Dark Energy equation of state: a principal component analysis

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Summary

- Dark energy
- Analysis
- Results
- Conclusions
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- Dark energy
  - general properties
  - time evolution

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  - time evolution
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  - binning strategy
  - dealing with correlations
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Dark Energy

Why we need it?

What is dark energy?

How can we treat it?
Dark Energy

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- $\Omega_{tot} \sim 1$
- $q_0 < 0$

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source: ESA, Planck collaboration
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What is dark energy?
- Cosmological Constant
- Modified Gravity
- Quintessence

How can we treat it?
Cosmological Constant shows issues, known as fine tuning and coincidence

\[ R_{\mu}^{\nu} - \frac{1}{2} g_{\nu}^{\mu} R = 8\pi G T_{\nu}^{\mu} \]
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Modified Gravity Theories:

- large scales modification of gravity;
- no exotic particle but change in gravitational action;
- must reproduce small scales gravity behavior and a matter domination epoch.
Cosmological Constant shows issues, known as fine tuning and coincidence

\[ R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = 8\pi G T^\mu_{\nu} \]

Scalar Field Theories:

- unknown field (or fields) responsible for acceleration;
- evolution driven by a potential (thawing or freezing models);
- requires a mechanism that explains coincidence.
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$$\rho_e = \rho_{e,0} \exp \left[ \int_0^z \frac{3(1+w_e)}{1+z} \, dz \right]$$

- $w_e = \frac{\rho_e}{\rho_{e,0}} < -\frac{1}{3}$ to allow $q_0 < 0$
- $w_e = \frac{\rho_e}{\rho_{e,0}} = -1$ for a cosmological constant $\Lambda$
Binning strategy

\[
\begin{cases}
  w(z) = w_i & z = z_i \\
  w(z) = w_i + \delta w + \delta_w \tanh \left( \frac{\delta z - z}{s} \right) & z \in [z_i, z_{i+1}] \\
  w(z) = -1 & z \geq z_6
\end{cases}
\]

- 6 redshift bins
- equally spaced in \( \ln(a) \)
- between \( z \in [0.0, 2.0] \)

Binning strategy

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\end{align*}
\]

- \( z = z_i \) \quad z = \tilde{z}_i \\
- \( z \in [z_i, z_{i+1}] \) \quad z \geq \tilde{z}_6 \\
- 6 redshift bins
- equally spaced in \( \ln(a) \)
- between \( z \in [0.0, 2.0] \)

We are trying to reconstruct $w_e$ using probes that depend on an integrated value of this quantity:

$$C = \langle \vec{w} \vec{w}^T \rangle - \langle \vec{w} \rangle \langle \vec{w}^T \rangle$$
Dealing with correlations: Principal Component Analysis

$D.~Huterer~and~G.~Starkman~-~Phys.Rev.Lett.90:031301$

\[ C^{-1} = F = O^T \Lambda O \Rightarrow \tilde{q} = \tilde{w} O \]

\[ O' = F^{1/2} = O^T \Lambda^{1/2} O \]
PCA

Dealing with correlations: Principal Component Analysis

D. Huterer and G. Starkman - Phys.Rev.Lett.90:031301

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![Graph showing weights and z values]
Bayesian analysis

We use \texttt{cosmomc} package with PPF \texttt{camb} version to determine pdfs for

\[
\{ \Omega_b h^2, \Omega_c h^2, H_0, n_s, A_s, w_i \}
\]

using a flat prior on \( w_i \in [-3.5; 0.33] \):

95% credible intervals - W7+SNLS+BAO
Results of the analysis

W7+SNIa

95% credible intervals

Results of the analysis

**W7+SNLS+BAO**

- **run1**: SDSS-dr7 at $z=0.20,0.35$ - WiggleZ at $z=0.44,0.60,0.73$
- **run2**: 6dFGRS at $z=0.10$ - WiggleZ at $z=0.44,0.60,0.73$
- **run3**: WiggleZ at $z=0.44,0.60,0.73$
- **run4**: 6dFGRS at $z=0.10$ - SDSS-dr7 at $z=0.20,0.35$ - WiggleZ at $z=0.44,0.60,0.73$

