Dark Energy
Why
What
Binning w
Binning strategy
Correlations
Analysis
Results
Conclusions

Dark Energy equation of state:

a principal component analysis

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16 January 2014

Summary

- Dark energy
- Analysis
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  - general properties
  - time evolution
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Dark Energy

Why we need it?

What is dark energy?

How can we treat it?

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- $\Omega_{tot} \sim 1$
- $q_0 < 0$

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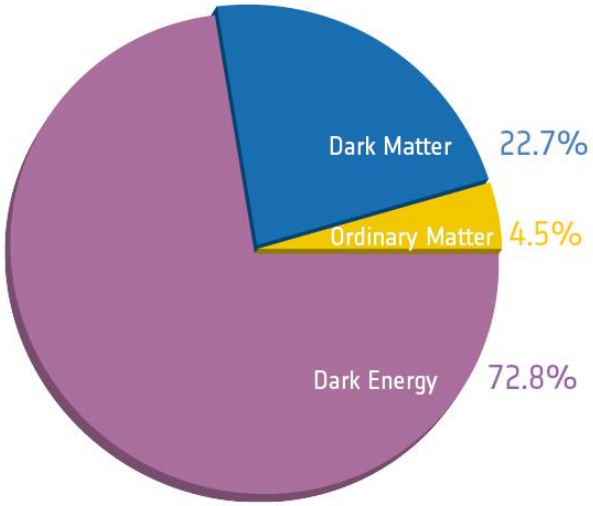
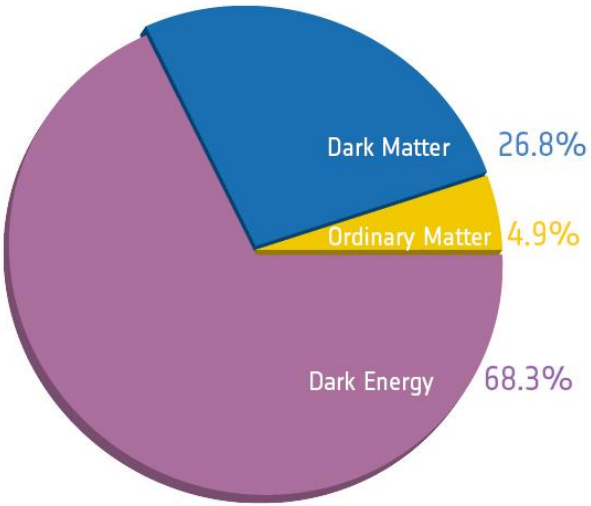
Why



Before Planck



After Planck

source: ESA, Planck collaboration

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- Modified Gravity
- Quintessence

How can we treat it?

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Cosmological Constant shows issues, known as *fine tuning* and *coincidence*

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Modified Gravity Theories:

- large scales modification of gravity;
- no exotic particle but change in gravitational action;
- must reproduce small scales gravity behavior and a matter domination epoch.

What

Cosmological Constant shows issues, known as *fine tuning* and *coincidence*

$$R^{\mu}_{\nu} - \frac{1}{2} g^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu}$$

Scalar Field Theories:

- unknown field (or fields) responsible for acceleration;
- evolution driven by a potential (thawing or freezing models);
- requires a mechanism that explain coincidence.

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$$\boxed{\rho_e = \rho_{e,0} \ \exp\left[\int_0^z \frac{3(1+w_e)}{1+z}\, dz\right]}$$

- $w_e = \frac{P_e}{\rho_e} < -\frac{1}{3}$ to allow $q_0 < 0$
- $w_e = \frac{P_e}{\rho_e} = -1$ for a cosmological constant Λ

Binning strategy

$$\begin{cases} w(z) = w_i & z = z_i \\ w(z) = w_i + \delta_w + \delta_w \tanh\left(\frac{\delta_z - z}{s}\right) & z \in [z_i, z_{i+1}] \\ w(z) = -1 & z \geq z_6 \end{cases}$$

