Thermal boson stars

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Motivation

- **Boson stars** are self-gravitating configurations of bosons at zero temperature.
- They are usually called **self-gravitating Bose-Einstein condensates**
- This would also mean that:
  - **Self-gravitating configurations** must exist, which would be in **thermal equilibrium**, and that should be able to **condensate at low enough temperatures**.
- Were first studied in the seminal paper by **Ruffini & Bonazzola, PR (1967)**
    - **Relativistic**: Einstein-Klein-Gordon equations (complex field)
    - **Non-relativistic**: Schrödinger-Poisson equations

Boson Stars (Newtonian approach)

- Equations of motion for **equilibrium configurations**:

$$\nabla^2\psi = 2(mU - \epsilon)\psi$$
$$\nabla^2 U = N_0|\psi|^2$$

- Normalization of the wavefunctions: $\int |\psi|^2 dV = 1$
- **Mass density**: $\rho = N_0|\psi|^2$
- **Particle number**: N_0
- **Conservation** of the total particle number: $N_T = N_0$

Thermal Boson Stars (Newtonian approach)

- Equations of motion for **equilibrium configurations**:

$$\nabla^2 \psi_j = 2(mU - \epsilon_j)\psi_j$$

$$\nabla^2 U = \sum_{j=0} N_j |\psi|_j^2$$

Multi- state configurations
Matos & U-L, GRG (2007)
Bernal et al, PRD (2010)
U-L & Bernal, PRD (2010)

- Normalization of the wavefunctions: $\int \psi_j \psi_l^* dV = \delta_{jl}$
- **Mass density**: $\rho = \sum_{j=0} N_j |\psi|_j^2$
- **Particle numbers**: $N_j = \dfrac{1}{e^{\beta(\epsilon_j - \mu)} - 1}$
- **Conservation** of the total particle number: $N_T = \sum_j N_j = \text{const.}$

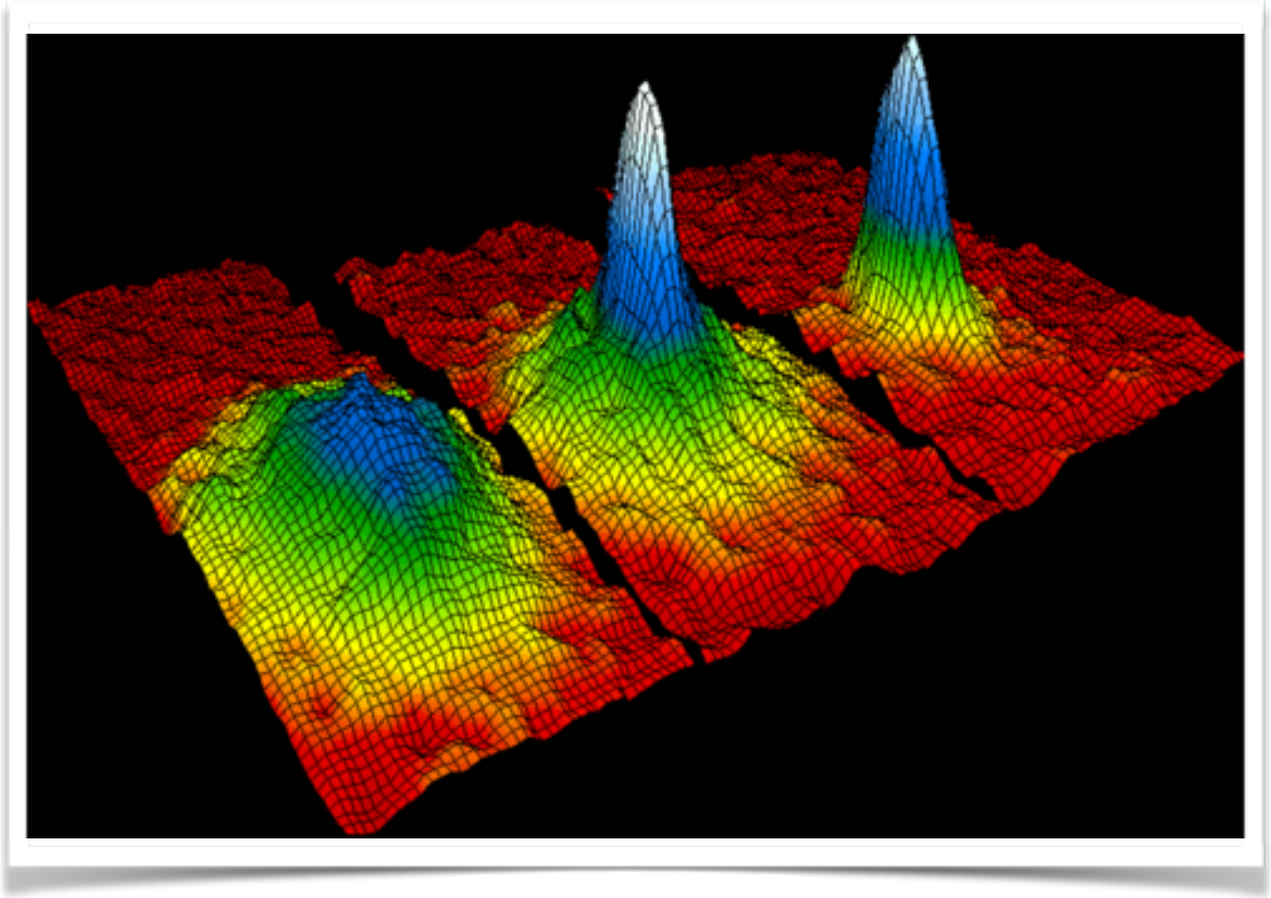
What must we expect?

