A history of CMB observations

- COBE (1992)
  - $\Delta T = 18 \, \mu$K
- WMAP (2003)
- Planck (2013)
WMAP and Planck have given us excellent measurements of the temperature power spectrum which (mostly) support LCDM cosmology...

(very) brief background

\[
\Delta T(n) = \sum_{l>0} \sum_{m=-l}^{l} a_{lm} Y_{lm}(n)
\]

\[
\langle a_{lm} a_{l'm'}^* \rangle = \delta_{ll'} \delta_{mm'} C_l
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Same info, reorganized

$$\tilde{C}^{TT}(\theta) \equiv \langle \Theta(\hat{n}_1) \Theta(\hat{n}_2) \rangle$$

$$C(\theta) = \sum_{l} \frac{l(l+1)}{4\pi} C_l P_l(\cos \theta)$$
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ACF measurements from WMAP

Defined a statistic:

\[ S_{1/2} = \int_{1/2}^{-1} d(\cos \theta) C(\theta)^2 \]
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 LCDM value: \( \sim 50,000 \ \mu K^4 \)
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Calculated WMAP and Planck values:

\{ \sim 1000 - 8000 \, \mu K^4 \} \hspace{1cm} .03 - 5\% \text{ likely depending on the choice of data set}
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This is an \textit{a posteriori} statistic

Calculated WMAP and Planck values:

\{ \sim 1000 - 8000 \ \mu K^4 \ \text{\(\leftarrow\)) .03 - 5\% likely depending on the choice of data set} \}
But we need to move beyond temperature to learn something new.

Lots of work has gone into characterizing the lack of correlation in temperature data...

- Recent analysis of Planck maps by Copi, Huterer, Schwarz, Starkman – arXiv: 1310.3831
- A nice (short) review of the lack of correlation at large angles arXiv: 1201.2459
- Detailed comparison of WMAP to Planck large angle anomalies arXiv: 1303.5083
Our goal:

Find an optimal \textit{a priori} measure for investigating the lack of correlation at large angles
The 2-point **Angular Correlation Function**

\[ \tilde{C}^{TT}(\theta) \equiv \langle \Theta(\hat{n}_1) \Theta(\hat{n}_2) \rangle \]

\[ \Theta(\hat{n}) = \Theta_{SW} + \Theta_{ISW} \]

\[ \Theta_{SW}(\hat{n}) = -\frac{1}{3} \Phi(\chi^* \hat{n}, \chi^*) \]

\[ \Theta_{ISW}(\hat{n}) = -2 \int_0^{\chi^*} d\chi \dot{\Phi}(\chi \hat{n}, \chi) \]
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\[ C^{TT}(\theta) = \text{Stuff} \times \langle \Phi(\chi^* \hat{n}_1) \Phi(\chi^* \hat{n}_2) \rangle + \int \text{Stuff}' \times \langle \Phi(\chi^* \hat{n}_1) \Phi(\chi^* \hat{n}_2) \rangle + \int \text{Stuff}'' \times \langle \Phi(\chi_1 \hat{n}_1) \Phi(\chi_2 \hat{n}_2) \rangle \]
Starting point: find cross correlation which traces same physics as CMB temperature data

Correlate with the **lensing potential:**

\[
\varphi = 2 \int_0^{\chi_*} d\chi \frac{\chi_* - \chi}{\chi_* \chi} \Phi(\chi \hat{n}, \chi) \\
C^T_{\varphi}(\theta) = \langle \varphi(\hat{n}_1) \Theta_{SW}(\hat{n}_2) \rangle + \langle \varphi(\hat{n}_1) \Theta_{ISW}(\hat{n}_2) \rangle
\]
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**Correlate with the **lensing potential**: **

\[
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\]

\[
C^T\varphi(\theta) = -\frac{2}{3} F(\eta_0 - \chi_1) \int_0^{\chi_*} d\chi_1 F(\eta_0 - \chi_1) \frac{\chi_* - \chi_1}{\chi_* \chi_1} \langle \Phi(\chi_1 \hat{n}_1) \Phi(\chi_* \hat{n}_2) \rangle
\]

\[-4 \int_0^{\chi_*} d\chi_1 \int_0^{\chi_*} d\chi_2 F(\eta_0 - \chi_1) \frac{dF}{d\eta}(\eta_0 - \chi_2) \frac{\chi_* - \chi_1}{\chi_* \chi_1} \langle \Phi(\chi_1 \hat{n}_1) \Phi(\chi_2 \hat{n}_2) \rangle
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If new physics is to blame, we should see it in the **lensing-temperature correlation**
Use this to test the **Null Hypothesis:**

