Full-sky CMB lensing reconstruction in presence of galactic residuals and point sources

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Estimation of the lensing potential from full-sky maps

- Full-sky quadratic estimator

\[
\hat{\phi}_L^M \propto A_L \int d\hat{n} Y_L^{M*} \left( \sum_{\ell_1 m_1} \frac{1}{C_{\ell_1}} \Theta_{\ell_1}^{m_1} Y_{\ell_1}^{m_1} \right) \nabla \left( \sum_{\ell_2 m_2} \widetilde{C}_{\ell_2} \Theta_{\ell_2}^{m_2} Y_{\ell_2}^{m_2} \right)
\]

- Estimator is unbiased

\[
\langle \hat{\phi}_L^M \rangle_{\text{lens}} = \phi_L^M
\]

- Estimator variance

\[
\langle \hat{\phi}_L^M \hat{\phi}_L^{M*} \rangle = C_{\phi \phi}^L + N_L^{(0)} + N_L^{(1)} + N_L^{(2)} + \ldots
\]

Gaussian noise

\[\nabla \rightarrow \text{high order biases}\]


Kesden et al. (2003), Hanson et al. (2011)
✓ Reconstruction in a perfect pessimistic Planck-like case:

- noise = 60\,\mu\text{k.arcmin}
- 5 arcmin beam
- \(l_{\text{max}}=2300\)

Perfect simulation:
- Homogeneous coverage
- White and known noise
- No sky cuts
- Known cosmology
Estimation of the lensing potential from full-sky maps

Lensing code: iLens  Basak et al. 2009

- Unbiased reconstruction of the lensing potential
- Non-zero curl modes
Galaxy

- Component separation will help but residuals expected
- Masking large regions of the sky is necessary

How to handle galactic mask?
- Analytically?
Complications

✓ Galaxy
  
  • Component separation will help but residuals expected
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✓ How to handle galactic mask?
  
  • Analytically?
    \[
    \langle \hat{\phi}^M_L \hat{\phi}^{M*}_L \rangle = \sum_{\lambda} K^{LM}_\lambda C^{\phi \phi}_\lambda + N^{(0)'}_L + \cdots
    \]

\[
K^{LM}_\lambda = \frac{1}{2^7} \int d^7 \cos \theta_i \lambda(\lambda + 1) \frac{\Pi^\lambda}{4\pi} d^\lambda_{11}(\theta_7) \left( \frac{\Pi_L}{\sqrt{4\pi}} (-1)^M d^L_{-M1}(\theta_1) \left( \frac{\Pi_L}{\sqrt{4\pi}} (-1)^M d^L_{-M1}(\theta_2) \right) \times \right.
\]

\[
\left( \sum_{l_1 m_1} \Pi^2_{l_1} f_1(l_1) d^{l_1}_{m_10}(\theta_1) d^{l_1}_{m_10}(\theta_3) \right) \left( \sum_{l_3 m_3} \Pi^2_{l_3} f_1(l_3) d^{l_3}_{m_30}(\theta_2) d^{l_3}_{m_30}(\theta_5) \right) \times \]

\[
\left( \sum_{l_2 m_2} \Pi^2_{l_2} \sqrt{l_2(l_2 + 1)} f_2(l_2) d^{l_2}_{m_20}(\theta_1) d^{l_2}_{m_20}(\theta_4) \right) \left( \sum_{l_4 m_4} \Pi^2_{l_4} \sqrt{l_4(l_4 + 1)} f_2(l_4) d^{l_4}_{m_40}(\theta_2) d^{l_4}_{m_40}(\theta_6) \right) \times \]

\[
\left( \sum_{l'_{m'1} m'_{1}} C^{\theta \theta}_{l'_{1} l_{1}} d^{l'_{1}}_{-m'_{1}0}(\theta_3) d^{l'_{1}}_{-m'_{1}0}(\theta_5) \right) \left( \sum_{l'_{2} m'_{2} 0} \Pi^2_{l'_{2}} d^{l'_{2}}_{-m'_{2}0}(\theta_4) d^{l'_{2}}_{-m'_{2}0}(\theta_6) d^{l'_{2}}_{00}(\theta_7) \right) \left( \sum_{\lambda'_{2}} C^{\theta \theta}_{\lambda'_{2} \lambda_{2}} \lambda'_{2}(\lambda'_{2} + 1) \frac{\Pi^2_{\lambda'_{2}}}{4\pi} d^{\lambda'_{2}}_{-1-1}(\theta_7) \right) \times \]

