Measurements of Primordial Non-Gaussianity and Gravitational Lensing in WMAP CMB Data

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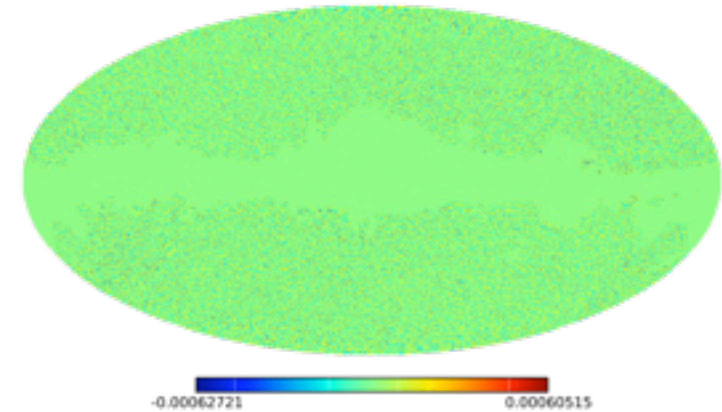
This Started With Measuring Non-Gaussianity.

- Common way to compute non-Gaussianity:

$$S_3 = f_{\rm NL} \sum_{l_1} \sum_{l_2} \sum_{l_3} \left\{ \frac{B_{l_1 l_2 l_3} B_{l_1 l_2 l_3}}{C_{l_1} C_{l_2} C_{l_3}} \right\}$$

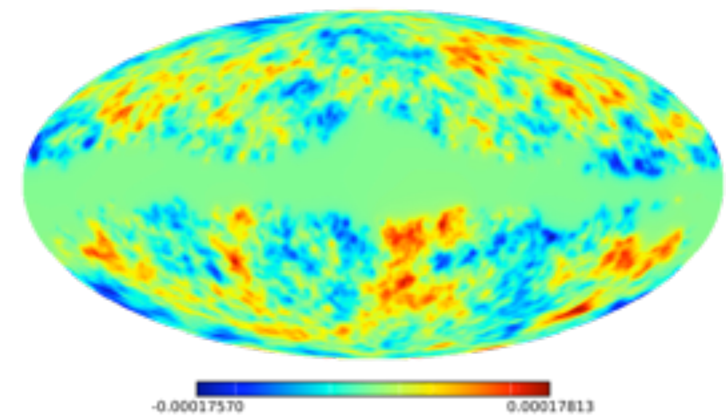


A Map

- Can extract from data:

$$S_3 \equiv \int r^2 dr \int d\hat{\Omega} A(r,\hat{\Omega}) B^2(r,\hat{\Omega})$$

Where A and B are weighted CMB maps.



B Map

- Alternatively:

$$C_l^{A,B^2} \equiv \frac{1}{2l+1} \int r^2 dr \sum_m A^*_{lm}(r) B^{(2)}_{lm}(r); \qquad C_l^{AB,A} \equiv \frac{1}{2l+1} \int r^2 dr \sum_m (AB)^*_{lm}(r) B_{lm}(r)$$

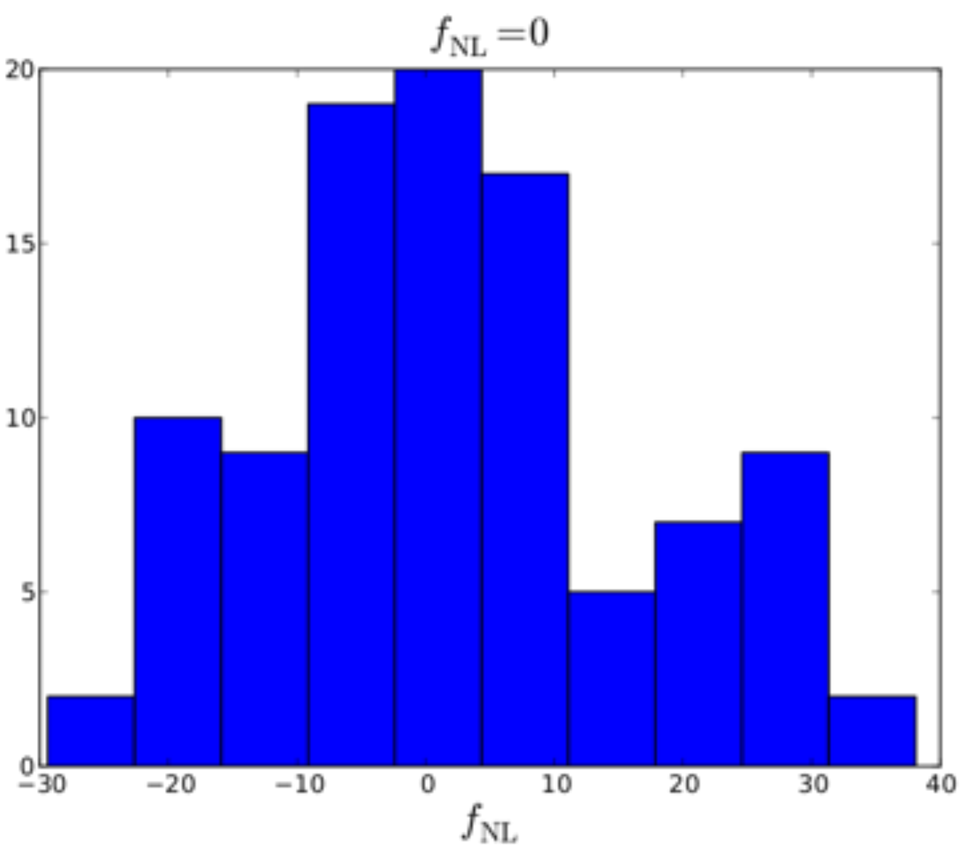
- Leading to a skewness power spectrum:

$$C_l^{2,1} \equiv \left(C_l^{A,B^2} + 2 C_l^{AB,B} \right) = \frac{f_{\rm NL}}{(2l+1)} \sum_{l_2} \sum_{l_3} \left\{ \frac{B_{l l_2 l_3} B_{l l_2 l_3}}{C_l C_{l_2} C_{l_3}} \right\}$$

