The Shape of the CMB Lensing Bispectrum

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Berkeley Lensing Workshop, April 22, 2011

“The shape of the CMB lensing bispectrum”
Lewis, Challinor, Hanson (2011) arXiv:1101.2234

“CMB lensing and primordial non-Gaussianity”
Hanson, Smith, Challinor, Liguori (2009) arXiv:0905.4732
OUTLINE

I INTRODUCTION
- Lensing Bispectra
- Overlap with $f_{\text{NL}}^{\text{local}}$

II BISPECTRUM GUIDE:
- Significance in the high S/N limit
- Effect of lensing on the shape of other bispectra (particularly $f_{\text{NL}}^{\text{local}}$)

III PROSPECTS:
- Current status, ultimate limits.
- Planck
The CMB Temperature is given by

\[ T(\hat{n}) = \Theta(\hat{n}) + \Theta^{ISW}(\hat{n}) + [\nabla \phi \cdot \nabla \Theta](\hat{n}) + \ldots, \]

where the ISW and lensing effects are

\[ \Theta^{ISW}(\hat{n}) = 2 \int_0^{\chi_*} d\chi \dot{\Psi}(\chi \hat{n}; \eta_0 - \chi) \]

\[ \phi(\hat{n}) = -2 \int_0^{\chi_*} d\chi \frac{\chi_* - \chi}{\chi_* \chi} \Psi(\chi \hat{n}; \eta_0 - \chi) \]

This leads to a non-Gaussian bispectrum:

\[ \langle TTT \rangle = \left\langle \Theta^{ISW}(\nabla \phi \cdot \nabla \Theta) \Theta \right\rangle \]
THE E-$\phi$ CORRELATION

There are also lensing bispectra in polarization. No longer from cross-correlation with ISW, but from overlap between potentials which source $\phi$ and quadrupoles which source reionization E-modes (Lewis, Challinor, Hanson 2011). Recently implemented in CAMB.
THE LENSING BISPECTRUM

- The T-φ and E-φ correlations are significant—\( O(30\%) \) on large scales, but fade quickly.
- The φ-induced T and E covariances are mostly at high-\( l \), while the cross-correlation is at low-\( l \), so get a squeezed shape.

\[
\begin{align*}
\tau = 0.09 & \quad \tau = 0
\end{align*}
\]

Lewis, Challinor, Hanson (2011)
**USE OF LENSING-ISW**

- The lensing-ISW bispectrum is a (relatively) direct probe of dark energy (Seljak and Zaldarriaga 1998, Goldberg and Spergel 1998).
- Particularly good at breaking the angular diameter distance degeneracy (Hu 2001).
- There is a significant overlap with the $f_{NL}^{\text{local}}$-type bispectrum $\Psi^\text{NG}(\vec{x}) = \Psi + f_{NL}(\Psi^2 - \langle \Psi^2 \rangle)$ (Smith and Zaldarriaga 2006).
**LENSING-ISW AND $f_{\text{local}}^L$**

Why does the lensing-ISW bispectrum project onto the $f_{\text{local}}^L$ bispectrum?

- Lensing convergence results in a local change of scale $\rightarrow$ local change of variance.
- $f_{\text{local}}^L$ corresponds to a local change in the amplitude of the power spectrum $\rightarrow$ local change of variance.

So there is an overlap between the local and ISW bispectra (although phases differ). The large amplitude of the ISW-lensing bispectrum results in significant contamination for $f_{\text{local}}^L$. 
THINGS TO WORRY ABOUT

Planck has the ability to detect this signal (plots to come later), but there are a few fundamental things to worry about:

▶ In the event of a significant detection, what is the effect of signal variance?
▶ What is the effect of lensing on other bispectra (claims of large effects from Cooray, Sarkar and Serra 2008), and is there a way to calculate the lensing of the bispectrum non-perturbatively?