\[
\left( \sum_{l'_{3} m'_{3} 0} w^{*}_{l'_{3} m'_{3}} \Pi^0_{l'_{3}} d^{0}_{m'_{3}0}(\theta_3) \right) \left( \sum_{l'_{4} m'_{4} 0} w^{*}_{l'_{4} m'_{4}} \Pi^0_{l'_{4}} d^{0}_{m'_{4}0}(\theta_4) \right) \left( \sum_{l'_{5} m'_{5} 0} w^{*}_{l'_{5} m'_{5}} \Pi^0_{l'_{5}} d^{0}_{m'_{5}0}(\theta_5) \right) \left( \sum_{l'_{6} m'_{6} 0} w^{*}_{l'_{6} m'_{6}} \Pi^0_{l'_{6}} d^{0}_{m'_{6}0}(\theta_6) \right)
\]

Benoit-Lévy et al., in prep
Complications

✓ Galaxy
  • Component separation will help but residuals expected
  • Masking large regions of the sky is necessary

✓ How to handle galactic mask?
  • Analytically?
  • Inverse covariance weighting is difficult
    • Planck resolution
    • Low noise, large dynamic
  • Inpainting techniques hard to control

✓ We want a simple, robust and linear pipeline
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✓ We want a simple, robust and linear pipeline
  • Apodized galactic cut
Apodized galactic mask

\[ f_{\text{sky}} = \frac{1}{N_{\text{pix}}} \sum_i w_i^A \]
Analytical treatment of masks

Azimuthal mask: some simplifications...

\[
\langle \hat{\Phi}^M \hat{\Phi}^{M*} \rangle = \sum_{\lambda} K^L_M C_\lambda^{\phi\phi} + N^{(0)}_L + \cdots
\]

\[
K^L_M = \frac{1}{2^7} \int d^7 \cos \theta \: \lambda(\lambda + 1) \frac{\Pi_L}{4\pi} d^1_{11}(\theta_1) \left( \frac{\Pi_L}{4\pi} d^L_{M1}(\theta_2) \right) \left( \frac{\Pi_L}{4\pi} d^L_{M1}(\theta_3) \right) \times
\]

\[
\left( \sum_{l_1m_1l_1'} C_{l_1}^{\gamma\gamma} \Pi^2_{l_1l_1'} f_1(l_1) f_1(l_1') d^{l_1}_{m_1}(\theta_1) d^{l_1}_{m_1}(\theta_3) d^{l_1}_{m_1}(\theta_5) \right) \times
\]

\[
\left( \sum_{l_2m_2l_2'} \Pi^2_{l_2l_2'} \sqrt{l_2(l_2 + 1)} f_2(l_2) \sqrt{l_4(l_4 + 1)} f_2(l_4) d^{l_2}_{m_2}(\theta_1) d^{l_2}_{m_2}(\theta_4) \right) \times
\]

\[
\left( \sum_{l_3m_3l_3'} w^{l_3}_{l_3} \Pi^{l_3}_{l_3} d^{l_3}_{m_3}(\theta_3) \right) \left( \sum_{l_4m_4l_4'} w^{l_4}_{l_4} \Pi^{l_4}_{l_4} d^{l_4}_{m_4}(\theta_4) \right) \left( \sum_{l_5m_5l_5'} w^{l_5}_{l_5} \Pi^{l_5}_{l_5} d^{l_5}_{m_5}(\theta_5) \right) \left( \sum_{l_6m_6l_6'} w^{l_6}_{l_6} \Pi^{l_6}_{l_6} d^{l_6}_{m_6}(\theta_6) \right)
\]

\[N^{(0)}\] is (a bit) simpler: only 6d integral

\[
\langle \hat{\Phi}^M \hat{\Phi}^{M*} \rangle = C_L^{\phi\phi} + N_L^{(0)} + N_L^{(1)} + N_L^{(2)} + \cdots
\]