We want to tell whether our Universe is a statistical fluke within LCDM or if it has more interesting physics.
Realizations for LCDM are straightforward... constrained realizations require a little more work.

We want to mimic the lack of large-angle auto correlation in temperature AND have a power spectrum that is consistent with measurements.
Realizations for LCDM are straightforward...

\textit{Constrained} realizations require a little more work.

We want to mimic the lack of large-angle auto correlation in temperature \textbf{AND} have a power spectrum that is consistent with measurements.

- Make realizations of Cls from the measured spectrum
- Draw coefficients from these Cls
- Calculate $S(1/2)$ on a cut sky and compare to experimental bound
- Keep only the realizations which have a smaller $S(1/2)$
- Use constrained temperature realizations to constrain $\phi$

Full detail about constrained realizations can be found in Copi, Huterer, Schwarz, Starkman arXiv:1303.4786
Use the harmonic coefficients from our realizations to calculate an input spectrum that we use here:

\[
C^{T\varphi}(\theta) = \sum_{\ell} \frac{\ell(\ell + 1)}{4\pi} C^{T\varphi}_{\ell} P_{\ell}(\cos \theta)
\]

And then we can calculate this:

\[
S_{1/2}^{T\varphi} = \int_{1/2}^{-1} d(\cos \theta) \ C^{T\varphi}(\theta)^2
\]

In order to get a distribution of the statistic for our model.
Statistic Distributions

WMAP 7 year

- Constrained
- Unconstrained $\Lambda$CDM

Counts

$\log S_{1/2}^{T\phi} (\mu K^2)$

99%  99.9%

1.39e-7

38.3%  % of LCDM above that

Constrained 99% value

WMAP 9 year

- Constrained
- Unconstrained $\Lambda$CDM

Counts

$\log S_{1/2}^{T\phi} (\mu K^2)$

99%  99.9%

1.48e-7

39.6%
Maybe $S(1/2)$ isn't the best choice. We don't know at what angle constrained realizations (for correlations other than TT) will be suppressed.

$$S_{a,b}^{XY} = \int_a^b d(\cos \theta) [C_{XY}^X(\theta)]^2$$

Marginalize over angle to find (a priori) the most definitive statistic between LCDM and constrained realizations.

Optimizing for 99% C.L.

Optimizing for 99.9% C.L.
Optimizing Statistics

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Optimizing for 99% C.L.

Constrained value

\[ \text{1.43e-7} \] % of LCDM above that

\[ \text{40.4\%} \]

Optimizing for 99.9% C.L.

\[ \text{1.50e-7} \] % of LCDM above that

\[ \text{27.8\%} \]
Summary

The lack of correlation at large angles may point to interesting physics

Numerical analysis of constrained LCDM realizations can give us a handle on likelihood of our universe

Can help characterize whether our realization is a statistical fluke
We calculate values for $S(1/2)$ for both LCDM and constrained realizations
Showed that measurement of a large $S(1/2)$ will allow us to rule out null hypothesis at the appropriate confidence level

Statistics can be optimized a priori
There may be a more optimal choice for the statistic
For Temperature and Lensing there is a some improvement over $S(1/2)$

Ongoing work
Checking viability of other cosmological quantities to provide better tests
Provide theoretical prediction for shape of ACF with a length-scale cutoff
An aside: Why a cut sky?

The reported values for $S(1/2)$ are very different from cut sky maps versus reconstructed ILC maps.

If you trust the full-sky reconstruction, ALL of the correlation for angles larger than 60 degrees comes from behind the galaxy.

$S_{1/2}$ on a cut-sky just means $C(\theta)$ is calculated with $\tilde{C}_\ell$. 