Higher order lensing terms in $C_{l}^{\phi\phi}$ are known to be important for Planck, SPT (e.g. Alex’s talk yesterday).
We’ll write the lensed temperature graphically:

\[ T(\hat{n}) = \Theta(\hat{n}) + \Theta^{ISW}(\hat{n}) + \nabla^i \phi \nabla_i \Theta(\hat{n}) + \nabla^i \phi \nabla^j \phi \cdot \nabla_{ij} \Theta(\hat{n}) + \ldots \]

Using the following glossary:

- **Observable field:** \( T \)
- **Gaussian field:** \( \Theta, \Theta^{ISW} \)
- **Product of fields:** \( \phi \)
In bispectrum-language, often think of the minimum-variance estimator for the lensing-ISW amplitude as

\[
\hat{A} = \frac{1}{\mathcal{F}} \sum_{l_1 l_2 l_3} \sum_{m_1 m_2 m_3} \sum \frac{T_{l_1 m_1} T_{l_2 m_2} T_{l_3 m_3}}{C_{l_1}^{TT} C_{l_2}^{TT} C_{l_3}^{TT}} B_{l_1 l_2 l_3}^{m_1 m_2 m_3}.
\]

It’s more natural to write

\[
\hat{A} = \frac{1}{\mathcal{F}} \sum_{l m} C_l^{T \phi} \hat{\phi}_{lm} T_{lm}^*. \]
In bispectrum-language, often think of the minimum-variance estimator for the lensing-ISW amplitude as

$$\hat{A} = \frac{1}{\mathcal{F}} \sum_{l_1l_2l_3} \sum_{m_1m_2m_3} T_{l_1m_1} T_{l_2m_2} T_{l_3m_3} C_{l_1}^{TT} C_{l_2}^{TT} C_{l_3}^{TT} B_{l_1l_2l_3}^{m_1m_2m_3}.$$ 

It's more natural to write

$$\hat{A} = \frac{1}{\mathcal{F}} \sum_{lm} C_l^T \hat{\phi}_{lm} T_{lm}^* N_l^{\phi \phi} C_l^{TT}.$$ 

\[ \Theta_\text{ISW}_{lm} \]

April 22, 2011 10 / 17
Treating $\hat{\phi}$ as Gaussian, suggests the prescription

$$\hat{A} = \frac{1}{F} \sum_{lm} \frac{C_l^{T\phi} \phi_{lm} T_{lm}^*}{N_l^{\phi\phi} C_l^{TT}} \rightarrow$$

$$\hat{A} = \frac{1}{F'} \sum_{lm} \frac{C_l^{T\phi} \phi_{lm} T_{lm}^*}{(N_l^{\phi\phi} + C_l^{\phi\phi}) C_l^{TT} + (C_l^{T\phi})^2}.$$ 

From simulations, describes well the increase in variance due to signal.
Signal Variance for $f_{NL}^{\text{local}}$

- The same picture applies to the squeezed triangles of $f_{NL}^{\text{local}}$ as well (sort of like Munshi and Heavens 2009).
- Can picture as correlation of two small-scale modes, modulated by large-scale mode.
- Potential improvement on the CSZ approach (Creminelli et al. 2007, Smith et al. 2011).
For a squeezed bispectrum, safe to use the "unlensed short-leg" approximation.

Lensing terms generate coupling between the two long-wavelength modes, e.g.

\[
\langle \Theta \phi \phi \phi \rangle = \langle \Theta \Theta \phi \phi \rangle = \Theta_{\text{ISW}}
\]
BISPECTRUM LENSING

- For a squeezed bispectrum, safe to use the “unlensed short-leg” approximation.
- Lensing terms generate coupling between the two long-wavelength modes, e.g.

\[
\left\langle \Theta \phi \phi \phi \phi \right\rangle = \left\langle \Theta \phi \phi \phi \right\rangle = \Theta ISW()
\]
Bispectrum Lensing

We want to calculate

\[
\begin{align*}
\nabla_i \phi \nabla_i \Theta & \quad + \quad \frac{1}{2} \nabla_i \phi \nabla_i \phi \nabla_{ij} \Theta & \quad + \quad \frac{1}{6} \nabla_i \phi \nabla_i \phi \nabla^k \phi \nabla_{ijk} \Theta \\
\langle & \quad T \quad \rangle & \quad + \quad \langle & \quad T \quad \rangle & \quad + \quad \langle & \quad T \quad \rangle & \quad + \quad \ldots
\end{align*}
\]

Always keeping one \( \phi \) free (grayed out) to correlate with ISW.