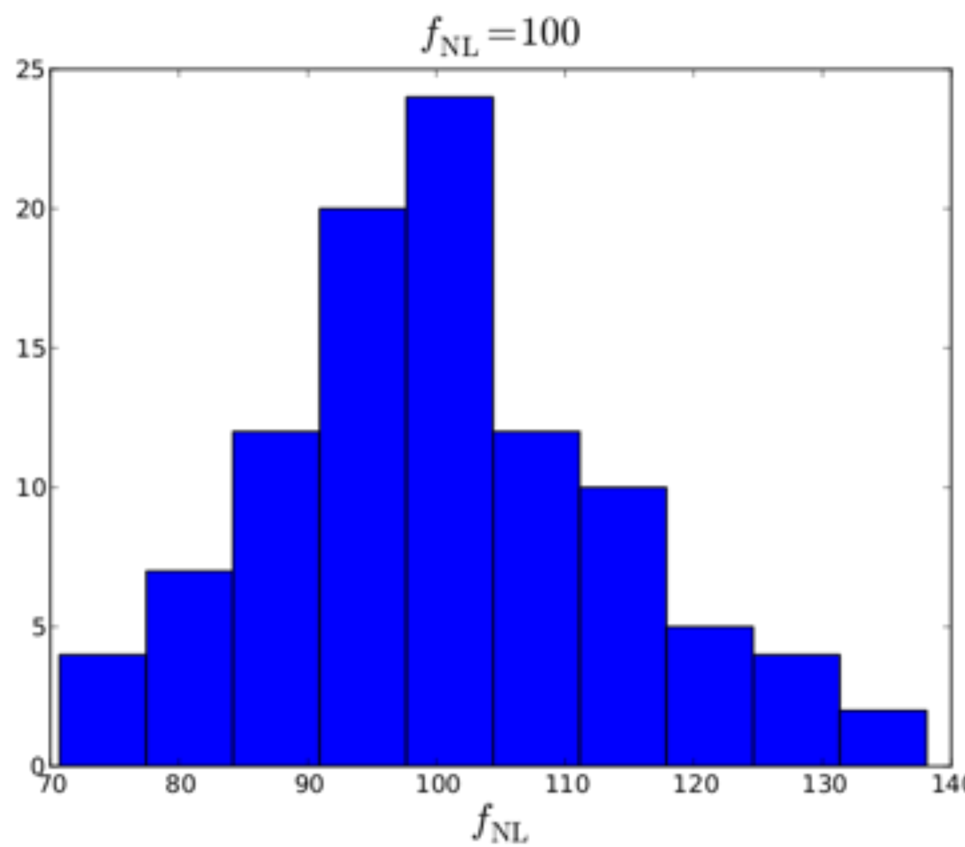
Cooray (2001)PRD, 64, 043516
Munshi et al. (2010) MNRAS, 401, 2406
Smidt et al. (2009) PRD 80, 123005

Testing Skewness Spectra on Simulations:

- Run estimator100 publicly available simulations:
  (From Franz Elsner and Benjamin D. Wandelt)
- Add desired level of NG: $a_{lm} = a_{lm}^{G} + f_{\rm NL} a_{lm}^{NG}$
- Good as fisher error estimate is +/- 13



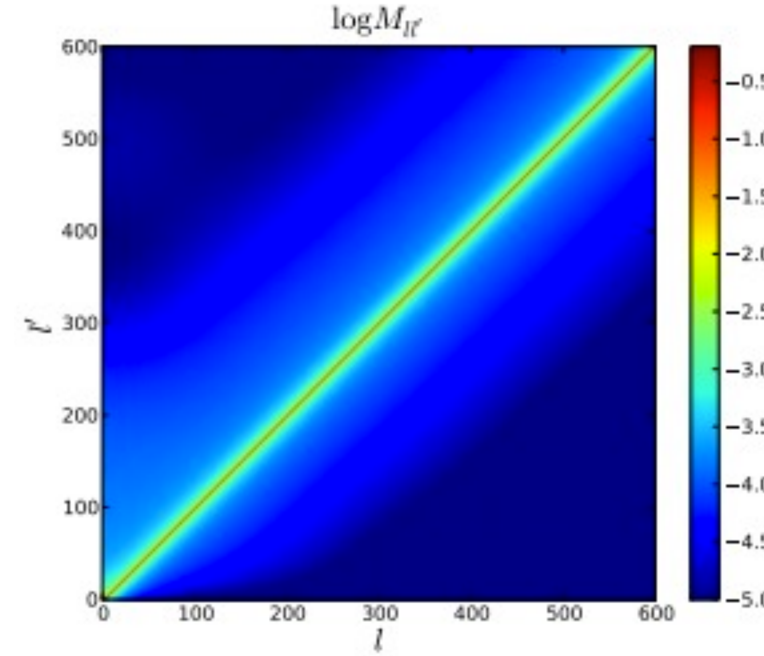
$f_{nl} = 2$ +/- 14



$f_{nl} = 101$ +/- 14

Works well on simulations.

Parameter Estimation



- We remove Masking Effects:
  (Master Alogrithm of Hivon 2001.)

$$\tilde{C}_l = \sum_{l'} M_{ll'} C_{l'} ,$$

- We want to minimize where C is covariance matrix:
  (C is obtained from Gaussian simulations.)