- All states are initially populated, and their **population is uniquely determined** by the temperature **T**, the total particle number **N**, and the **trapping potential**.

- The **energy levels are fixed** by the trapping potential, and remain fixed all throughout.

- There is a **critical temperature** below which the ground state is **massively populated**.

  - This is exactly the famous **Bose-Einstein condensation**



High T ———-> Low T

BEC, *Wikipedia*

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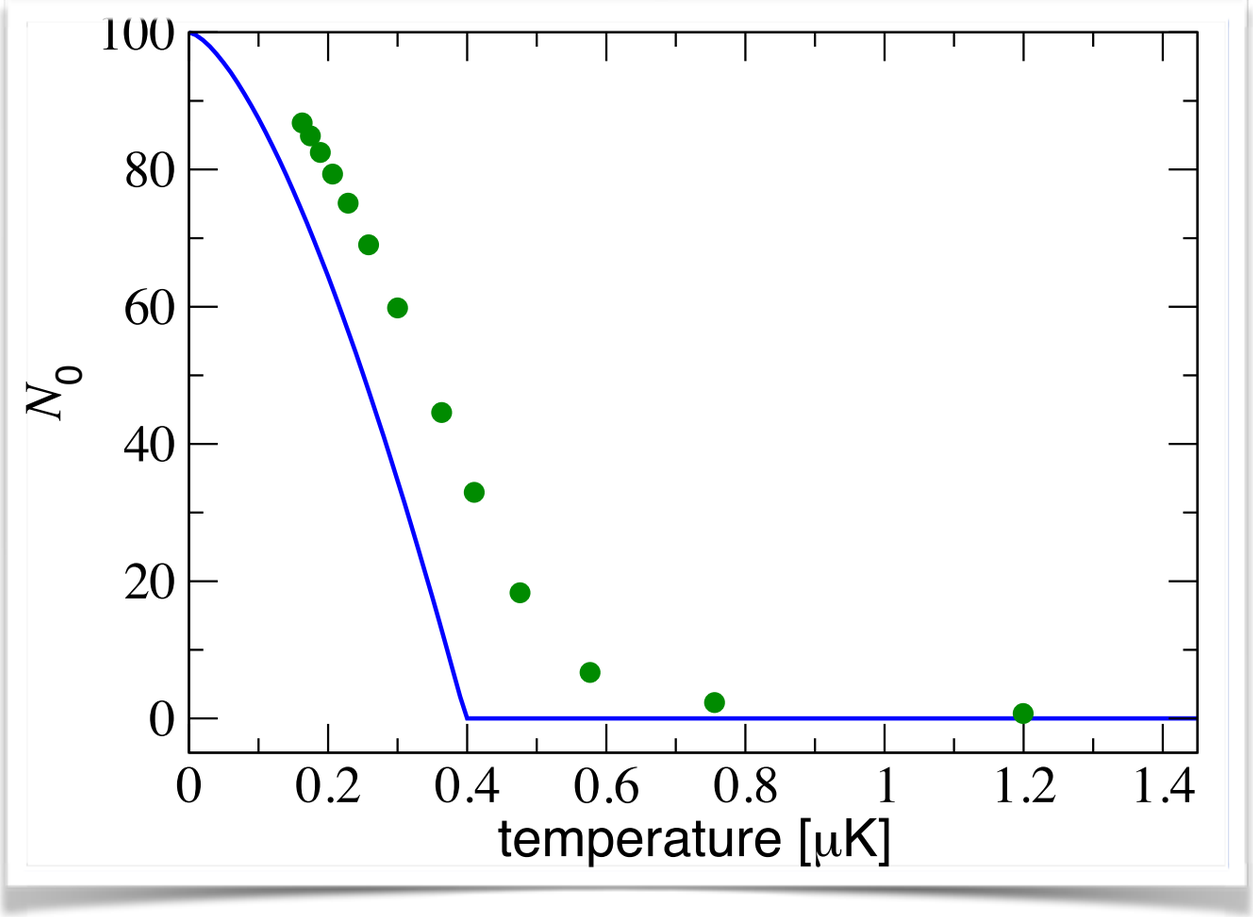