- 6 redshift bins
- equally spaced in $\ln(a)$
- between $z \in [0.0, 2.0]$

D. Huterer and A. Cooray - Phys.Rev.D71:023506; P. Serra et al.: Phys.Rev.D80:121302

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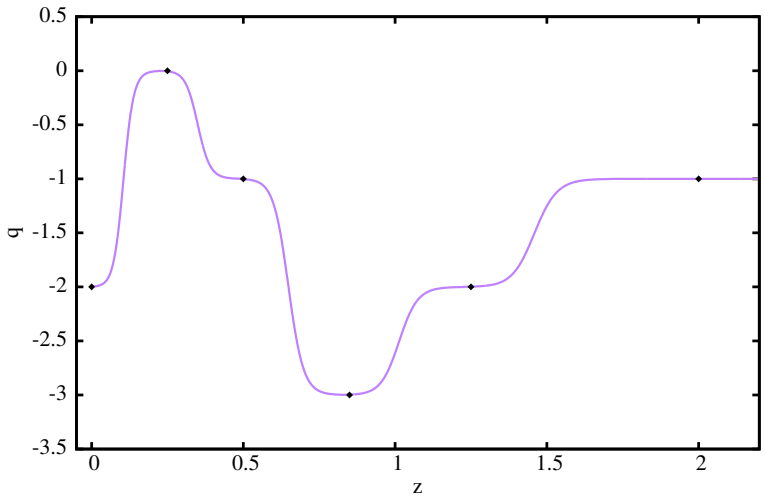
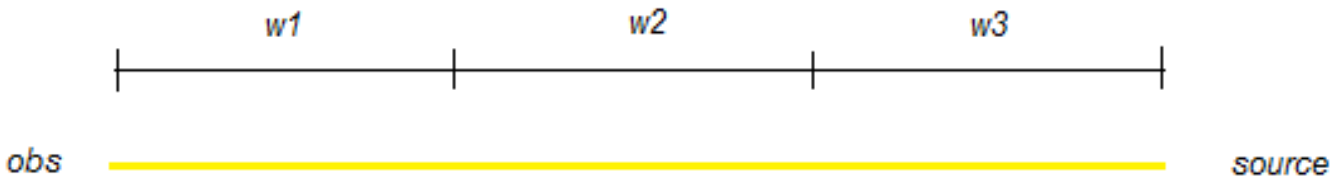


Correlations

We are trying to reconstruct w_e using probes that depend from an integrated value of this quantity:



this implies that the covariance matrix between different values of w_e is not diagonal:

$$C = < \vec{w}\vec{w}^T > - < \vec{w} >< \vec{w}^T >$$

Dark Energy equation of state:

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Dealing with correlations: Principal Component Analysis

D. Huterer and G. Starkman - Phys.Rev.Lett.90:031301

$$C^{-1} = F = O^T \Lambda O \Rightarrow \vec{q} = \vec{w}\, O$$

$$\boxed{O' = F^{\frac{1}{2}} = O^T \Lambda^{\frac{1}{2}} O}$$

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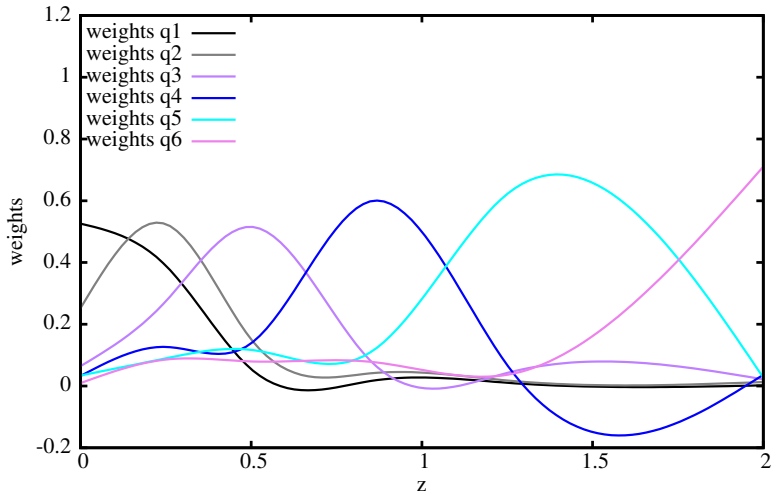
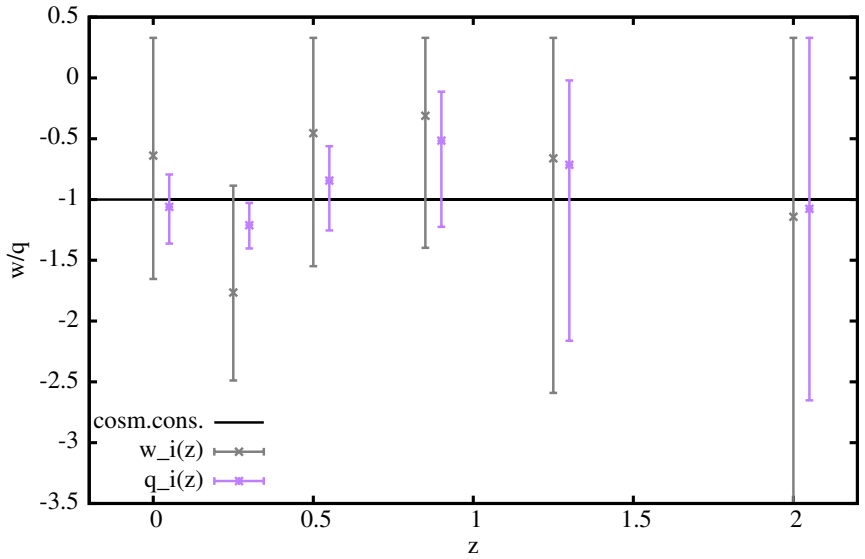