Note that in real space

\[
\frac{\delta}{\delta \nabla \phi(\hat{n})} T(\hat{n}) = (\nabla \Theta)[\hat{n} + \nabla \phi(\hat{n})] = \nabla T(\hat{n}),
\]

where \( \nabla T \) is the lensed gradient of the CMB.
BISPECTRUM LENSING

We want to calculate

\[ \nabla_i \phi \nabla_i \Theta + \frac{1}{2} \nabla_i \phi \nabla_j \phi \nabla \Theta + \frac{1}{6} \nabla_i \phi \nabla_j \phi \nabla k \phi \nabla \Theta \ldots \]

Always keeping one \( \phi \) free (grayed out) to correlate with ISW.

In Fourier space we have (again with \( \hat{n} \))

\[ \left\langle T(\vec{l}_1) \frac{\delta}{\delta \phi(\vec{l}_2)^*} T(\vec{l}_3) \right\rangle = -\frac{i}{2\pi} \delta(\vec{l}_1 + \vec{l}_2 + \vec{l}_3) \vec{l}_2 \cdot \left\langle T(\vec{l}_1) \nabla T(\vec{l}_1)^* \right\rangle \]

\[ \approx -\frac{1}{2\pi} \delta(\vec{l}_1 + \vec{l}_2 + \vec{l}_3) (\vec{l}_1 \cdot \vec{l}_2) C_{l_1}^{TT}. \]

Where the validity of \( \approx \) is determined by the extent to which gradients and lensing commute.
**Bispectrum Lensing**

- So lensing of squeezed bispectra may be approximated simply by accurately lensing the long legs, e.g. using CAMB.
- Works well for $f_{\text{NL}}^{\text{local}}$.
- Corrects an $O(10\%)$ suppression at low-$l$.
- Also explains the $N_l^{(2)}$ bias for $\hat{\phi}_{lm}$ power spectrum at low-$l$.

Slice through $f_{\text{NL}}^{\text{local}}$

Hanson et al. (2009)
Detection on Data

- Currently null results on WMAP (e.g. Calabrese et. al. 2009).
- Ultimate detection significance with T and E limited by signal variance to $\approx 9\sigma$, although non-linear effects could make this higher (Mangilli and Verde 2009).

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{f_{NL}}$</th>
<th>$\sigma_{\text{lens}}$</th>
<th>Corr.</th>
<th>$f_{NL}^{\text{Bias}}$</th>
<th>$\sigma_{f_{NL}^{\text{marg}}}$</th>
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<tr>
<td>T</td>
<td>4.31</td>
<td>0.19</td>
<td>0.24</td>
<td>9.5</td>
<td>4.44</td>
</tr>
<tr>
<td>T+E</td>
<td>2.14</td>
<td>0.12</td>
<td>0.022</td>
<td>2.6</td>
<td>2.14</td>
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<tr>
<td>Planck T</td>
<td>5.92</td>
<td>0.26</td>
<td>0.22</td>
<td>6.4</td>
<td>6.06</td>
</tr>
<tr>
<td>Planck T+E</td>
<td>5.19</td>
<td>0.22</td>
<td>0.13</td>
<td>4.3</td>
<td>5.23</td>
</tr>
</tbody>
</table>

Lewis, Challinor, Hanson (2011)
**Planck Prospects**

- Good match for Planck, with full-sky coverage.
- Predict $\approx 4\sigma$ from temperature alone for linear ISW.
- Difficulty in that low-$l$ in $\phi$ is also where many scan-strategy associated systematics live.
- Also need to worry about things like peculiar velocity dipole.

A challenge, but exciting!
CONCLUSIONS

- ISW-\(\phi\) directly traces dark energy / matter at \(z \sim 2\).
- E-\(\phi\) is significant as well, and should be sensitive to even higher redshifts.
- Signal variance can be treated by approximating \(\hat{\phi}\) as Gaussian.
- “non-Perturbative” lensed bispectra can be calculated in the squeezed limit, using unlensed short-legs.
- Aim for detection with Planck, which has sky-coverage and S/N for \(\approx 4\sigma\).