Finite particle number effects vs standard textbook approx.

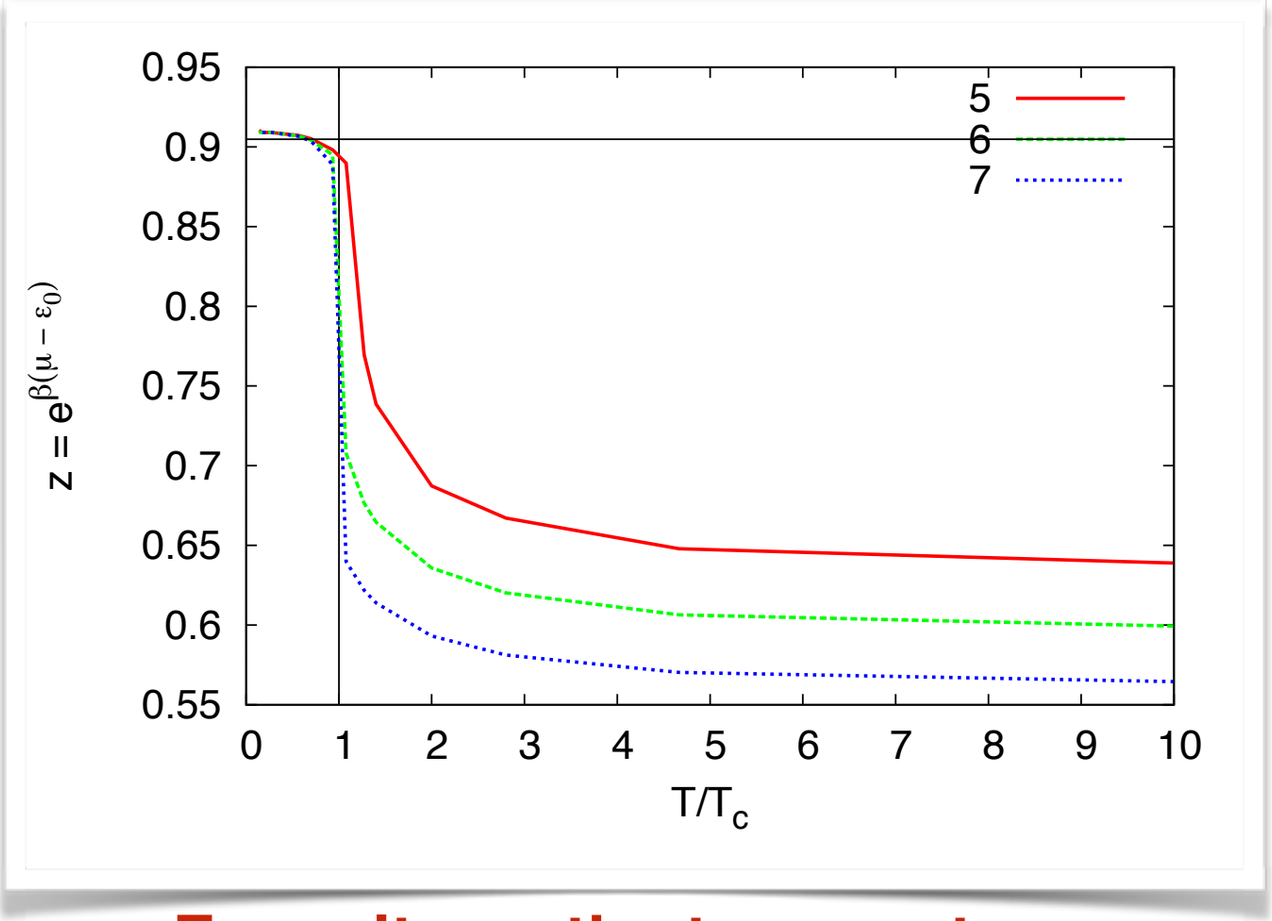
What did we find?