Said et al. - Phys.Rev.D88:043515
Results of the analysis

W7+SNLS+BAO

95% credible intervals

Results of the analysis

W7+SNLS+BAO(run2)+H(z)+H_0

95% credible intervals

A qualitative comparison

Only a qualitative comparison, must check that perturbation evolution is not strongly modified but can be assumed classical

95% credible intervals

Conclusions

- cosmological constant is inside the 95% confidence limit;

- lower values of \( w_e \) seem to be preferred at lower redshift;

- qualitative comparison can limit parameter spaces for different DE models.
Future work

- new datasets for SNIa, BAO and CMB;
- CMB shift parameters to perform quantitative analysis for modified gravity scenarios;
- forecasts of the impact of future sky surveys.


A. Conley et al. - Supernova Constraints and Systematic Uncertainties from the First 3 Years of the Supernova Legacy Survey; ApJS, 192, 1; 2011.


B.A. Reid et al. - The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: measurements of the growth of structure and expansion rate at z=0.57 from anisotropic clustering; arXiv:1203.6641; 2012.


F. Beutler et al. - The 6dF Galaxy Survey: z ≈ 0 measurement of the growth rate and σ8; arXiv:1204.4725; 2012.

M. Moresco et al. - New constraints on cosmological parameters and neutrino properties using the expansion rate of the Universe to z 1.75 - JCAP07 053; 2012.
Cosmological Constant

\[ S = K \int d^4x \sqrt{-g} (R - 2\Lambda) + S_m \]

\[ H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3} \]

\[ w_e = -1 \]
Cosmological Constant

\[ S = K \int d^4x \sqrt{-g} (R - 2\Lambda) + S_m \]

\[ H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3} \]

\[ w_e = -1 \]

Problems

- **fine-tuning** → \( \rho_\Lambda \sim 10^{-47} \text{GeV}^4 \ll \rho_{\text{vac}} \sim 10^{74} \text{GeV}^4 \)
- **coincidence** → \( \mathcal{O}(\rho_\Lambda) = \mathcal{O}(\rho_m) \)
Quintessence

**THAWING**

\[ V(\phi) \propto V_0 + \alpha \phi^n \]

**FREEZING**

\[ V(\phi) \propto \alpha \phi^{-n} \]

**Quintessence**

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**THAWING**

\[ V(\phi) \propto V_0 + \alpha \phi^n \]

**FREEZING**

\[ V(\phi) \propto \alpha \phi^{-n} \]

\[
\begin{align*}
P_e &= \frac{1}{2} \dot{\phi}^2 - V(\phi) \\
\rho_e &= \frac{1}{2} \dot{\phi}^2 + V(\phi) \\
W_e &= \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}
\end{align*}
\]

Modified Gravity

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f(R), f(T), f(G) MODELS

HIGHER DIMENSIONS MODELS

Linder, 2004; Cognola, 2006; Ferraro, 2011; Rubin, 2009; Deffayet, 2002; Dvali, 2000.
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f(R), f(T), f(G) MODELS

HIGHER DIMENSIONS MODELS

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(X) + S_m \]

\[ S = \frac{\tilde{M}^3}{\tilde{G}} \int d^5x \sqrt{-\tilde{g}} \tilde{R} + \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g} R \]

\[ W_e = W_{\text{eff}} \]

Linder, 2004; Cognola, 2006; Ferraro, 2011; Rubin, 2009; Deffayet, 2002; Dvali, 2000.
Supernovae Ia

SN Ia are defined as standard candles $\rightarrow$ recalibrating the light-curves leads to a $L_{max} = cost$
Supernovae Ia

