with instrumental noise:

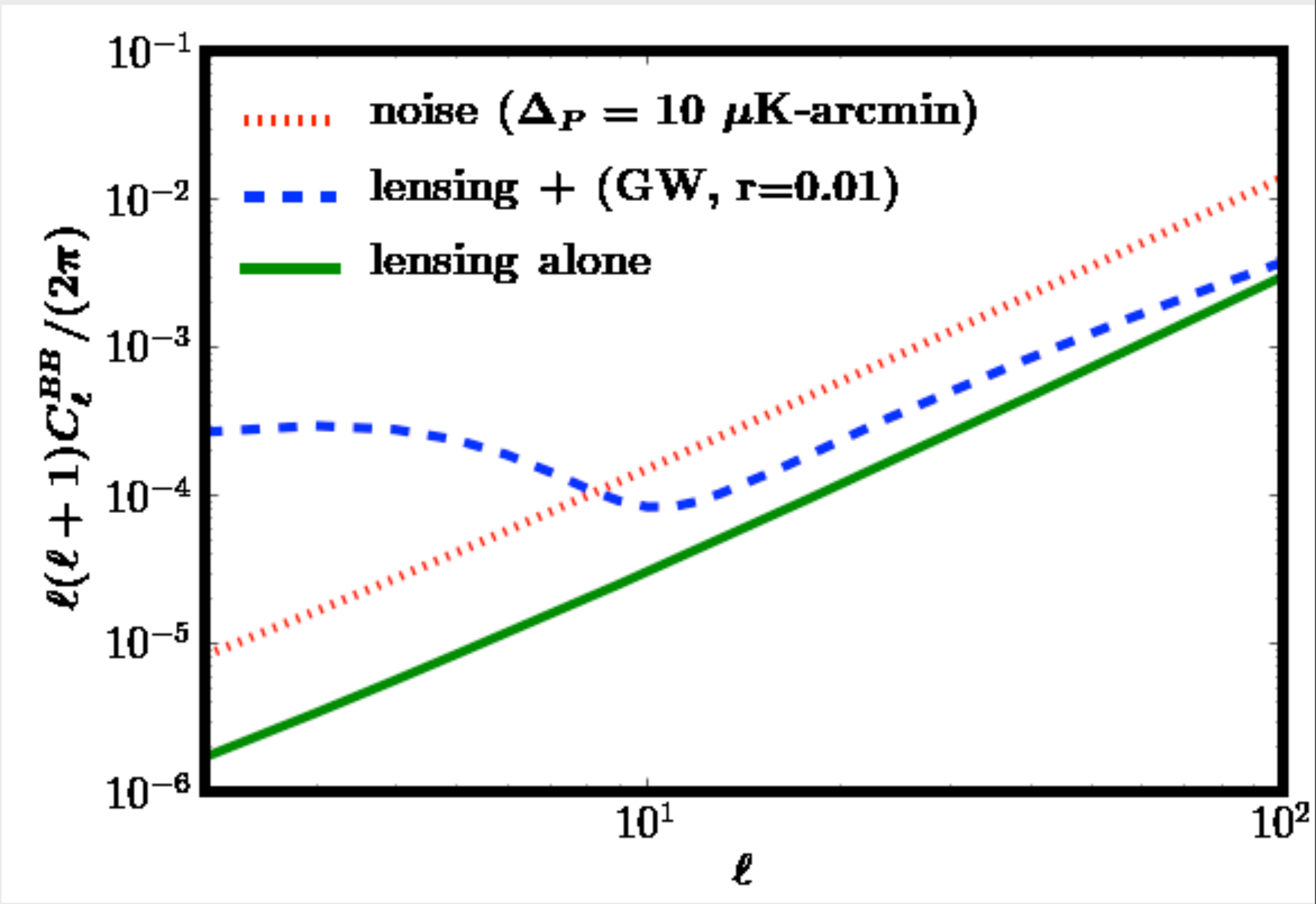
$$(\Delta_P)_{\rm eff} = \left[(\Delta_P)^2_{\rm lensing} + (\Delta_P)^2_{\rm instr}\right]^{1/2}$$

$(\Delta_P)_{\rm instr} \gtrsim 5\ \mu$K-arcmin

$\Rightarrow$ gravity wave measurement is noise-limited

$(\Delta_P)_{\rm instr} \lesssim 5\ \mu$K-arcmin

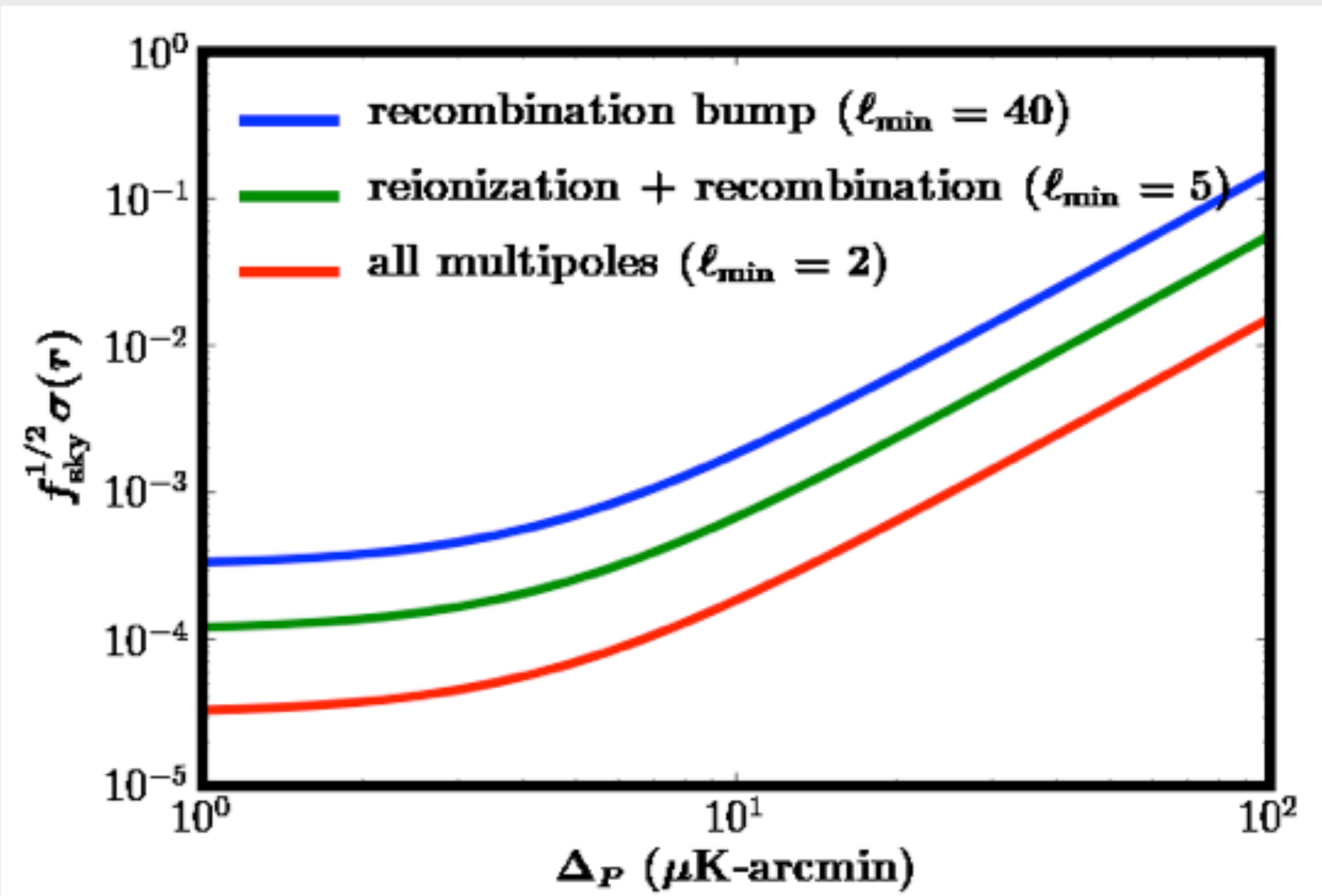
$\Rightarrow$ gravity wave measurement is lensing-limited



Foregrounds and "r"

Forecasts on r are very sensitive to assumptions about foregrounds!
e.g. consider simple mode-counting forecast,

$$\sigma(r) = \left[\frac{f_{\rm sky}}{2} \sum_\ell (2\ell+1) \left(\frac{\partial C_\ell^{BB}/\partial r}{C_\ell^{BB} + N_\ell^{BB}} \right)^2 \right]^{-1/2}$$