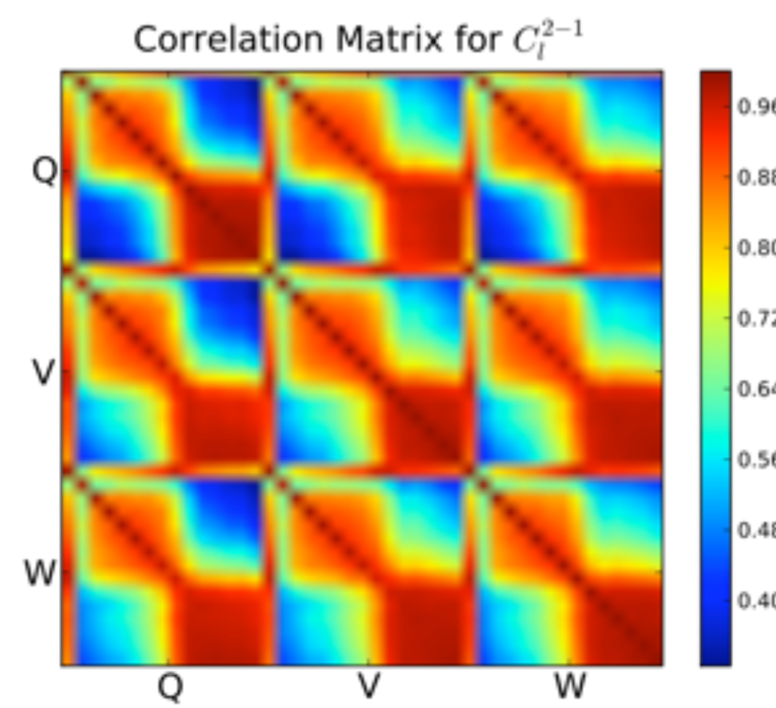
$$\chi^2 = (y - M \cdot p)^T C^{-1} (y - M \cdot p)$$

- We set the above's derivative to zero and solve for our parameters

$$p = (M^T C^{-1} M)^{-1} M^T C^{-1} \cdot y$$

- Our error for each parameter is

$$\Delta p = (M^T C^{-1} M)^{-1}$$



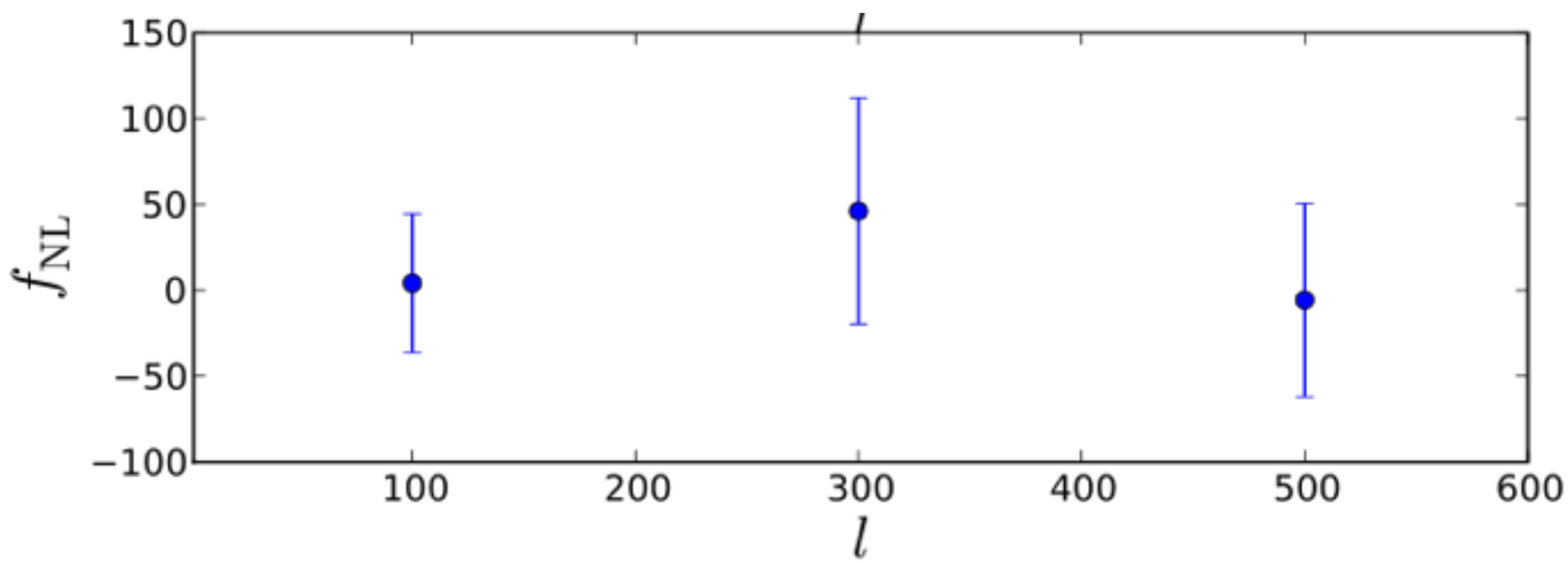
Fit for f_{nl} with the skewness power spectra :

Smidt et al. (2009) PRD 80, 123005

- f_{nl} from $C_l^{(2,1)}$ estimator :

V: 16.7 ± 27.1, W: 18.7 ± 27.2, V+W: 11.0 ± 24

- f_{nl} variation with scales :



Let's Complicate It: Trispectrum

- We can look at four point function for non-Gaussianity:

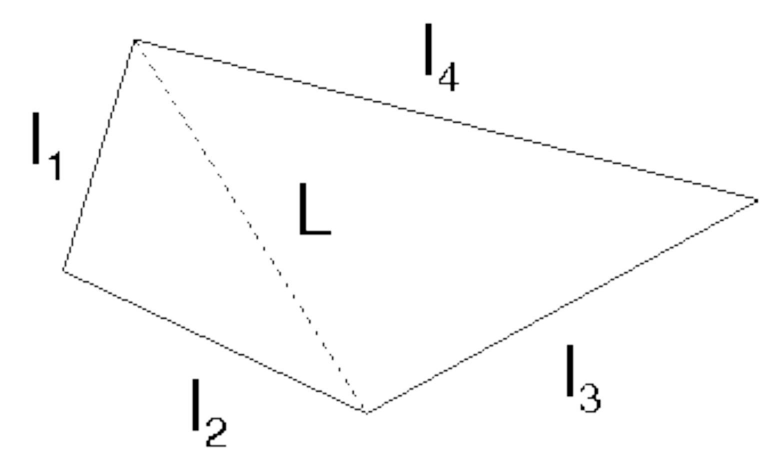
$$< a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} a_{l_4 m_4} >_c = \sum_{LM} T^{l_3 l_4}_{l_1 l_2}(L) \begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & M \end{pmatrix} \begin{pmatrix} l_3 & l_4 & L \\ m_1 & m_2 & -M \end{pmatrix}$$

- Where T has the form:

$$T^{l_1 l_2}_{l_3 l_4}(L) = \tau_{\rm NL} F(l_1, l_2, l_3, l_4, L) + g_{\rm NL} G(l_1, l_2, l_3, l_4, L)$$

- And construct Kurtosis spectra to constrain this non-Gaussianity.