SN Ia are defined as standard candles → recalibrating the light-curves leads to a $L_{\text{max}} = \text{cost}$

$$d_L = \sqrt{\frac{L_{\text{max}}}{4\pi\Phi}}$$

$$d_L(z) = a_0(1+z)f_K \left( \int_0^z \frac{c}{a_0 H_0} \frac{dz'}{E(z')} \right)$$

$$E = \frac{H(z)}{H_0} = \left[ \Omega_{r,0}a^{-4} + \Omega_{m,0}a^{-3} + \Omega_{e,0} \left( \int_0^1 -3(1+w_e) \frac{da}{a} \right) + \Omega_k a^{-2} \right]^{1/2}$$
Barionic Acoustic Oscillations

BAO are defined as standard rulers → The 2-points correlation function shows a peak at a comoving separation equal to the sound horizon.
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\[ d_A = \frac{x}{\theta} \]

\[ d_A(z) = \frac{a_0}{(1+z)^2} f_K \left( \int_0^z \frac{c}{a_0 H_0} \frac{dz'}{E(z')} \right) \]

\[ E = \frac{H(z)}{H_0} = \left[ \Omega_{r,0} a^{-4} + \Omega_{m,0} a^{-3} + \Omega_{\text{e,0}} a \int_0^1 -3(1+w_e) \frac{da}{a} + \Omega_k a^{-2} \right]^{1/2} \]
Cosmic Microwave Background

CMB shows temperature anisotropies → linked to the length of density perturbation at decoupling
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\[
\delta(\vec{k}) = A \int d^3 r \delta(\vec{r}) \exp[-i \vec{k} \cdot \vec{r}]
\]

\[
\delta(\vec{r}) = \frac{\delta \rho(\vec{r})}{\rho}.
\]

\[
P(k) = \int d^3 r \xi(r) \exp[-i \vec{k} \cdot \vec{r}]
\]

\[
C_l = \frac{1}{(2l+1)} \sum_{m=-l}^{l} |a_{lm}|^2
\]
Other probes: Cosmic Chronometers

- expansion rate from differential evolution of massive early-type galaxies;
- bias from degenerate parameters reduced by the differential quantity;
- different methods to estimate age for galaxies.

Moresco, 2012.
In DE comoving coordinate system

\[
\begin{align*}
\delta \rho' &= \delta \rho_e + 3 \rho_e \frac{u_e}{k_H} \\
\delta p' &= \delta p_e + 3 \frac{p'_e}{p_e} \rho_e \frac{u_e}{k_H}
\end{align*}
\]

\((') = \frac{d}{d \ln(a)}\)

\(k_{ij} = k / aH\)

Fang, 2008.
Crossing the Phantom Divide

In DE comoving coordinate system, the conservation of momentum \((\rho_\text{e} \vec{u}_\text{e}) = T^0_i\) yields

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\begin{align*}
\delta \rho' &= \delta \rho_\text{e} + 3 \rho_\text{e} \frac{u_\text{e}}{k_H} \\
\delta p' &= \delta p_\text{e} + 3 \frac{p'_\text{e}}{\rho_\text{e}} \rho_\text{e} \frac{u_\text{e}}{k_H}
\end{align*}
\]

\[u'_\text{e} = 3 \left( w_\text{e} + c_s^2 - \frac{p'_\text{e}}{\rho'_\text{e}} - \frac{1}{3} \right) u_\text{e} + k_H c_s^2 \delta_\text{e} + (1 + w_\text{e}) k_H A\]

\[\left( ' \right) = \frac{d}{d \ln(a)}\]

\[k_{ii} = k / aH\]

\[A = \text{grav pot a.g.}\]

\[c_s^2 = \frac{\delta p'}{\delta \rho_\text{e}}\]

\[\delta_\text{e} = \frac{\delta \rho_\text{e}}{\rho_\text{e}}\]