- All states are initially populated, and their population is uniquely determined by the temperature **T**, the total particle number **N**, and the **trapping *gravitational* potential**.

- The **energy levels are *not* fixed** by the trapping gravitational potential, and *do not* remain fixed afterwards.

- There is a **critical temperature** below which the ground state is massively populated.

  - This is exactly the *gravitational* **Bose-Einstein condensation**



Fugacity vs the temperature, for different number of states

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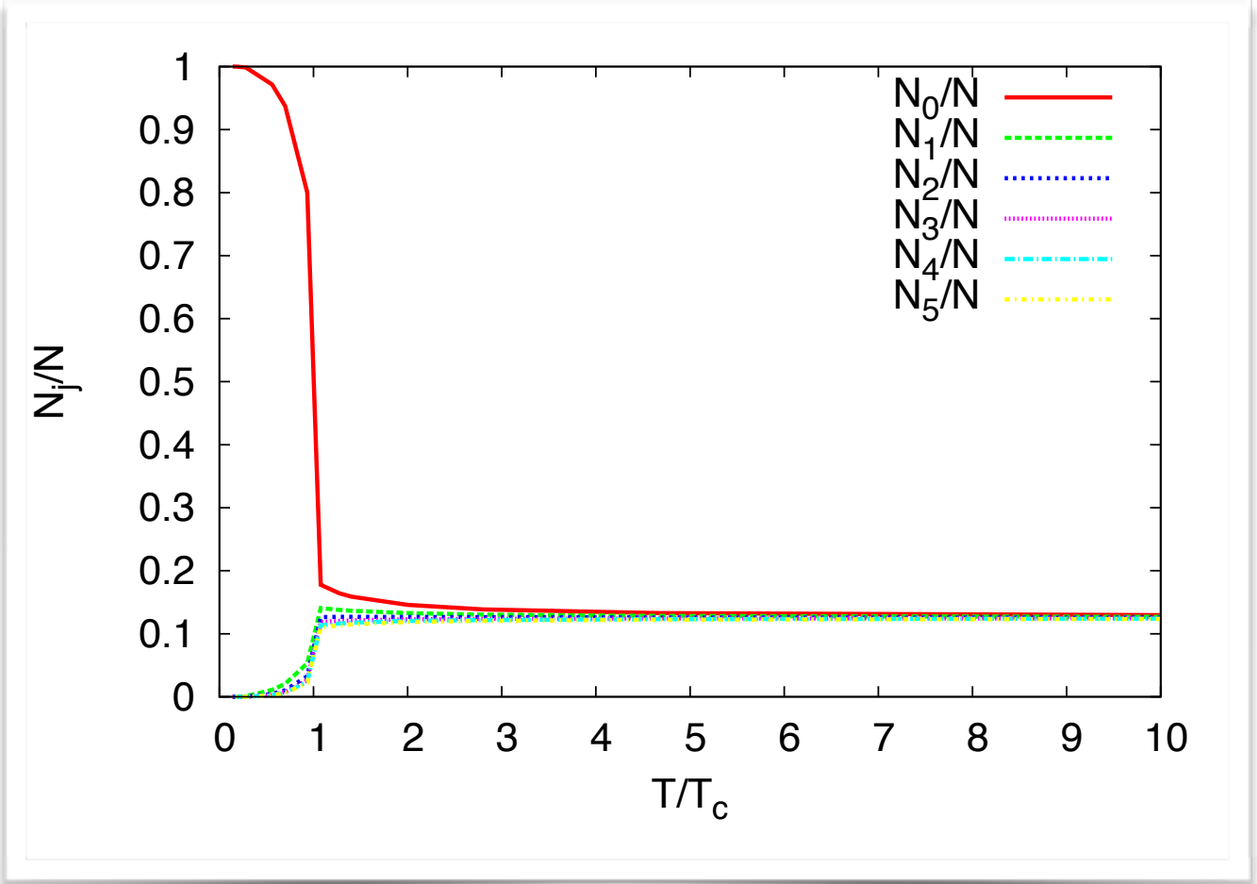