Reionization bump has 10 times more S/N than the recombination bump

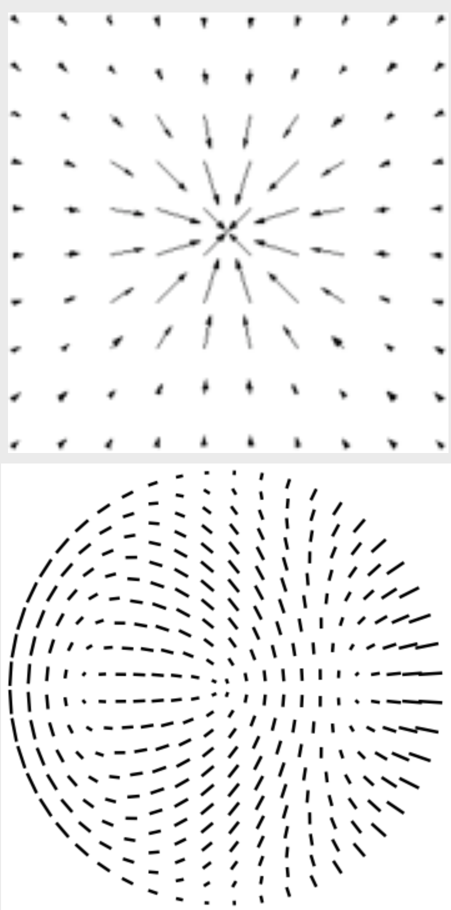
B-mode quadrupole has same S/N as all $\ell \geq 3$ modes combined

We will avoid quoting values for $\sigma(r)$, will instead quote foreground-independent quantities (e.g. ratio between two values of $\sigma(r)$ with same foreground assumptions)

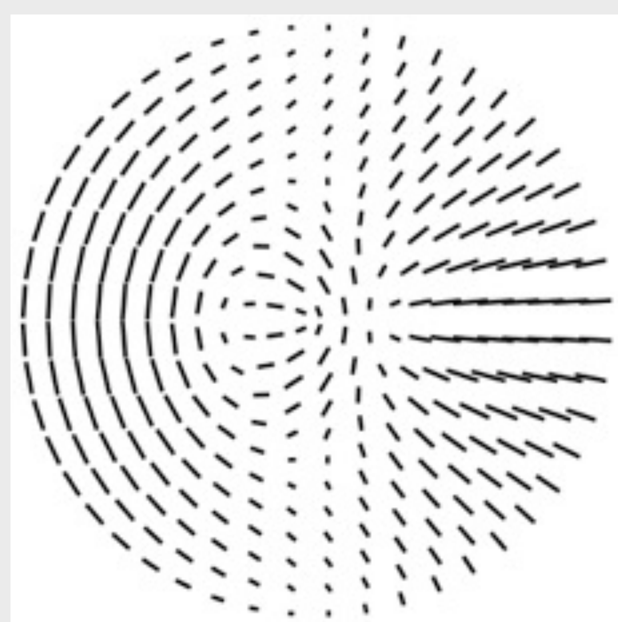
Lens reconstruction: polarization

Quadratic estimators are defined for all pairs (TT, TE, TB, EE, EB) but EB estimator dominates in low-noise limit:

$$\hat{\mathbf{d}}(\mathbf{l}) \propto \int \frac{d^2\mathbf{l}'}{(2\pi)^2} i\mathbf{l}' \sin[2(\varphi_{l'} - \varphi_{l-l'})] \frac{E(\mathbf{l}')B(\mathbf{l}-\mathbf{l}')}{(C^{EE}_{\ell'} + N^{EE}_{\ell'})(C^{BB}_{l-l'} + N^{BB}_{l-l'})}$$

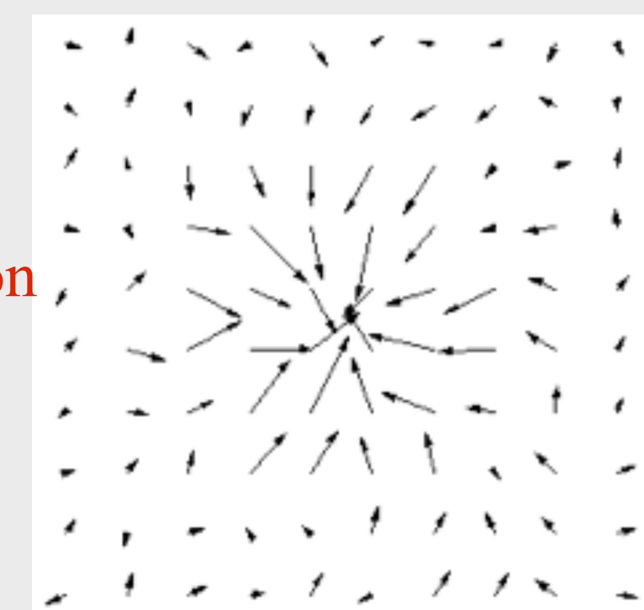


deflection + unlensed polarization (unobservable)



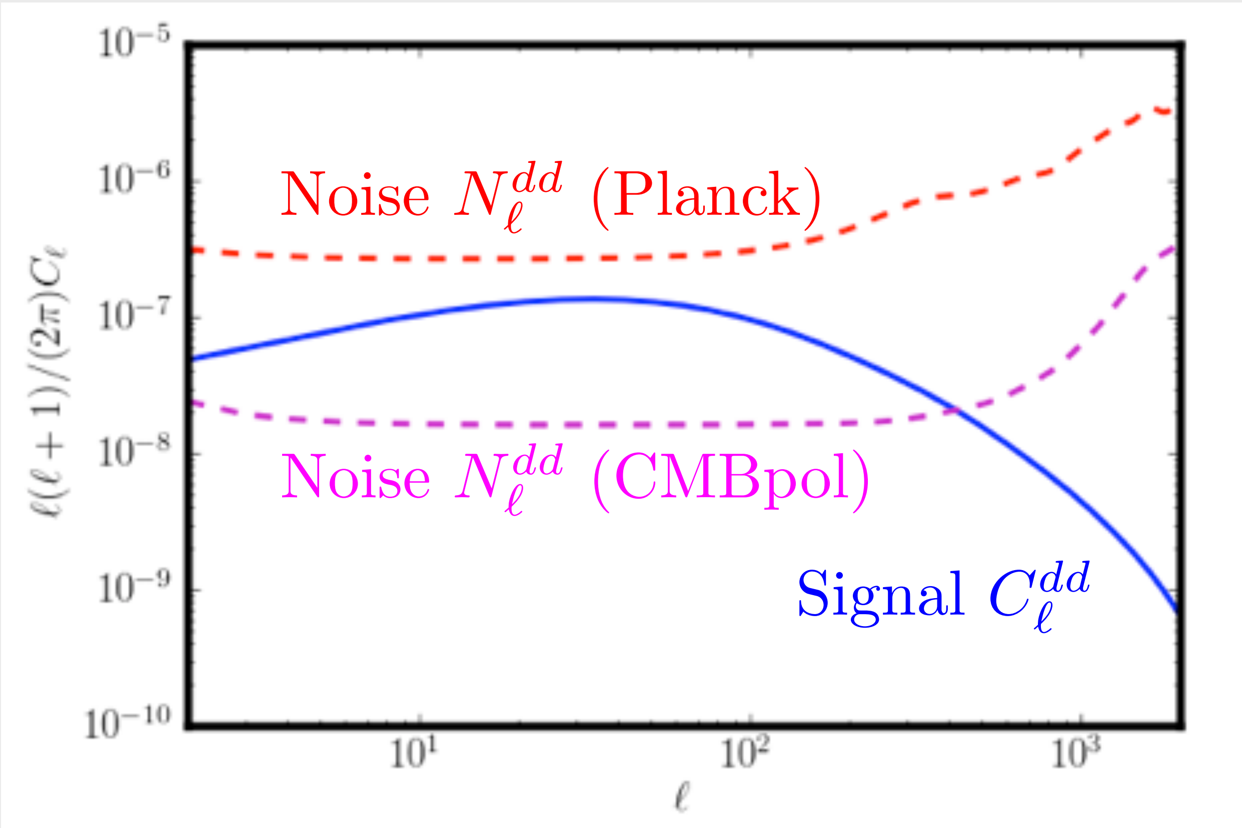
lensed polarization (observed)