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$$\left\{ \begin{array}{ll} \delta \rho' = \delta \rho_e + 3 \rho_e \frac{u_e}{k_H} \\ \delta p' = \delta p_e + 3 \frac{p'_e}{\rho_e} \rho_e \frac{u_e}{k_H} \end{array} \right. \quad (') = \frac{d}{\ln(a)}$$

$$k_{ii} = k/aH$$

$$A = \text{grav pot a.g.}$$

$$c_s^2 = \frac{\delta p'}{\delta \rho_e}$$

$$\delta_e = \frac{\delta \rho_e}{\rho_e}$$

$$B = \text{space time piece } \delta g_{\mu \nu} \text{ a.g.}$$

$$H_L = \text{space space piece } \delta g_{\mu \nu} \text{ a.g.}$$

$$c_K = 1 - 3K/k^2$$

$$u'_e = 3 \left( w_e + c_s^2 - \frac{p'_e}{\rho_e} - \frac{1}{3} \right) u_e + k_H c_s^2 \delta_e + (1 + w_e) k_H A$$

$$\delta'_e + 3(c_s^2 - w_e) \delta_e + 9 \left( c_s^2 - \frac{p'_e}{\rho_e} \right) \frac{u_e}{k_H} = k_H u_e - (1 + w_e)(k_H B + 3H_L')$$

$$c_K k^2 \Phi = 4\pi G \alpha^2 \sum_l \rho_l \delta_l'$$

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\[c_k k^2 \Phi = 4 \pi G a^2 \sum \rho_i \delta_i'\]

\[W_e = -1 \Rightarrow \frac{p'_e}{\rho'_e} \to \infty\]

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\[
\begin{aligned}
u'_e &= 3 \left( w_e + c_s^2 - \frac{p'_e}{\rho'_e} - \frac{1}{3} \right) u_e + kH c_s^2 \delta e + (1 + w_e) kH A \\
\delta'_e + 3(c_s^2 - w_e) \delta e + 9 \left( c_s^2 - \frac{p'_e}{\rho'_e} \right) \frac{u_e}{kH} &= kH u_e - (1 + w_e)(kH B + 3 H'_L) \\
c_K k^2 \Phi &= 4\pi G a^2 \sum_i \rho_i \delta'_i
\end{aligned}
\]

\[
W_e = -1 \Rightarrow \frac{p'_e}{\rho'_e} \rightarrow \infty \Rightarrow \delta_e \text{ diverges if } c_s^2 = \text{cost}
\]

Fang, 2008.
PPF Prescription

$\sigma_s^2$ can be variable in multifield theory
$c_s^2$ can be variable in multifield theory $w(\alpha)$ is difficult to compute in these conditions.
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This can be solved working in the PPF framework

- momentum and density components within a unified dynamical variable
  \[ \Gamma = \frac{4\pi G\alpha^2}{c_k k^2} \rho_x (\delta_x + 3 u_x / k_H) - \Phi \]

- first closure relation makes the anisotropic stress vanish

- second closure relation relates matter and dark energy momentum densities, making the latter be smooth under a given transition scale

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Principal Component Analysis

This methodology is called PCA, and we will follow the procedure shown by *Huterer&Starkman-2008*:
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- compute Fisher matrix $F = C^{-1}$;
- find eigenvalues and eigenvectors for $F$, so that $F = O^T \Lambda O$, with $\Lambda$ diagonal;
- the uncorrelated parameter are now $\tilde{q} = \tilde{w}O$ (principal components) and the row of $O$ are the weights that tells us how the $q$’s relate to $w$’s;
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- the uncorrelated parameter are now $\vec{q} = \vec{w} O$ (principal components) and the row of $O$ are the weights that tells us how the $q$’s relate to $w$’s;
- we chose another weight matrix here, $W^T W = F$, as to say $W = F^{1/2}$, because it ends up with weights mostly positive, and we normalize every row to unity;
- we can compute this weight matrix as $W = O^T \Lambda^{1/2} O$. 
CMB shift parameters

\[ R = \sqrt{\Omega_m H_0^2} r(z_*) / c \]

\[ l_\alpha = \pi r(z_*) / r_s(z_*) \]

\[ \theta = 100 r_s(z_*) / D_A(z_*) \]

where:

- \( r(z_*) \rightarrow \) comoving distance to photon-decoupling surface
- \( r_s(z_*) \rightarrow \) comoving sound horizon at photon-decoupling epoch
- \( D_A(z_*) \rightarrow \) angular diameter distance to the photon-decoupling surface

Wang & Mukherjee, 2007