Bayesian analysis

We use `cosmomc` package with PPF `camb` version to determine pdfs for

$$\{\Omega_b h^2, \Omega_c h^2, H_0, n_s, A_s, w_i\}$$

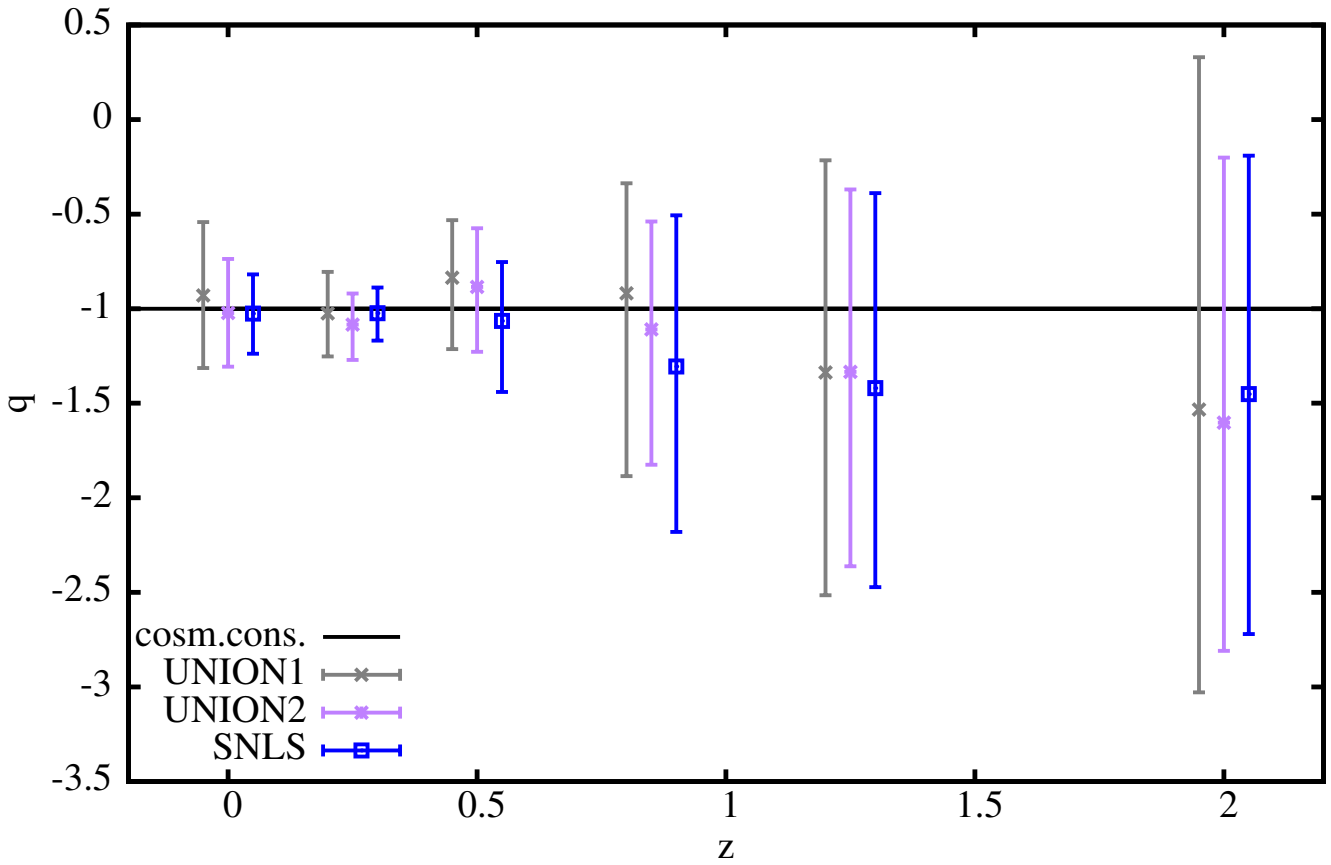
using a flat prior on $w_i \in [-3.5; 0.33]$:



95% credible intervals - W7+SNLS+BAO

Results of the analysis

W7+SNIa



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Said et al.-Phys.Rev.D88:043515