Particle number vs the temperature, for different number of states

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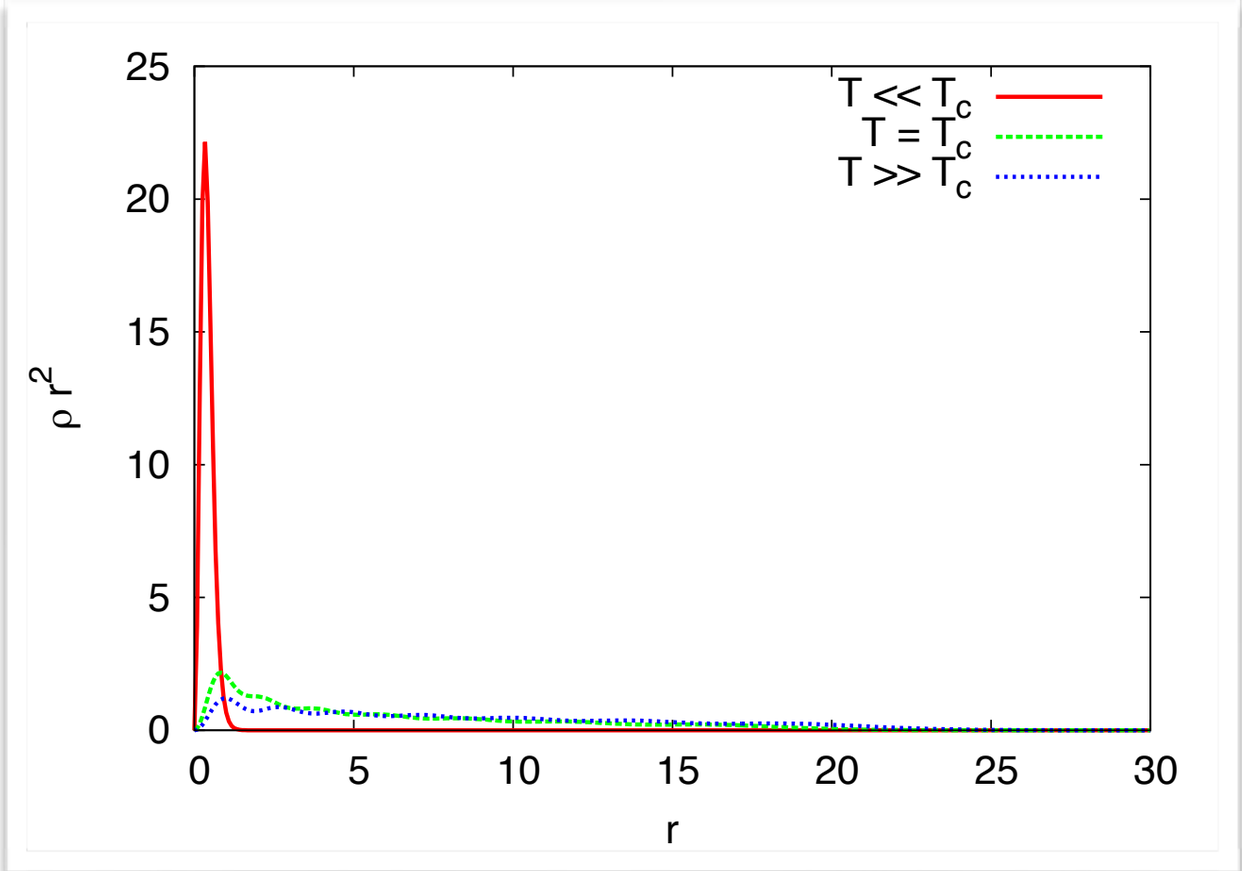


Particle number vs the temperature, for different number of states

Conclusions

- We have found, for the first time, **numerical solutions for the BEC transition of a bunch of self-gravitating bosons**.