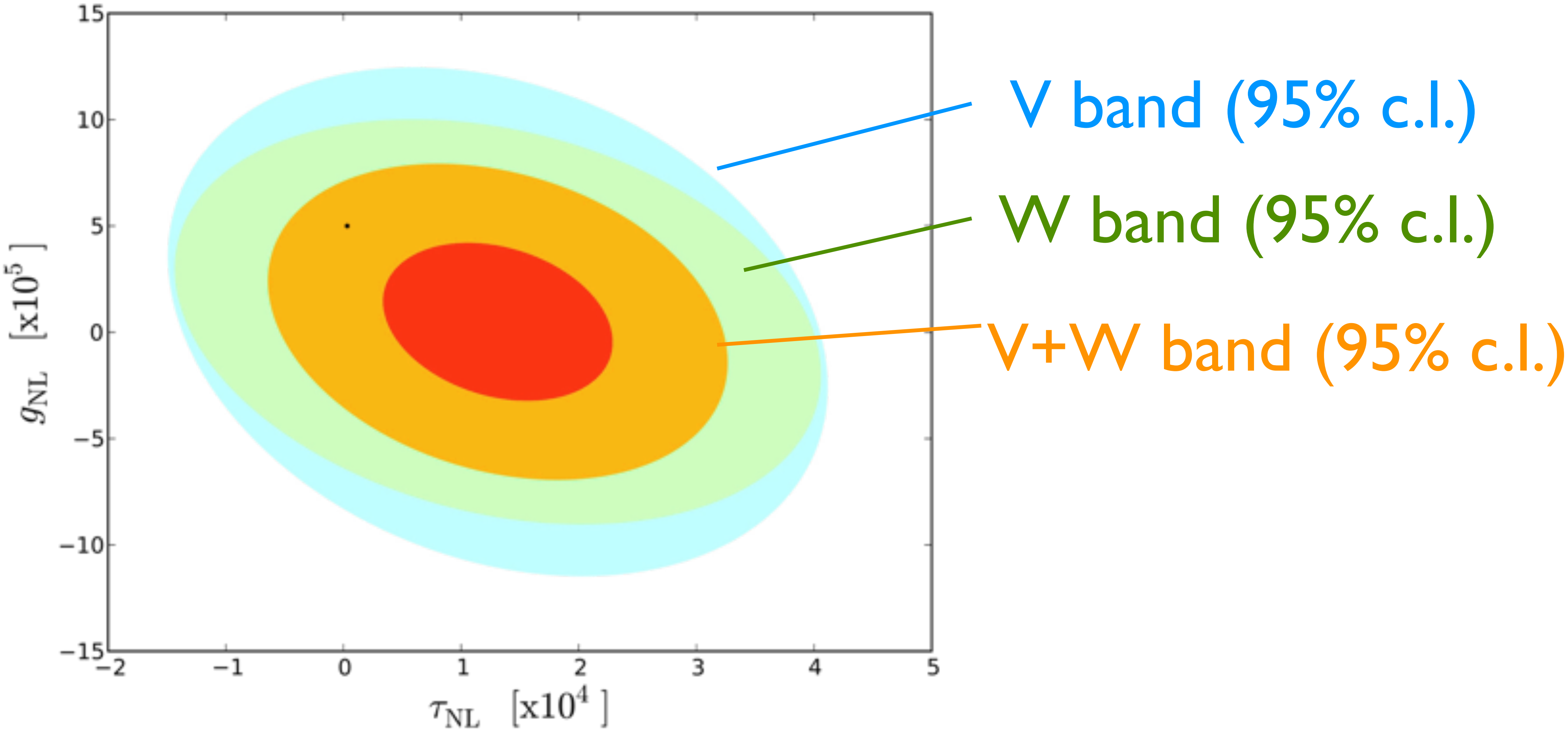
$$K_l^{(2,2)} = \sum_{l_i} \frac{T^{l_3 l_4}_{l_1 l_2}(l) T^{l_3 l_4}_{l_1 l_2}(l)}{C_{l_1} C_{l_2} C_{l_3} C_{l_4}} \qquad K_l^{(3,1)} = \sum_{l_i L} \frac{T^{l_1 l_2}_{l_3 l}(L) T^{l_1 l_2}_{l_3 l}(L)}{C_{l_1} C_{l_2} C_{l_3} C_l}$$



This time, 2x2 and 3x1 weighted maps are used.

Results:

Smidt et al.(2010) PRD 81, 123007



	V	W	V+W
g_{nl}	$4.2 \pm 53 \times 10^4$	$4.1 \pm 59 \times 10^4$	$4.2 \pm 39 \times 10^4$
τ_{nl}	$1.32 \pm 1.27 \times 10^4$	$1.39 \pm 1.31 \times 10^4$	$1.35 \pm 0.98 \times 10^4$

previous constraint on τ_{nl} from COBE was $<10^8$

Measuring $C_l^{\Phi\Phi}$ directly from the trispectrum of the CMB.