Results of the analysis

W7+SNLS+BAO

run1= SDSS-dr7 at z=0.20,0.35 - WiggleZ at z=0.44,0.60,0.73

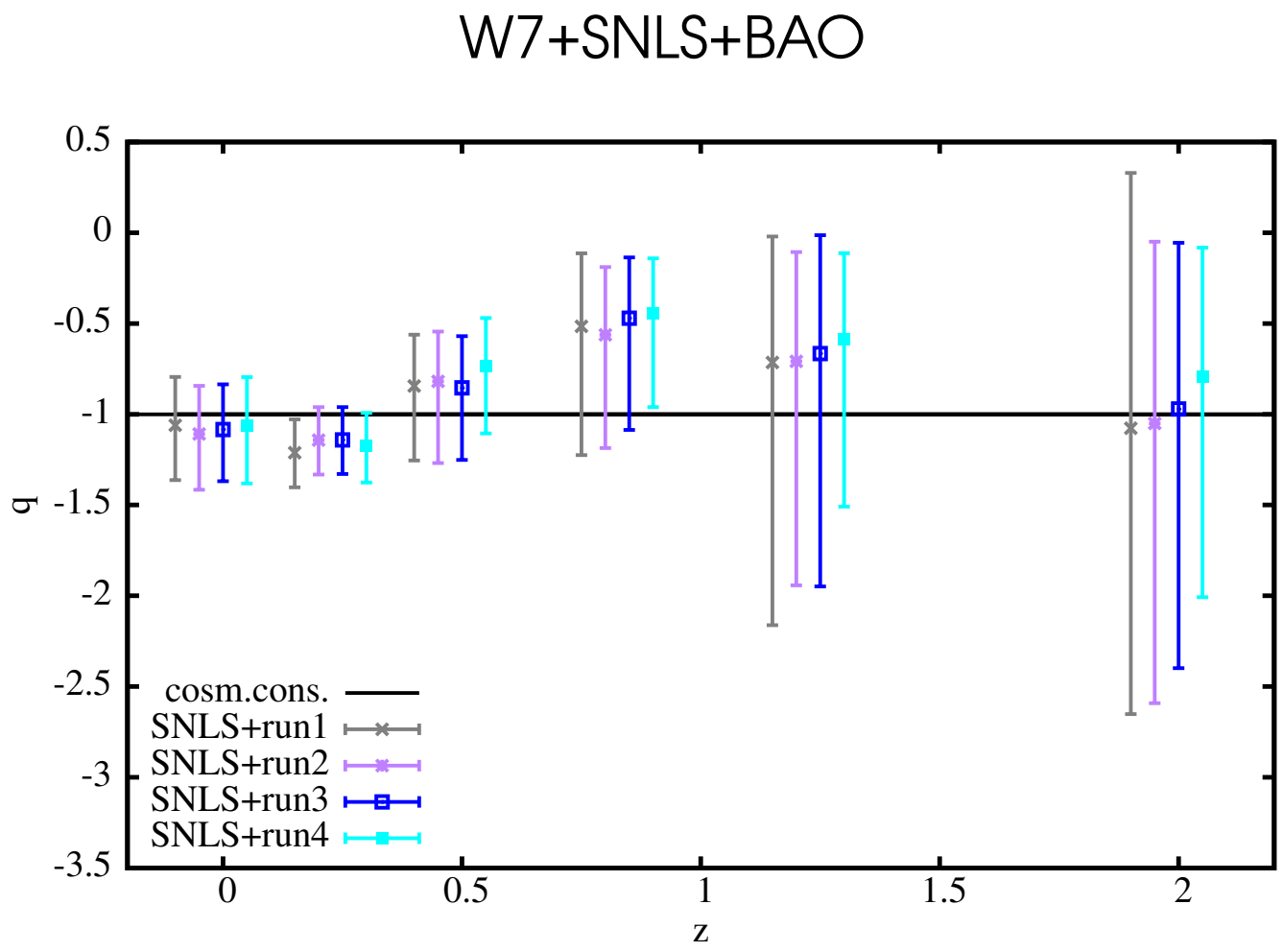
run2= 6dFGRS at z=0.10 - WiggleZ at z=0.44,0.60,0.73