- There are strong indications of a **sharp phase transition** for the formation of the BEC.

- Interestingly enough, the **critical temperature** does depend only upon the **total particle number**.

$$k_{\mathrm{B}} T_c' \simeq 7 \times 10^{-3} (m/m_{\mathrm{Pl}})^4 N'^3$$

- There are *indications* that **equilibrium configurations** must be **stable**.

- Leftover: Would it be useful to describe **rotation curves in scalar field galaxy halos**?

Low & High Temperatures

- **Low:**

$$\nabla^2 \psi_j = 2(mU - \epsilon_j)\psi_j$$

$$\nabla^2 U = N|\psi_0|^2$$

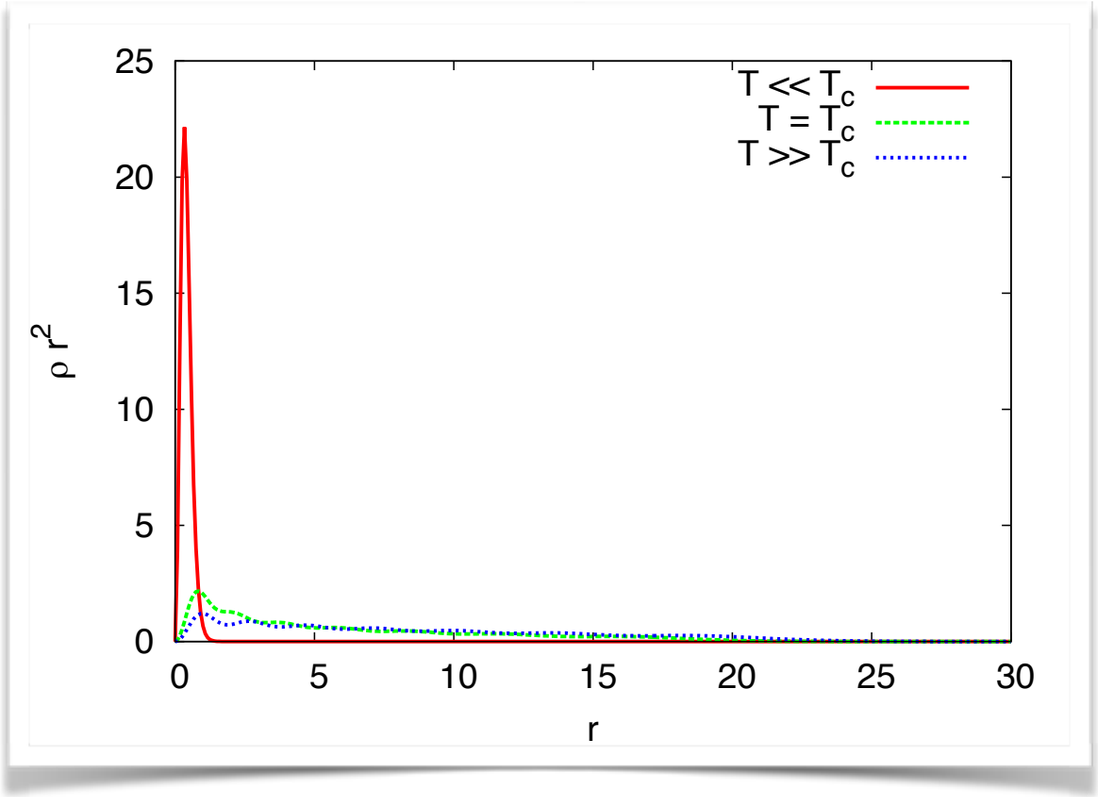
- Chemical potential:

$$\epsilon_0 - \mu \simeq T/N$$

- ***Ground-state*** configuration!