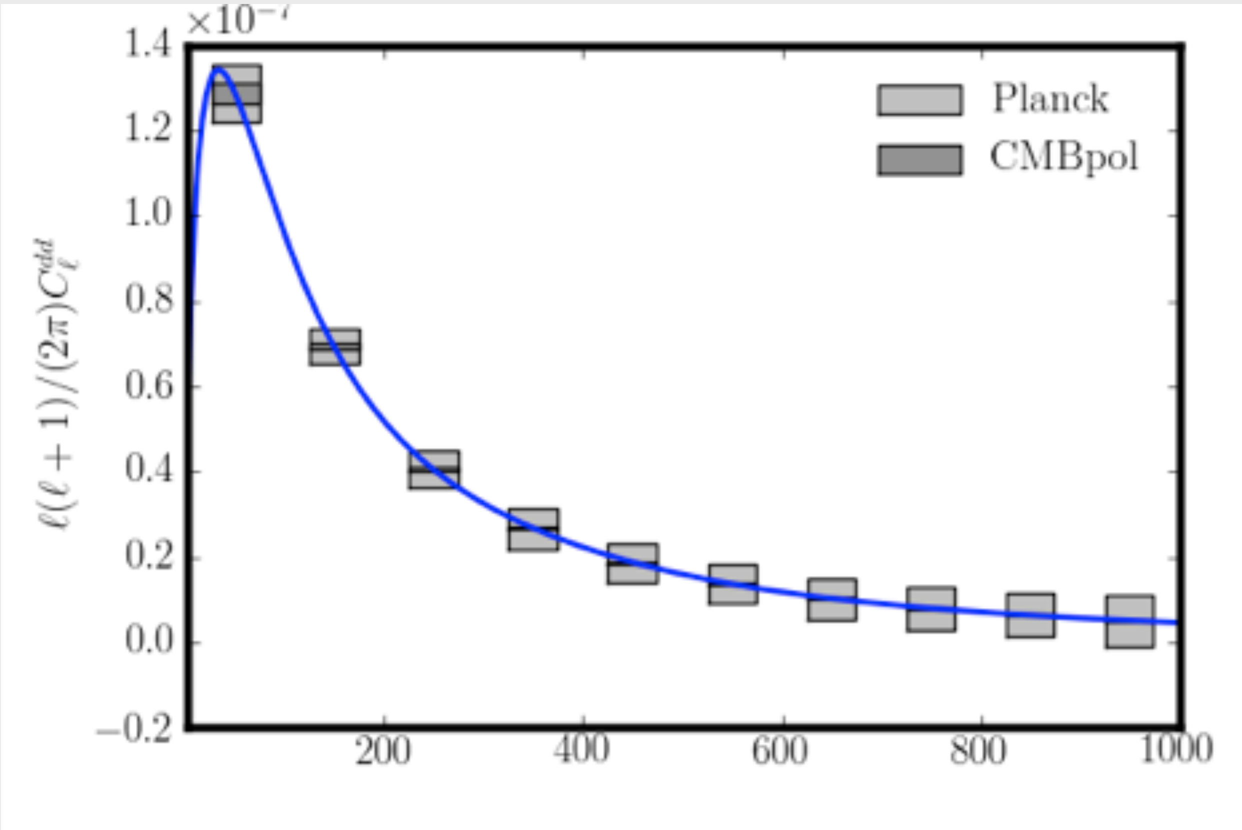
lens reconstruction



lens + noise (statistical reconstruction)

Lens reconstruction: polarization



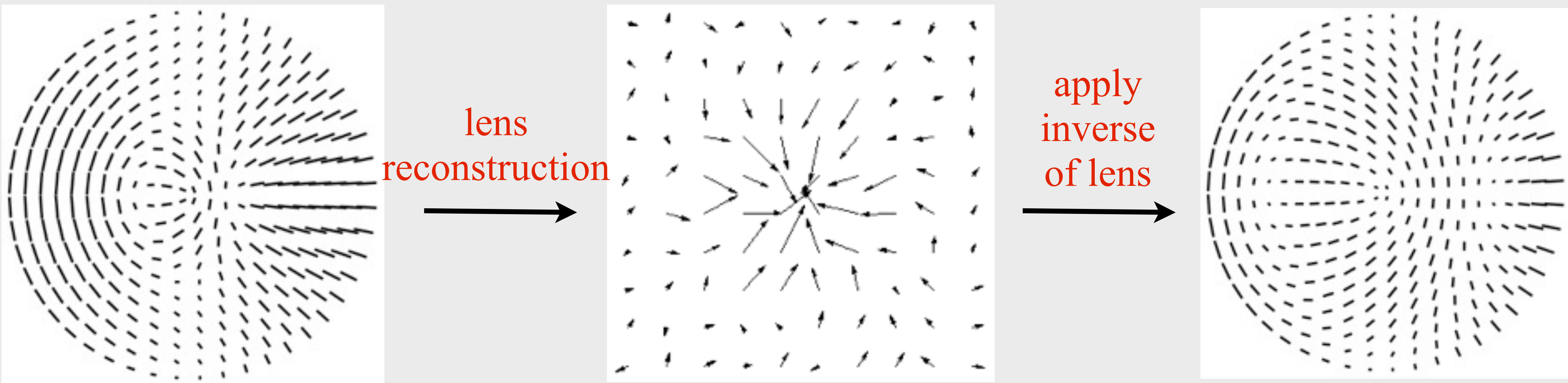


CMB polarization can ultimately provide a much more sensitive probe of lensing than temperature, especially on small angular scales

Increased statistical power since B-mode is all lensing

Delensing: idea

Estimate unlensed CMB, by combining observed (lensed) CMB with statistical reconstruction of lens



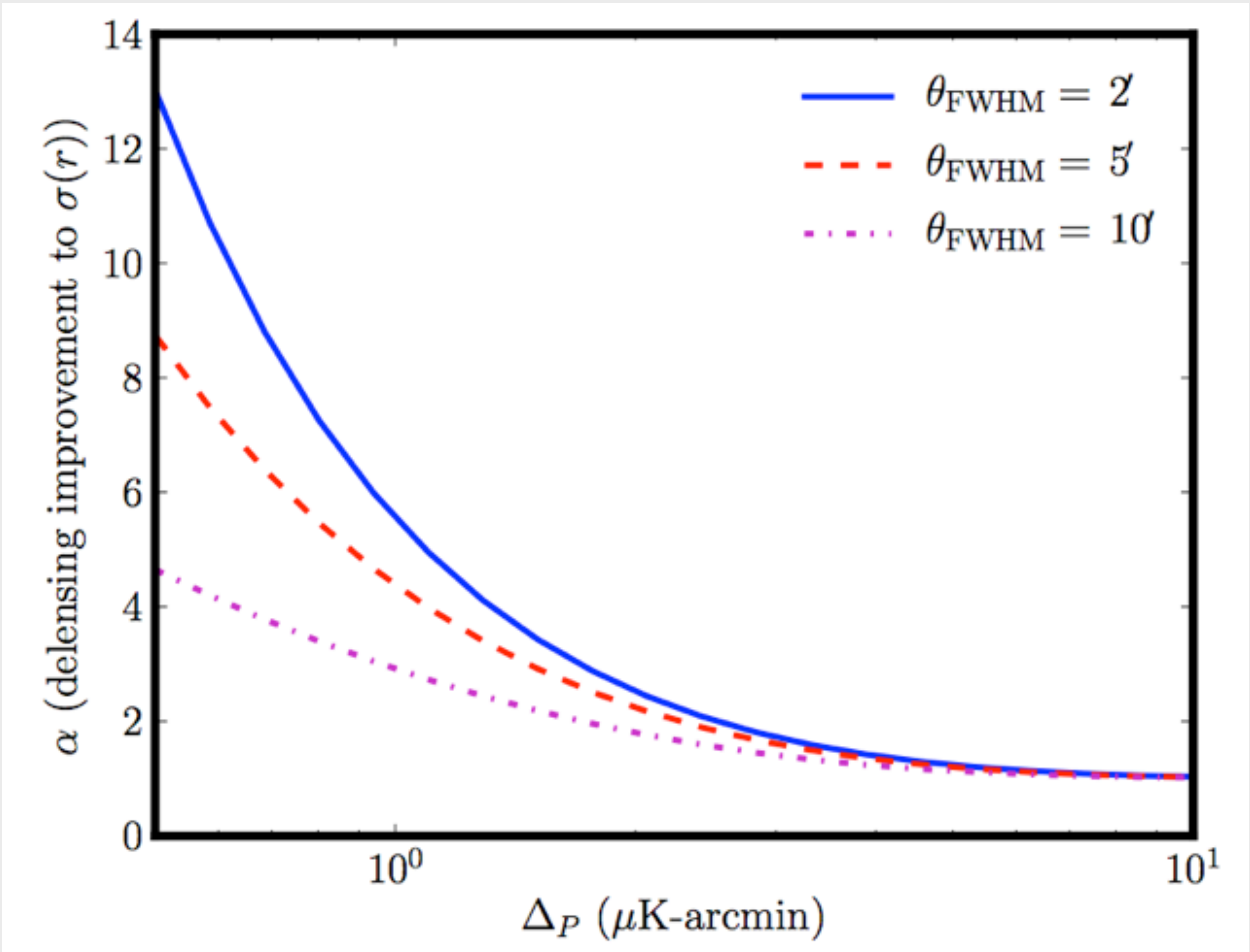
lensed polarization lens + noise "delensed" polarization

Delensed CMB has smaller lensed B-mode than original lensed CMB => error on r is improved (if lensing-limited)

B-modes on small scales are used to "clean" the large scales

Delensing: improvement on r

For instrumental noise significantly better than 5 μK-arcmin, delensing with a few-arcmin beam allows one to beat the noise floor from lensing