Smidt et al. 2011

(ApJL, Volume 728, Issue 1, L1 (2011))

arxiv:1012:1600

A CONSTRAINT ON THE INTEGRATED MASS POWER SPECTRUM OUT TO $z = 1100$ FROM LENSING OF THE COSMIC MICROWAVE BACKGROUND

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(Dated: December 7, 2010)
Draft version December 7, 2010

ABSTRACT

The temperature fluctuations and polarization of the Cosmic Microwave Background (CMB) are now a well-known probe of the Universe at an infant age of 400,000 years. During the transit to us from the surface of last scattering, the CMB photons are expected to undergo modifications induced by the intervening large-scale structure. Among the expected secondary effects is the weak gravitational lensing of the CMB by the foreground dark matter distribution. We derive a quadratic estimator that uses the non-Gaussianities generated by the lensing effect at the four-point function level to extract the power spectrum of lensing potential fluctuations integrated out to $z \sim 1100$ with peak contributions from potential fluctuations at z of 2 to 3. Using WMAP 7-year temperature maps, we report the first direct constraints of this lensing potential power spectrum and find that it has an amplitude of $A_L = 0.96 \pm 0.60$, 1.06 ± 0.69 and 0.97 ± 0.47 using the W, V and W+V bands, respectively.

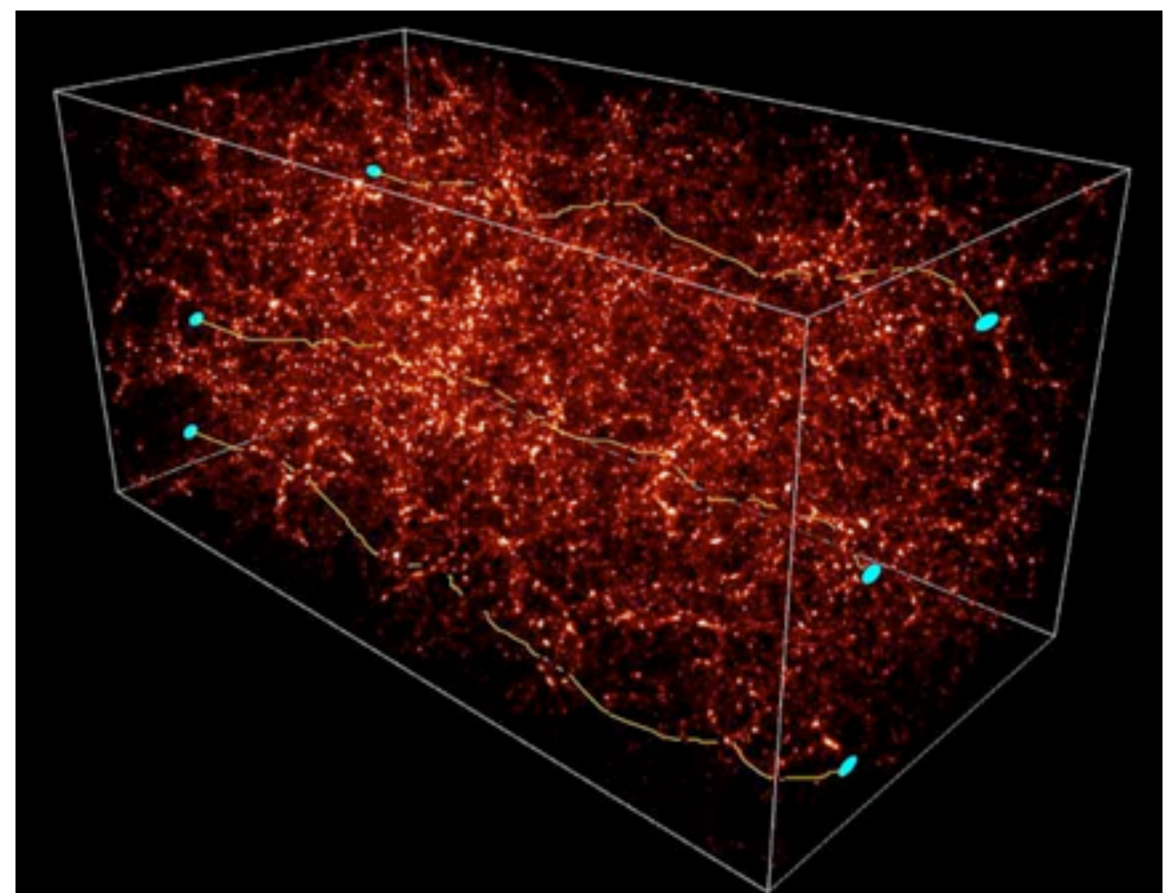
Subject headings: cosmology: cosmic microwave background — cosmology: observations — cosmology: theory — gravitational lensing

Weak Lensing

- Photons leave surface of last scattering.
- Deflected by large scale structure.

Gravitational Potential ϕ

Deflection Angle $\alpha = \nabla\phi$



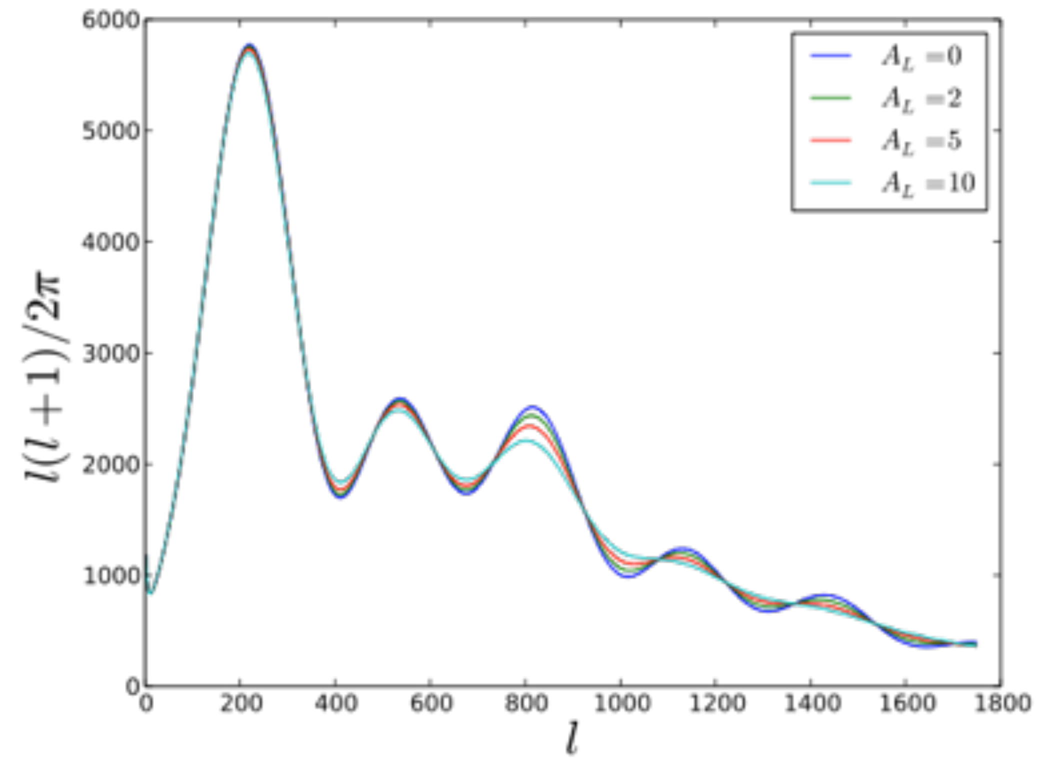
(Credit: S. Colombi (IAP), CFHT Team)