- Scaling properties:

Ground-state boson stars are *stable*!



$$\{r, \psi, U, N, \epsilon_j, T, \mu\} \quad \rightarrow \{\lambda^{-1} r, \lambda^{3/2}\psi, \lambda^2 U, \lambda N, \lambda^2 \epsilon_j, \lambda^3 T, \lambda^2 \mu\}$$

Low & High Temperatures

- **High:**
  $\nabla^2\psi_j = 2(mU - \epsilon_j)\psi_j$

$$\nabla^2 U = (N/j_{max} + 1) \cdot \sum_0^{j_{max}} |\psi_j|^2$$

  - Chemical potential:

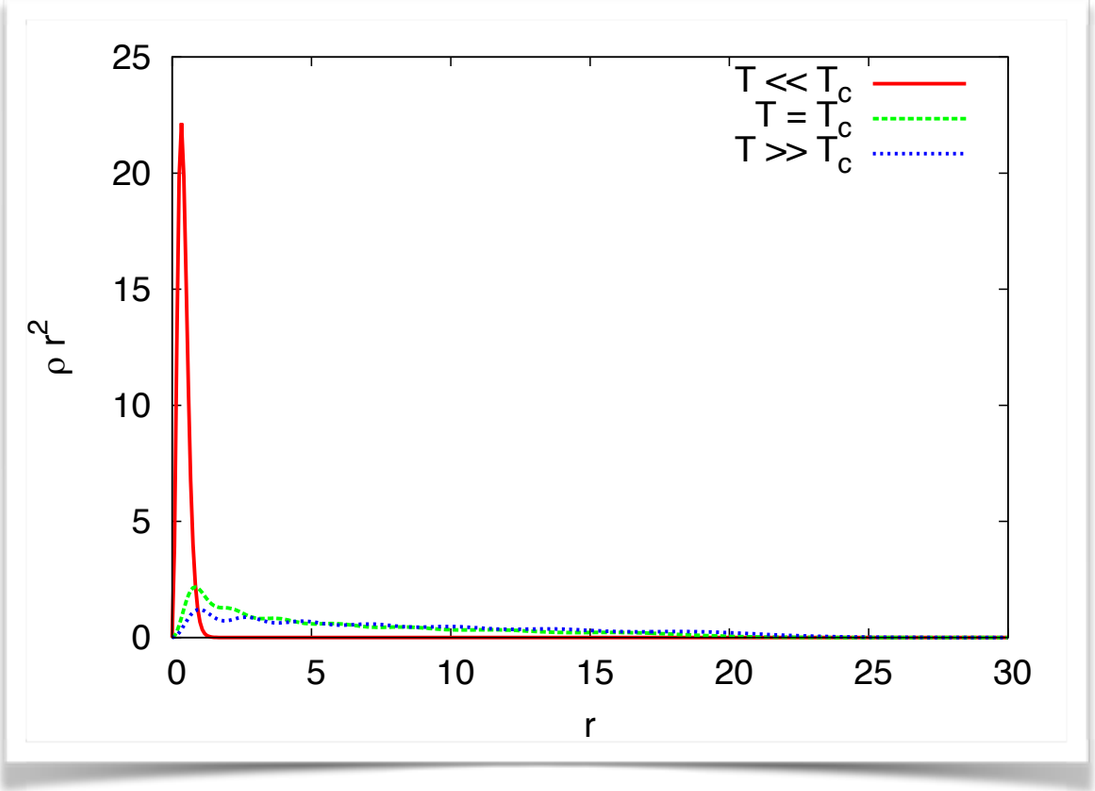
$$\epsilon_0 - \mu \simeq T(j_{max} + 1)/N$$

  - ***Multi-state*** configuration!

  - Scaling properties:

$$\{r, \psi, U, N, \epsilon_j, T, \mu\} \quad \rightarrow \{\lambda^{-1} r, \lambda^{3/2}\psi, \lambda^2 U, \lambda N, \lambda^2 \epsilon_j, \lambda^3 T, \lambda^2 \mu\}$$

Multi-state boson stars are stable if *excited states are less populated* than the *ground state*



What is the critical temperature?

- The **equations of motion** have the **same scaling properties** at **high & low temperatures**.

$$\{r, \psi, U, N, \epsilon_j, T, \mu\} \quad \rightarrow \{\lambda^{-1} r, \lambda^{3/2}\psi, \lambda^2 U, \lambda N, \lambda^2 \epsilon_j, \lambda^3 T, \lambda^2 \mu\}$$

- This means in particular that: $T' = T(N'/N)^3$
- … but this property should also hold for the **critical temperature**. From **our numerical experiments** we find that:

$$k_{\mathrm{B}} T_c' \simeq 7 \times 10^{-3} (m/m_{\mathrm{Pl}})^4 N'^3$$