run3= WiggleZ at z=0.44,0.60,0.73

run4= 6dFGRS at z=0.10 - SDSS-dr7 at z=0.20,0.35 - WiggleZ at z=0.44,0.60,0.73

Said et al.-Phys.Rev.D88:043515

Results of the analysis

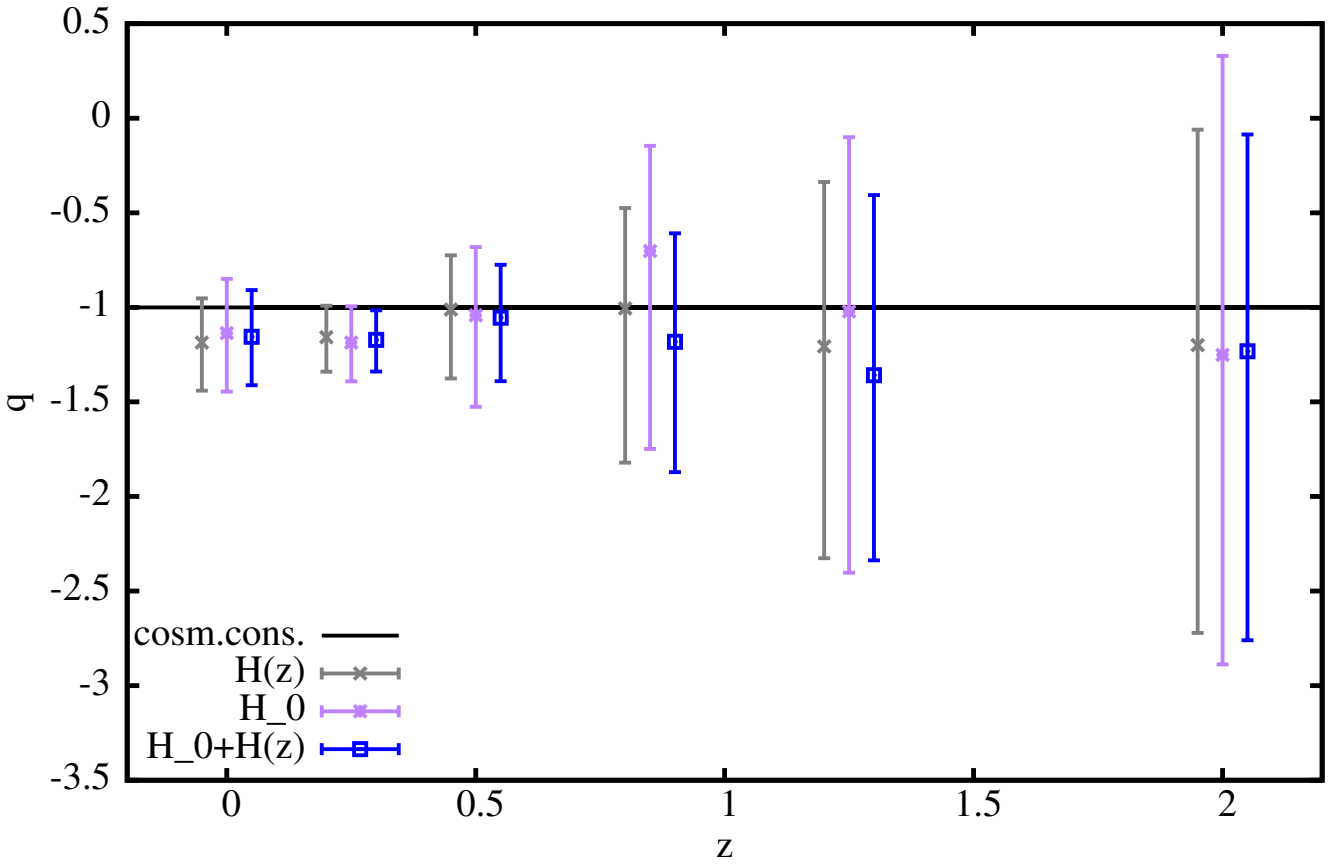


95% credible intervals

Said et al.-Phys.Rev.D88:043515

Results of the analysis

W7+SNLS+BAO(run2)+H(z)+H_0



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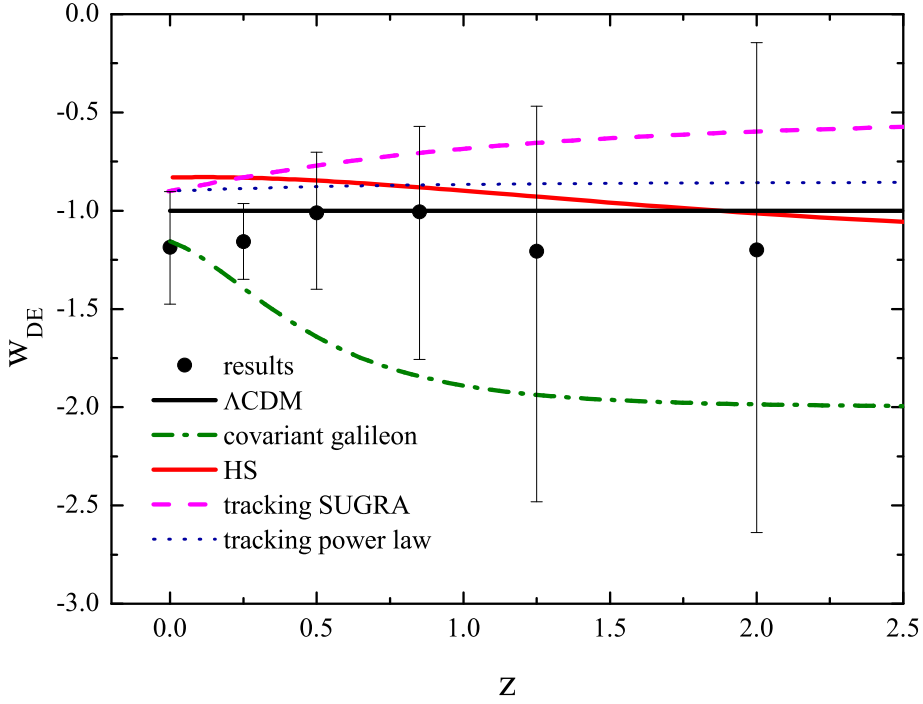
A qualitative comparison