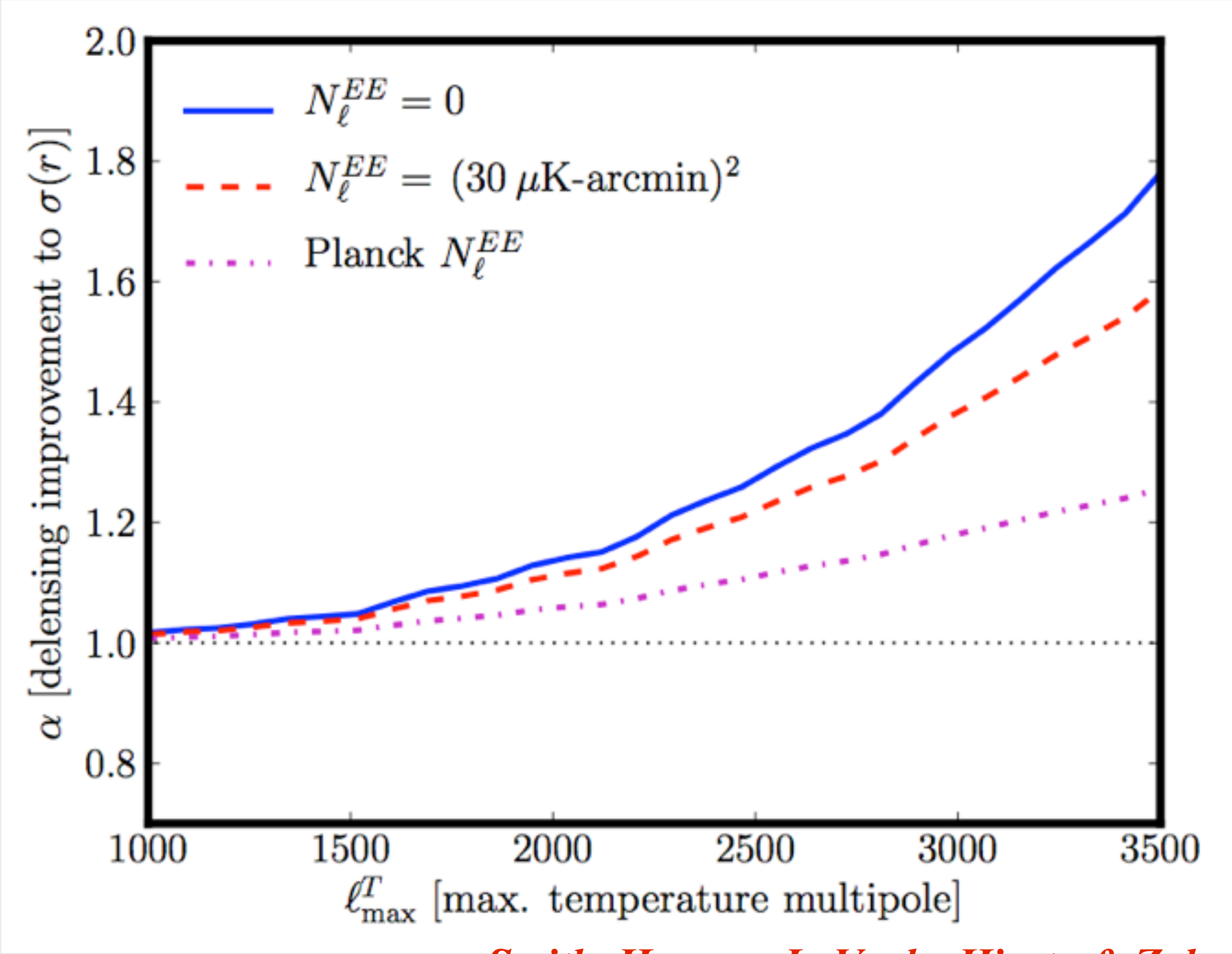


Smith, Hanson, LoVerde, Hirata & Zahn (2010)

Delensing using temperature?

No-go result: cannot use CMB temperature to delens polarization

CV-limited temperature ($\ell \leq \ell^T_{\rm max}$) + noisy E-mode measurement

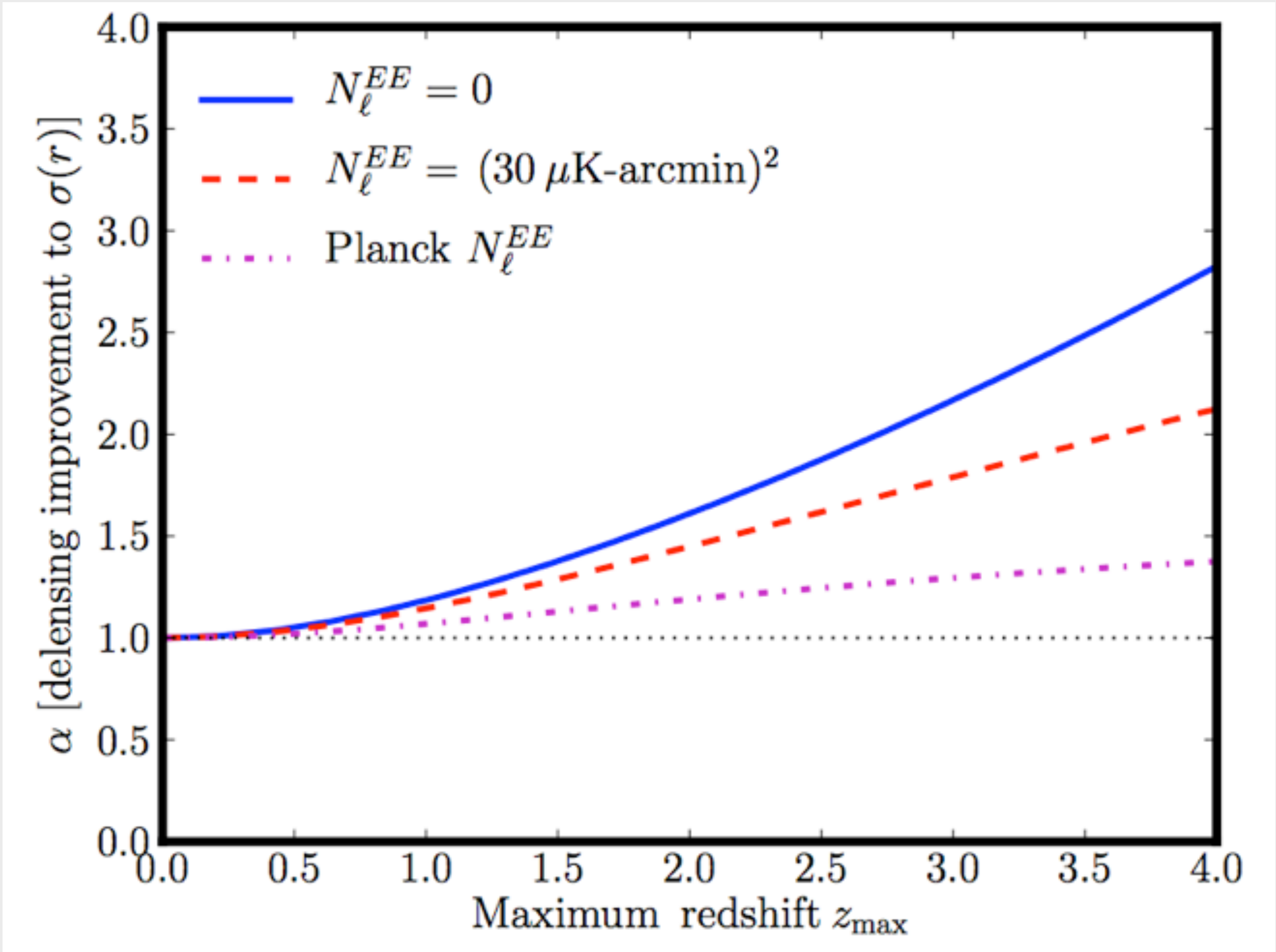


Smith, Hanson, LoVerde, Hirata & Zahn (2010)

Delensing with large-scale structure?

No-go result: cannot use large-scale structure to delens the CMB

Ideal LSS measurement $(z \leq z_{\rm max})$ + noisy E-mode measurement

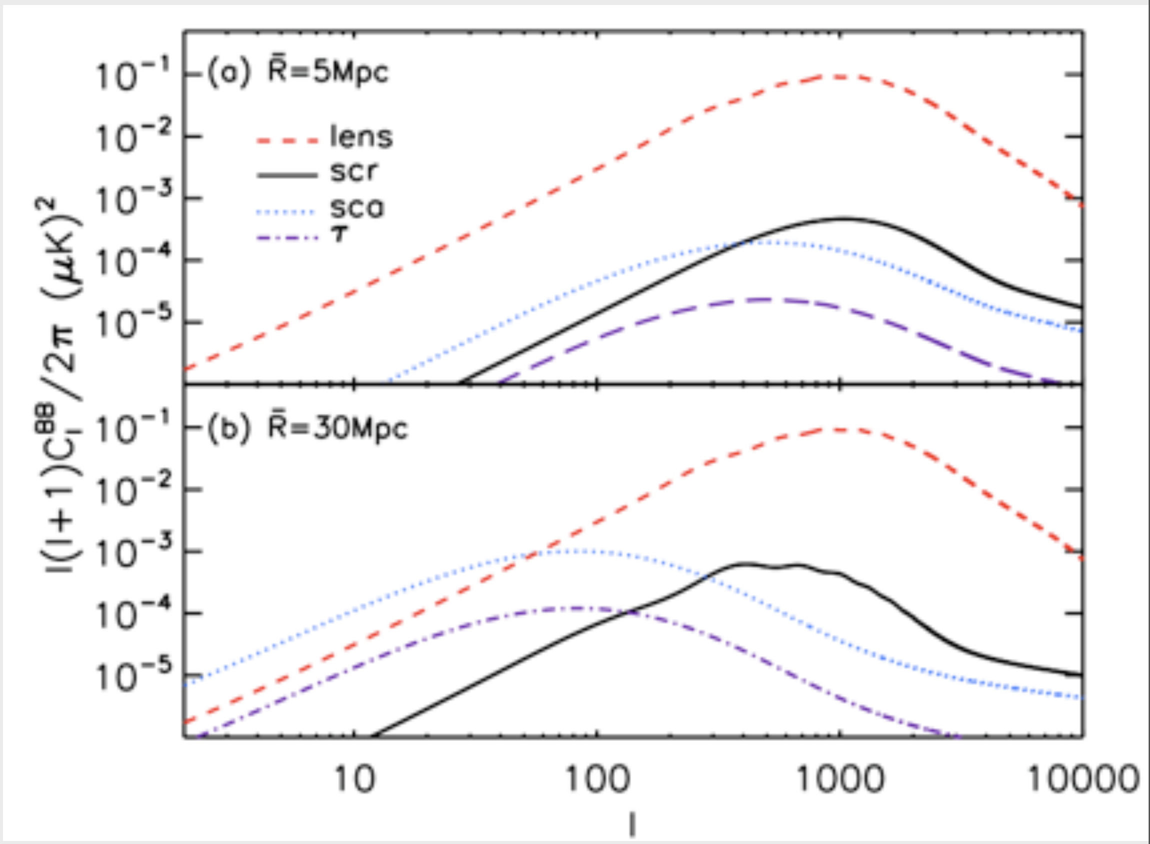


Smith, Hanson, LoVerde, Hirata & Zahn (2010)