Extracting lensing

$$\Theta(\mathbf{n}) = \left(\frac{\Delta T}{T}\right)(\mathbf{n}) = \sum_{lm} \Theta_{lm} Y^*_{lm}(\mathbf{n})$$

$$\Theta(\mathbf{n}) = \Theta(\mathbf{n} + \nabla\phi)$$

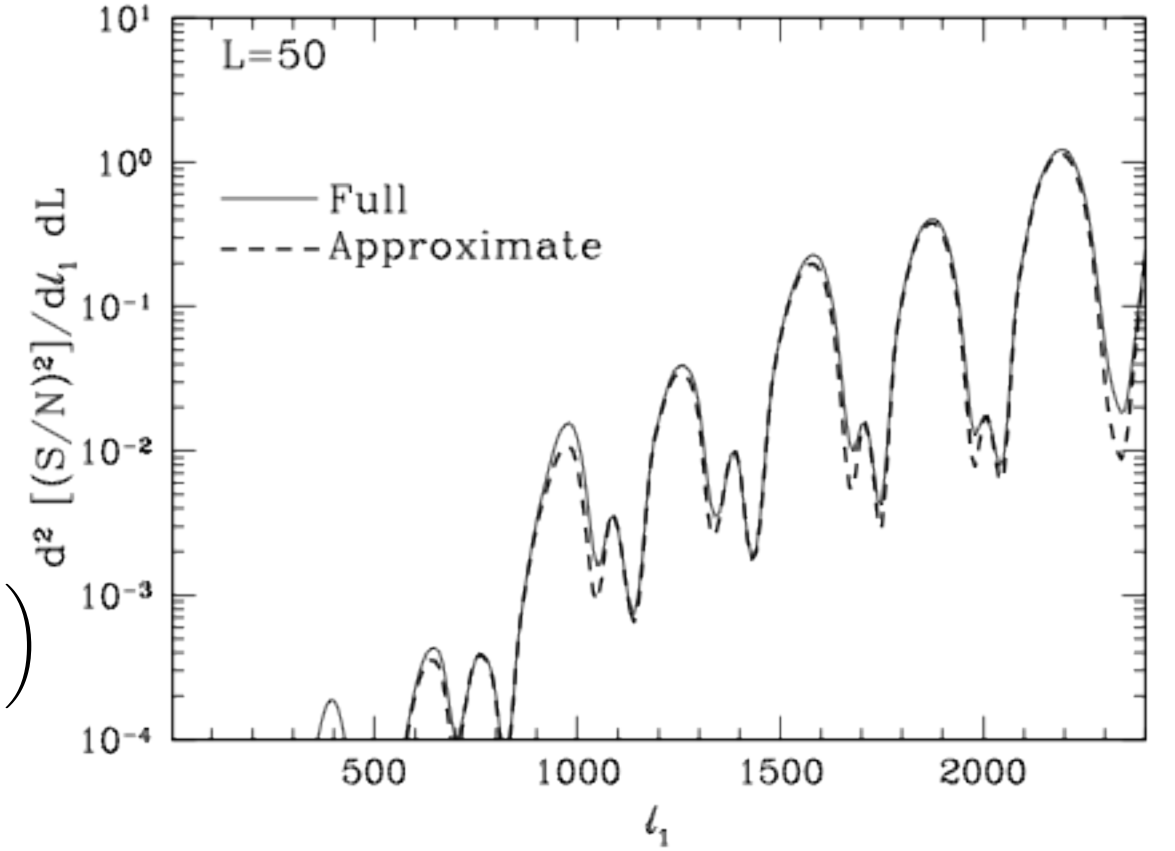
$$\sim \tilde{\Theta}(\mathbf{n}) + \nabla_i \phi(\mathbf{n}) \nabla^i \tilde{\Theta}(\mathbf{n})$$



$$\delta\Theta_{lm} = \Theta_{l'm'} + \sum_{LM} \sum_{l'm'} \phi_{LM} \tilde{\Theta}_{l'm'} (-1)^m \begin{pmatrix} l & l' & L \\ m & m' & -M \end{pmatrix} F_{ll'L}$$

$$F_{ll'L} = \sqrt{\frac{(2l+1)(2l'+1)(2L+1)}{4\pi}} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix} \frac{1}{2} [L(L+1) + l'(l'+1) - l(l+1)]$$

The Trispectrum



(Hu (2001) Phys.Rev.D64:083005)

$$\langle \Theta_{l_1 m_1} \Theta_{l_2 m_2} \Theta_{l_3 m_3} \Theta_{l_4 m_4} \rangle_c \sim$$

$$\sum_{LM} (-1)^M T^{l_1 l_2}_{l_3 l_4}(L) \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & -M \end{pmatrix} \begin{pmatrix} l_3 & l_4 & L \\ m_3 & m_4 & M \end{pmatrix}$$