Only a qualitative comparison, must check that perturbation evolution is not strongly modified but can be assumed classical

95% credible intervals

Said et al.-Phys.Rev.D88:043515

Conclusions

- cosmological constant is inside the 95% confidence limit;
- lower values of w_e seem to be preferred at lower redshift;
- qualitative comparison can limit parameter spaces for different DE models.

Future work

- new datasets for SNIa, BAO and CMB;
- CMB shift parameters to perform quantitative analysis for modified gravity scenarios;
- forecasts of the impact of future sky surveys.

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Cosmological Constant

$$S = K \int d^4x \sqrt{-g}(R - 2\Lambda) + S_m$$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\boxed{w_e = -1}$$

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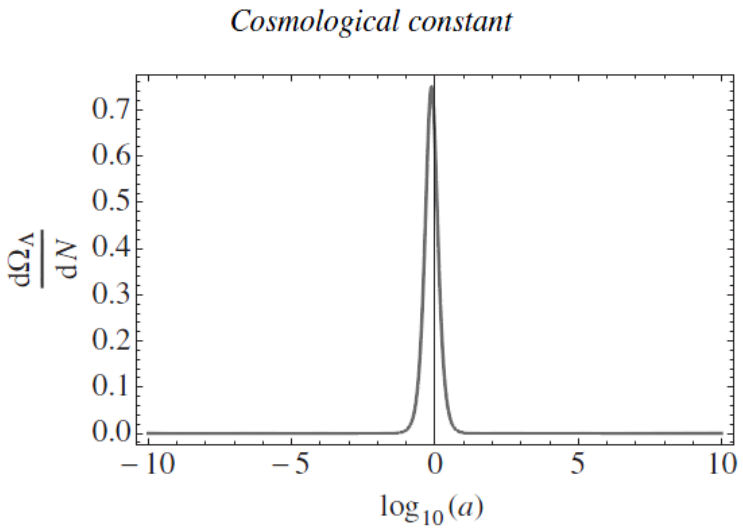
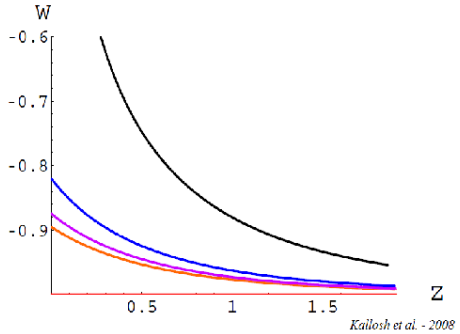


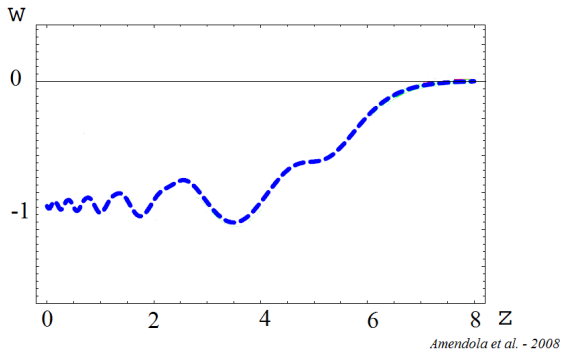
Amendola&Tsujikawa, 2010.

Problems

- *fine-tuning*→ $\rho_\Lambda \sim 10^{-47} GeV^4 \ll \rho_{vac} \sim 10^{74} GeV^4$
- *coincidence*→ $\mathcal{O}(\rho_\Lambda) = \mathcal{O}(\rho_m)$

Quintessence





THAWING

$$V(\phi) \propto V_0 + \alpha\phi^n$$

FREEZING

$$V(\phi) \propto \alpha\phi^{-n}$$

Kallosh, 2003; Linder, 2003; Linder, 2008; Doran&Robbers, 2006; Wetterich, 2007; Amendola, 2008.