B-modes from patchy reionization

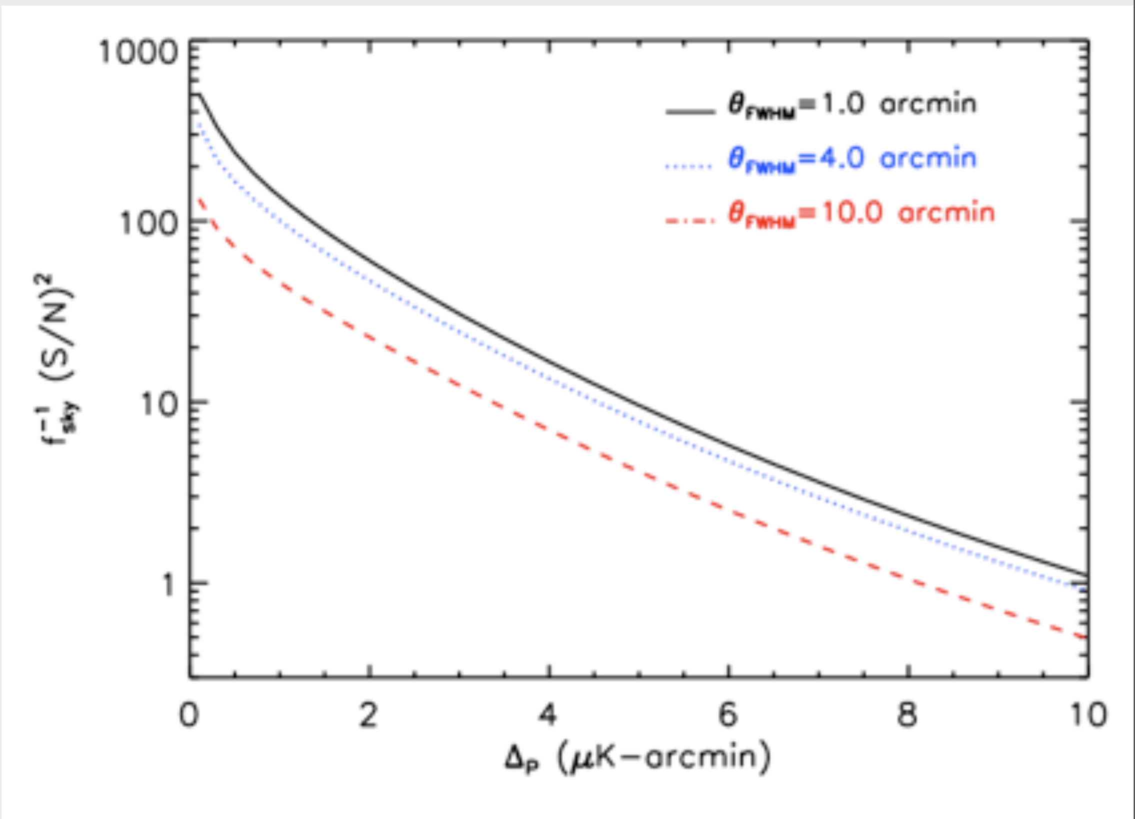
Reionization bubbles generate B-modes

via scattering (dominates at low l)
via screening (dominates at high l)



Dvorkin, Hu & Smith 0902.4413

Can construct quadratic estimator to reconstruct bubbles (analogous to lens reconstruction, with deflection field $\mathbf{d}(\mathbf{n})$ replaced by optical depth anisotropy $\Delta\tau(\mathbf{n})$)



Dvorkin & Smith, 0812.1566

1. CMB lensing: general picture

2. Non-Gaussian statistics

3. B-modes

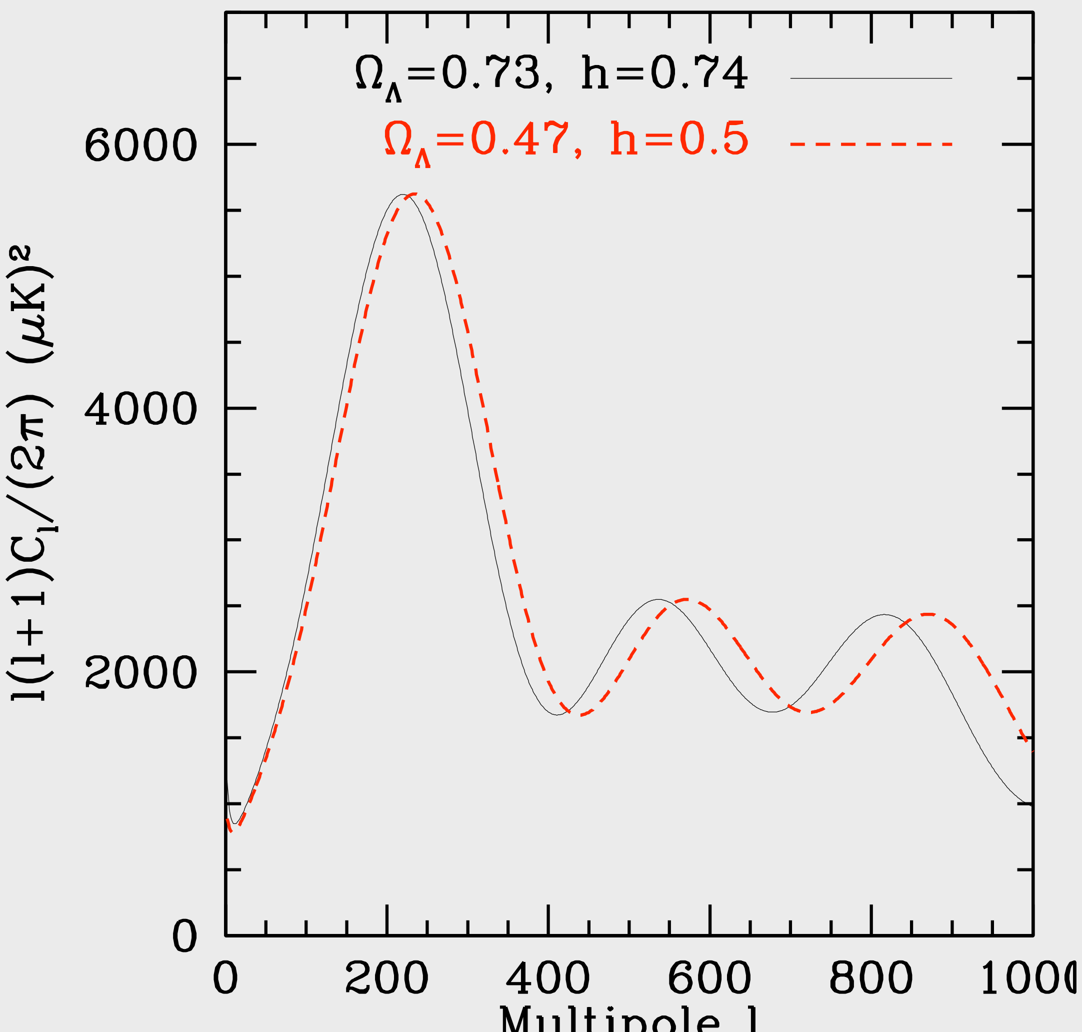
4. Cosmological information from lensing

Unlensed CMB: distance degeneracy

Consider the WMAP six-parameter space $\{\Omega_b h^2, \Omega_m h^2, A_s, \tau, n_s, \Omega_\Lambda\}$
First 5 parameters are well-constrained through power spectrum shape
Constraint on Ω_Λ comes entirely through angular peak scale:

$$\ell_a = \pi \frac{D_*}{s_*}$$

D_* ⟵ Angular diameter distance to last scattering
s_* ⟵ Distance sound travels before last scattering



Suppose that N "late universe" parameters are added (e.g. Ω_K, m_ν, w)