Where:

$$T^{l_1 l_2}_{l_3 l_4}(L) = \underline{C^{\phi\phi}_L} \left(\tilde{C}_{l_2} F_{l_1 l_2 L} + \tilde{C}_{l_1} F_{l_2 l_1 L} \right) \left(\tilde{C}_{l_4} F_{l_3 l_4 L} + \tilde{C}_{l_3} F_{l_4 l_3 L} \right)$$

- Has same form as local non-Gaussianity:

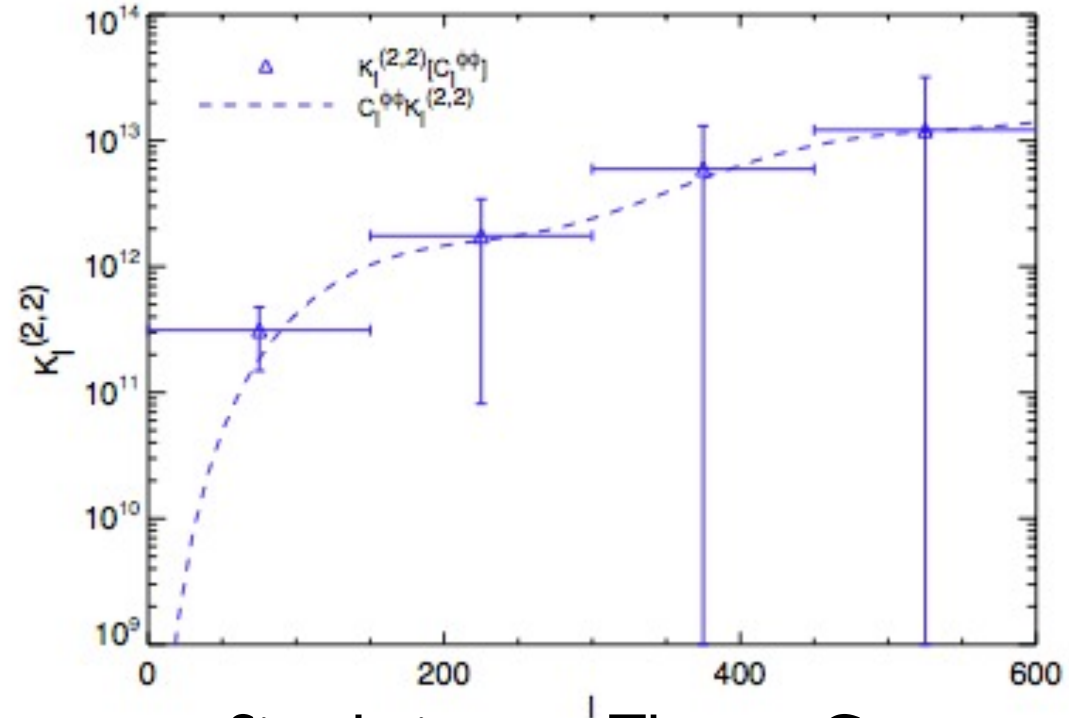
$$T^{l_1 l_2}_{l_3 l_4}(L) = \tau_{\mathrm{NL}} h_{l_1 l_2 L} h_{l_3 l_4 L} F(l_1, l_2, l_3, l_4, L)$$

- This means we can measure $C_l^{\Phi\Phi}$ similar to how we measured τ_{nl}.

So We Play The Same Game

- Weight maps appropriately.
- Compare to theoretical 2-2 estimator:

$$\mathcal{K}^{(2,2)}_{l(\mathrm{Lens})} = \frac{C_l^{\phi\phi}}{(2l+1)} \sum_{l_i} \frac{1}{(2l+1)} \frac{T^{l_3 l_4}_{l_1 l_2}(l) \hat{T}^{l_1 l_2}_{l_3 l_4}(l)}{\mathcal{C}_{l_1}\mathcal{C}_{l_2}\mathcal{C}_{l_3}\mathcal{C}_{l_4}}$$