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\langle \hat{\Phi}_L^M \hat{\Phi}_L^{M^*} \rangle = \sum_{\lambda} K_{\lambda}^{LM} C_{\lambda}^{\phi \phi} + N_L^{(0)} + \cdots
\]

\[
K_{\lambda}^{LM} = \frac{1}{2\pi} \int d^2 \cos \theta \quad \lambda(\lambda+1) \frac{\Pi_0}{4\pi} d_{11}(\theta_1) \left( \frac{\Pi_L}{4\pi} d_{LM}(\theta_2) \right) \left( \sum_{\lambda'} \frac{C_{\lambda'2}^{\phi \phi} \lambda_2^2 + 1}{4\pi} \frac{\Pi_{L2}^2}{d_{-1-1}^2(\theta_7)} \right) \times
\]

\[
\left( \sum_{l_1m_1l_1'} C_{l_1}^{\theta \theta} \Pi_{l_1l_1'} f_1(l_1) f_1(l_3) d_{m_10}(\theta_1) d_{m_10}(\theta_3) d_{m_10}(\theta_5) \right) \times
\]

\[
\left( \sum_{l_2m_2l_2'} \sqrt{l_2(l_2+1)} f_2(l_2) \sqrt{l_4(l_4+1)} f_2(l_4) d_{m_21}(\theta_1) d_{m_20}(\theta_4) \right) \times
\]

\[
\left( \sum_{l_3m_3l_3'} \Pi_{l_3l_3'} \right) \left( \sum_{l_4m_4l_4'} \Pi_{l_4l_4'} \right) \left( \sum_{l_5m_5l_5'} \Pi_{l_5l_5'} \right) \left( \sum_{l_6m_6l_6'} \Pi_{l_6l_6'} \right)
\]

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\[
\langle \hat{\Phi}_L^M \hat{\Phi}_L^{M^*} \rangle = \Phi_L^{\phi \phi} + N_L^{(0)} + X_L^{(1)} + X_L^{(2)} + \cdots
\]

Applying the estimator on unlensed maps gives \(N_0\): computation of the bias by Monte-Carlo
Pipeline description

Data map → Masking → Masked data map → Lensing estimation

Spice with apodized galactic mask

Empirical spectrum → Masking → Masking → Lensing estimation

N unlensed simulated maps → N unlensed simulated masked maps → \( N^{(0)}_{MC} \)

\[ C_{\phi\phi}^{(0)} + N_L^{(0)} + N_L^{(1)} + N_L^{(2)} + \cdots \]
Null-tests of the pipeline

Tests are made on 50 ‘data’ maps.

Using empirical spectrum from the data do not induce bias.

Uncut unlensed maps

Uncut lensed maps

Using empirical spectrum from the data do not induce bias.
Apodized galactic mask allows for unbiased reconstruction

Loss of large scale-modes

Coupling matrix close to identity. Band covariances under study
Apodization parameter dependance

Large apodization length increases missing fraction but reduces contamination
Inpainting by local constrained Gaussian realizations

Aim: fill the holes with pure Gaussian CMB

\[ P(T_1 | T_2) = \mathcal{N}(C_{12}C_{22}^{-1}T_2, C_{11} - C_{12}C_{22}^{-1}C_{21}) \]

\[ T_1 = \tilde{T}_1 + C_{12}C_{22}^{-1}(T_2 - \tilde{T}_2) \]

We expect a loss of power due to missing lensing in inpainted holes
Pipeline description

Data map

Inpainted data map

Local Gaussian constrained inpainter

Empirical spectrum

Masking

Masked inpainted data map

Lensing estimation

\[ \langle \hat{\phi}_L^M \hat{\phi}_L^{M*} \rangle = C_L^{\phi\phi} + N_L^{(0)} + N_L^{(1)} + N_L^{(2)} + \cdots \]

N unlensed simulated maps

Spice with apodized galactic mask

N unlensed simulated masked maps

Masking

Lensing estimation

\[ N_{MC}^{(0)} \]
As Gaussian CMB is restored, one would expect a correction by a factor $f_{\text{PS}}$, not $f_{\text{PS}}^2$
Reconstruction within theoretical error bars
Modes coupling become non-trivial
Galactic mask and point sources inpainting

Reconstruction within theoretical error bars
Conclusions

✓ Dealing with galaxy in full-sky lensing reconstruction
  • Dealing with galaxy can be done using a simple apodized mask
  • Unbiased reconstruction but loss of precision at large scales

✓ Gaussian inpainting of resolved point sources recovers most of the lensing power
  • Non-trivial mode coupling
  • Effect on $N^{(1)}$ and $N^{(2)}$

✓ Under study: error bars correlations due to modes coupling

Thank you!
Point sources inpainting on unlensed maps

$C_\ell \ell (\ell + 1)/2\pi$

$\ell$
Lensing potential reconstruction

Unlensed masked maps

Unlensed masked and inpainted maps