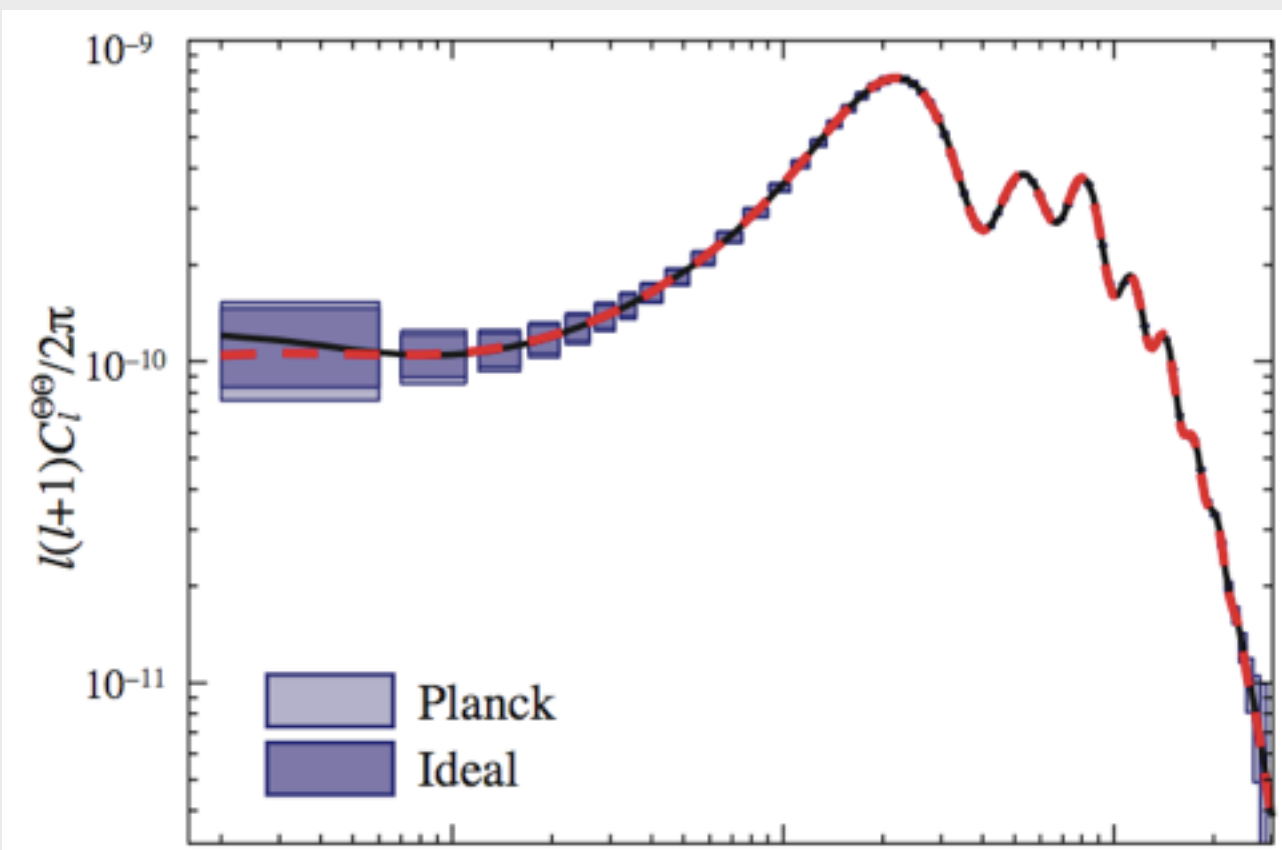
Then only one combination (corresponding to D_*) is constrained