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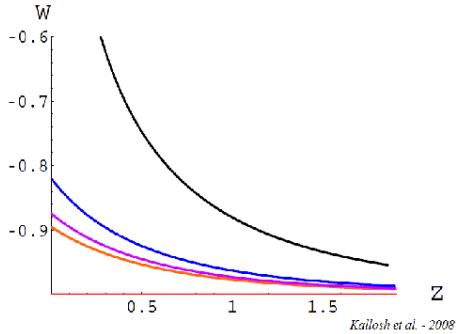
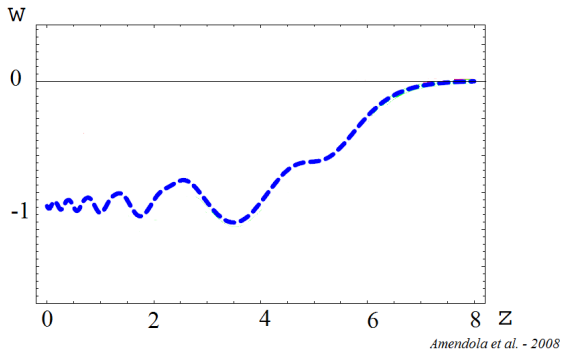




THAWING

$$V(\phi) \propto V_0 + \alpha\phi^n$$

FREEZING

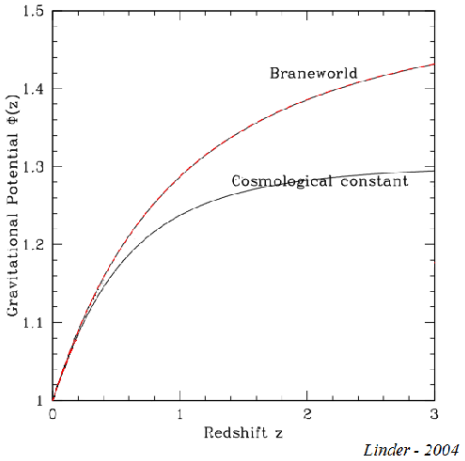
$$V(\phi) \propto \alpha\phi^{-n}$$

$$P_e = \frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad \rho_e = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$\boxed{w_e = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}}$$

Kallosh, 2003; Linder, 2003; Linder, 2008; Doran&Robbers, 2006; Wetterich, 2007; Amendola, 2008.

Modified Gravity

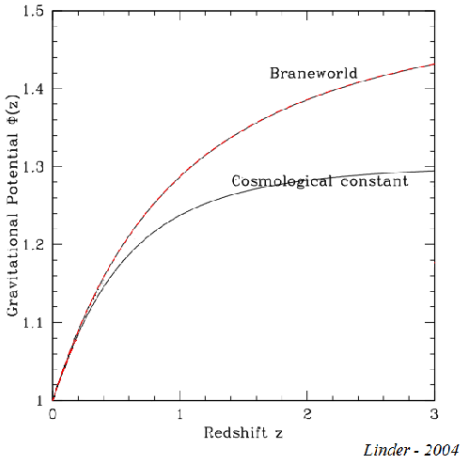


f(R), f(T), f(G) MODELS

HIGHER DIMENSIONS MODELS

Linder, 2004; Cognola, 2006; Ferraro, 2011; Rubin, 2009; Deffayet, 2002; Dvali, 2000.

Modified Gravity



f(R), f(T), f(G) MODELS

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(X) + S_m$$

HIGHER DIMENSIONS MODELS

$$S = \frac{\tilde{M}^3}{2}\int d^5x\sqrt{-\tilde{g}}\tilde{R} + \frac{M_{pl}^2}{2}\int d^4x\sqrt{-g}R$$

$$\boxed{w_e = w_{eff}}$$

Linder, 2004; Cognola, 2006; Ferraro, 2011; Rubin, 2009; Deffayet, 2002; Dvali, 2000.

Supernovae Ia

SNIa are defined as standard candles $\rightarrow$ recalibrating the light-curves leads to a $L_{max} = cost$

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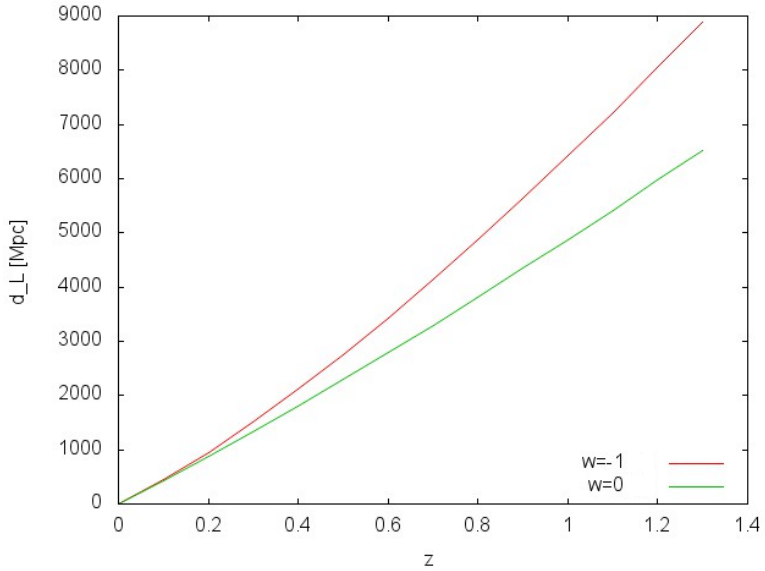
SNIa are defined as standard candles → recalibrating the light-curves leads to a $L_{max} = cost$