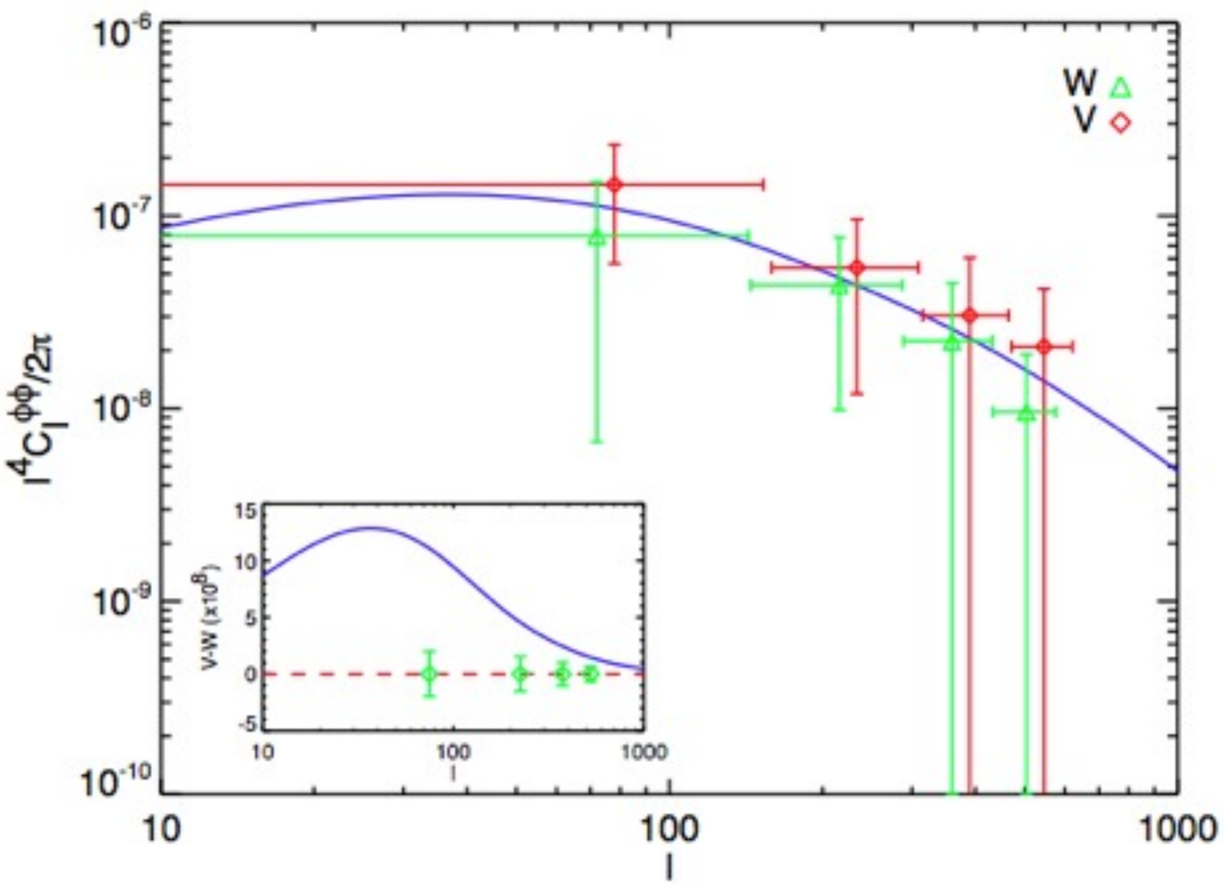


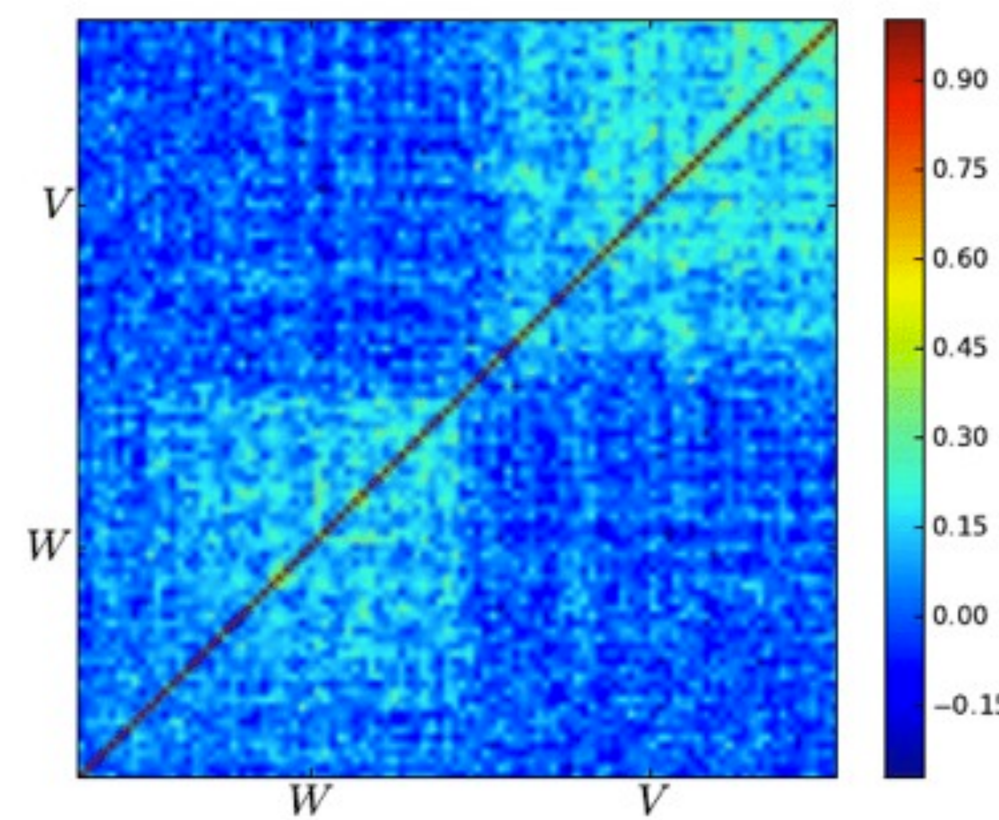
Simulations vs. Theory Curve.

- Make 400 Gaussian Maps
- Used Lenspix to seed 400 Gaussian maps with arbitrary $C_l^{\Phi\Phi}$.
- Use Gaussian Maps to subtract off Gaussian piece.

Results From WMAP Data

- Results from V and W band WMAP 7 data.
- Also a null test V - W.
- Use χ^2 and the covariance matrix as before.





A 2σ Excess For Lensing.

- To measure the lensing amplitude we constrain $A_l\ C_l^{\Phi\Phi}$
- $A_l = 0$ is for unlensed sky. $A_l = 1$ is fiducial.
- Use CosmoMC. (2 sigma measurement of lensing amplitude.)

Params.	WMAP7	WMAP7+A_L	WMAP7+A_L+$C_l^{\phi\phi}$
$10^3\Omega_b h^2$	22.51 ± 0.62	22.59 ± 0.63	22.60 ± 0.58
$10^2\Omega_{DM} h^2$	11.08 ± 0.57	11.04 ± 0.54	11.09 ± 0.54
τ	0.089 ± 0.016	0.090 ± 0.015	0.089 ± 0.015
n_s	0.967 ± 0.015	0.968 ± 0.014	0.968 ± 0.014
Ω_Λ	0.734 ± 0.031	0.737 ± 0.028	0.735 ± 0.027
Age/Gyr	13.8 ± 0.14	13.7 ± 0.14	13.7 ± 0.13
H_0^1	71.0 ± 2.7	71.3 ± 2.5	71.1 ± 2.4
$\mathbf{A_L}$	**1.0**	$\mathbf{0.87 \pm 1.05}$	$\mathbf{0.97 \pm 0.47}$

(Smidt et al. (2011) ApJL 728, Issue 1, L1)

In Conclusion

- These new estimators are powerful.
- Can constrain scale dependent quantities in the bispectrum and trispectrum.
- So far have been applied to non-Gaussianity and lensing.
- Thanks for listening.