Generates N-fold angular diameter distance degeneracy in parameter space

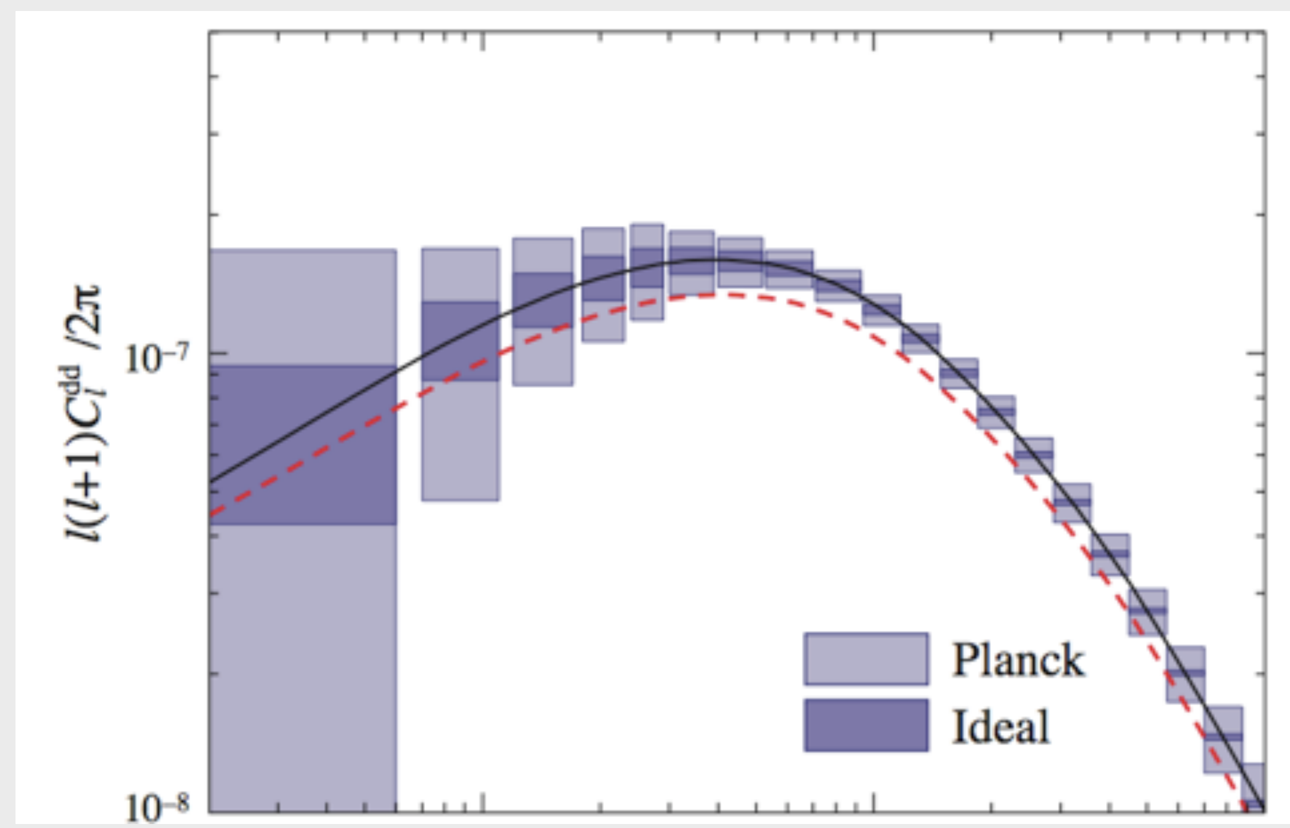
Lensing breaks distance degeneracy

Example from Hu 2001: models with $w = -1$ and $w = -2/3$

Ω_Λ chosen so that models have same D_*



unlensed C_l^{TT}



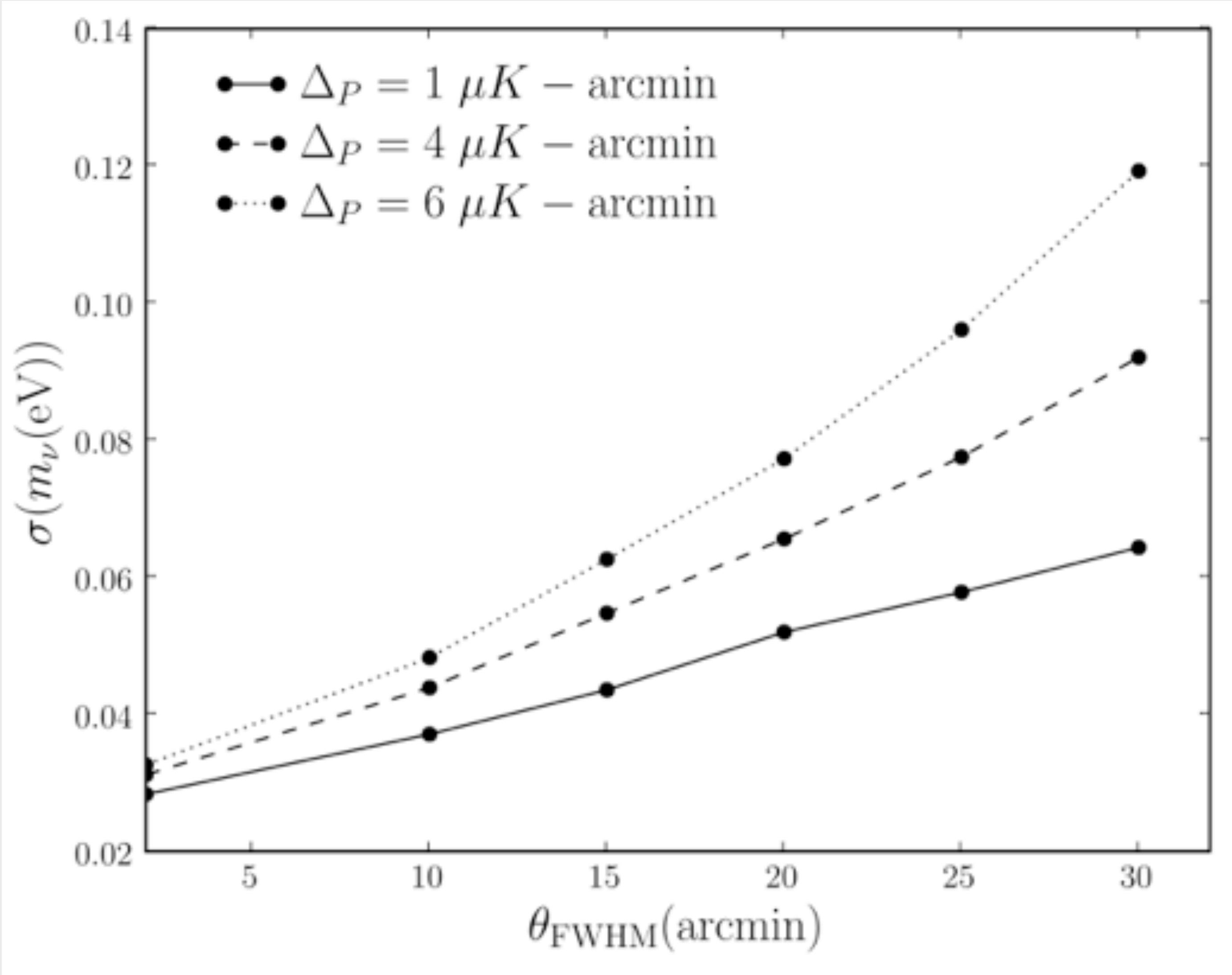
deflection power spectrum C_l^{dd}

Neutrino mass

Neutrino oscillation experiments measure Δm^2_ν between species

Current analysis of world data: $$\Delta m^2_{31} = (0.049 \pm 0.0012 \text{ eV})^2$$ $$\Delta m^2_{21} = (0.0087 \pm 0.00013 \text{ eV})^2$$

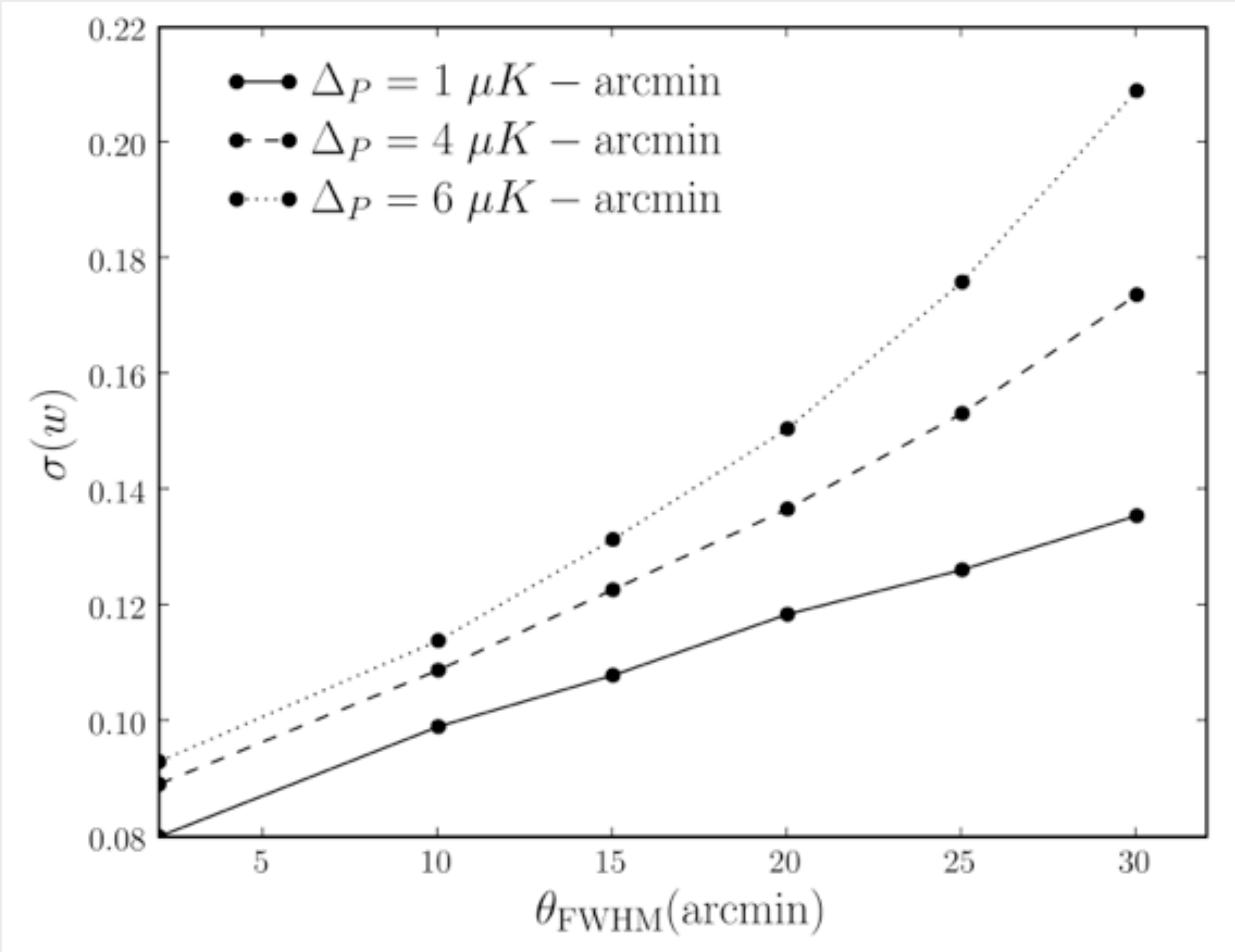
Cosmology is complementary: lensing is mainly sensitive to $\sum_\nu m_\nu$



Smith et al (2008)

Dark energy

In many parameterizations (e.g. w=constant), CMB lensing constrains dark energy weakly because redshift kernel (peaked at $z \approx 2$) is poorly matched to redshifts where dark energy is important ($z \lesssim 1$)



Smith et al (2008)

Early dark energy

Doran & Robbers parameterization (2006):

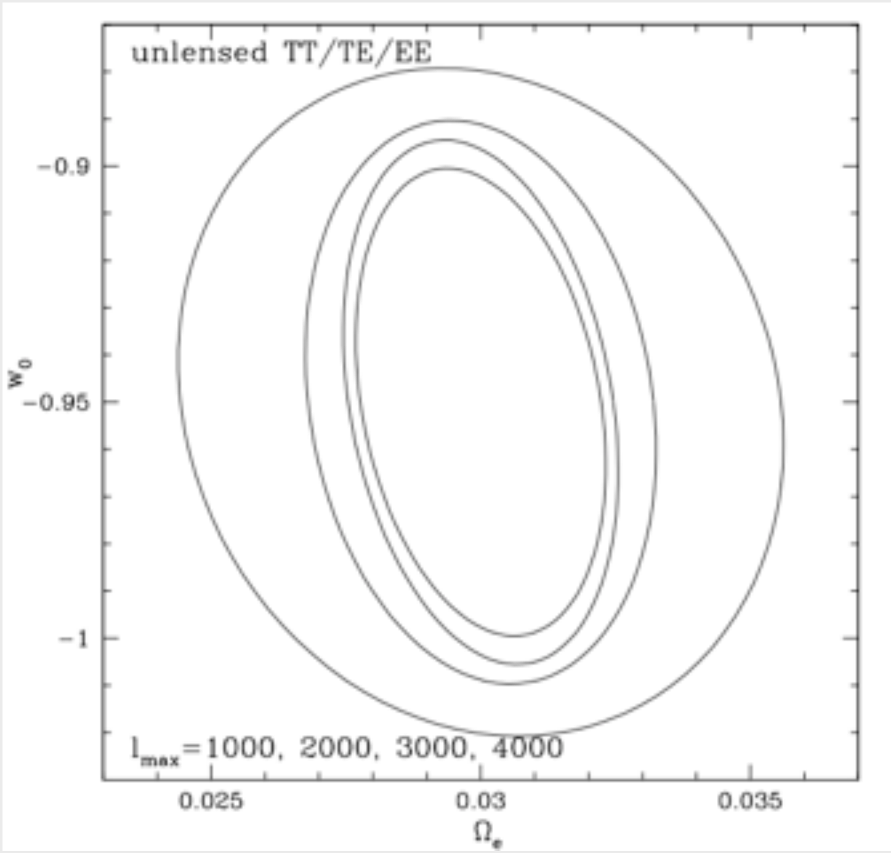
$$\Omega_\Lambda(a) = \frac{\Omega_\Lambda^0 - \Omega_\Lambda^e(1 - a^{-3w_0})}{\Omega_\Lambda^0 + (1 - \Omega_\Lambda^0)a^{3w_0}} + \Omega_\Lambda^e(1 - a^{-3w_0})$$

Tracker model:

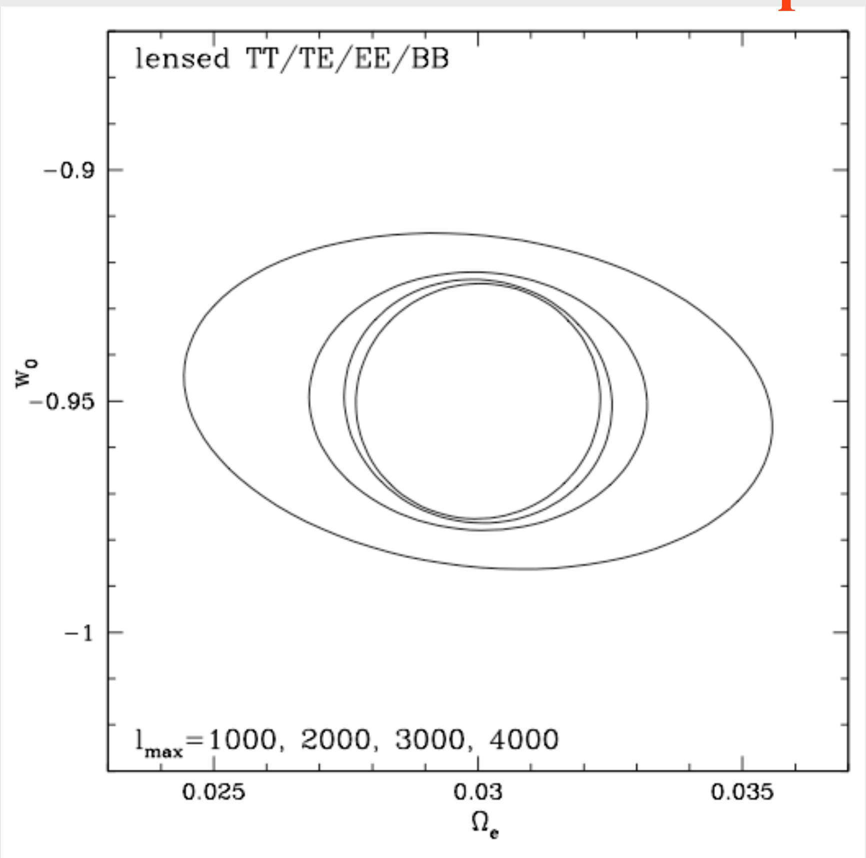
As $z \to 0$, $\Omega_\Lambda(z) \to \Omega_\Lambda^0$ and $w(z) \to w_0$

As $z \to \infty$, $\Omega_\Lambda(z) \to \Omega_\Lambda^e$ and $w(z) \to 0$

SNAP + unlensed CMBpol



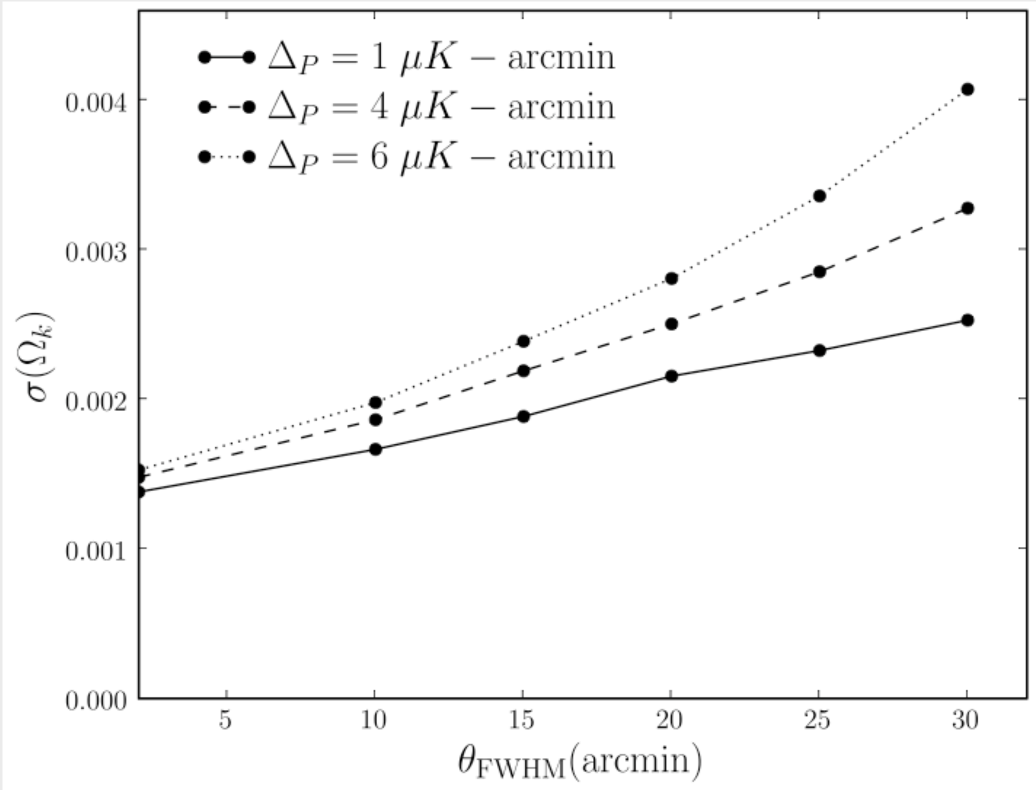
SNAP + lensed CMBpol



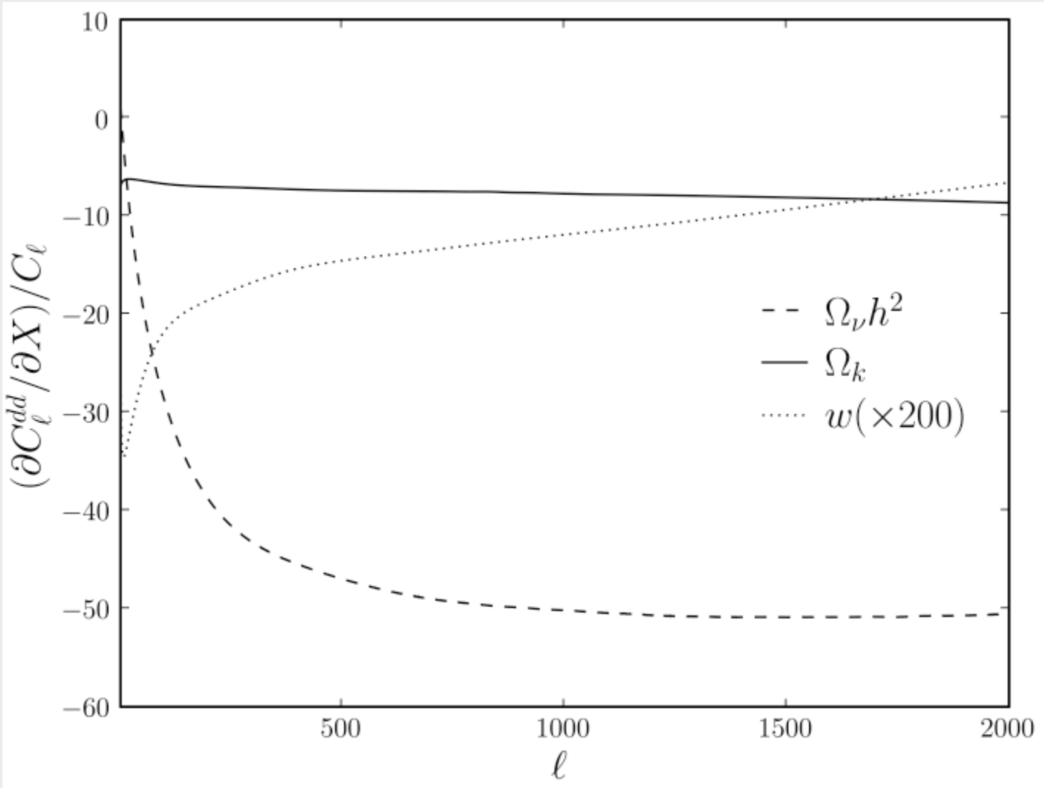
De Putter, Zahn & Linder (2009)

Curvature and joint constraints

Because full deflection power spectrum is measured, can constrain multiple "late universe" parameters simultaneously



$$
\begin{array}{c} \\ \sum_\nu m_\nu \\ w \\ \Omega_K \end{array}
\begin{array}{c} \begin{array}{ccc} \sum_\nu m_\nu & w & \Omega_K \end{array} \\
\left(\begin{array}{ccc} 1 & 0.34 & -0.82 \\ 0.34 & 1 & -0.63 \\ -0.82 & -0.63 & 1 \end{array}\right) \end{array}
$$



Smith et al (2008)

Summary

Gravitational lensing imprints characteristic non-Gaussian correlations on the CMB which can be extracted via higher-point statistics

Few-sigma detections of CMB lensing via several methods (C_l^{dg} three-point, C_l^{TT} two-point, C_l^{dd} four-point)

Polarization ultimately allows CV-limited lens reconstruction to $l \sim 1000$, lensing "noise floor" on r can be beaten via delensing

Lensing breaks distance degeneracy in unlensed CMB, maps gravitational potentials at high-z on largest observable scales