$$d_L = \sqrt{\frac{L_{max}}{4\pi\Phi}}$$

$$d_L(z) = a_0(1+z) f_K\left(\int_0^z \frac{c}{a_0 H_0}\frac{dz'}{E(z')}\right)$$

$$E = \frac{H(z)}{H_0} = \left[\Omega_{r,0}a^{-4} + \Omega_{m,0}a^{-3} + \Omega_{e,0}\, e^{\int_0^1 -3(1+w_e)\frac{da}{a}} + \Omega_k a^{-2}\right]^{1/2}$$



Barionic Acoustic Oscillations

BAO are defined as standard rulers $\rightarrow$ The 2-points correlation function shows a peak at a comoving separation equal to the sound horizon.

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$$d_A = \frac{x}{\theta}$$

$$d_A(z) = \frac{a_0}{(1+z)^2} f_K\left(\int_0^z \frac{c}{a_0 H_0} \frac{dz'}{E(z')} \right)$$

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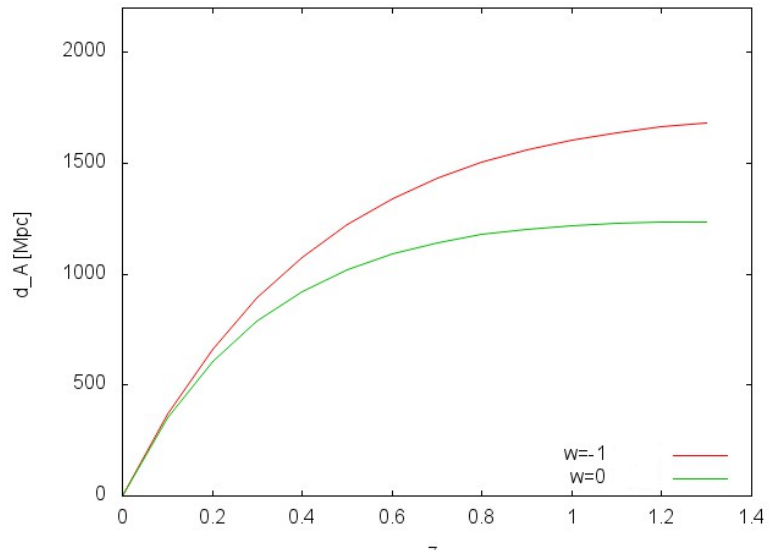

Cosmic Microwave Background

CMB shows temperature anisotropies $\rightarrow$ linked to the length of density perturbation at decoupling

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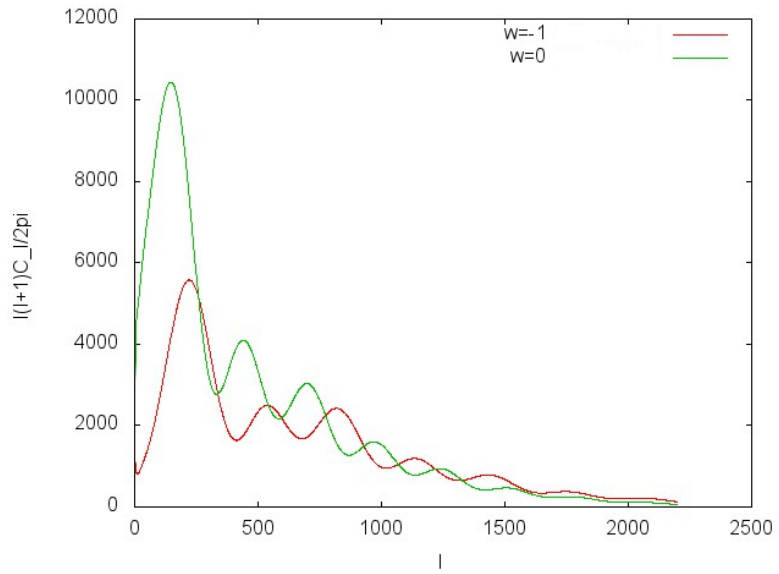
CMB shows temperature anisotropies → linked to the length of density perturbation at decoupling

$$\delta(\vec{k}) = A \int d^3 r \delta(\vec{r}) exp[-i\vec{k} \cdot \vec{r}]$$

$$\delta(\vec{r}) = \frac{\delta\rho(\vec{r})}{\rho}$$

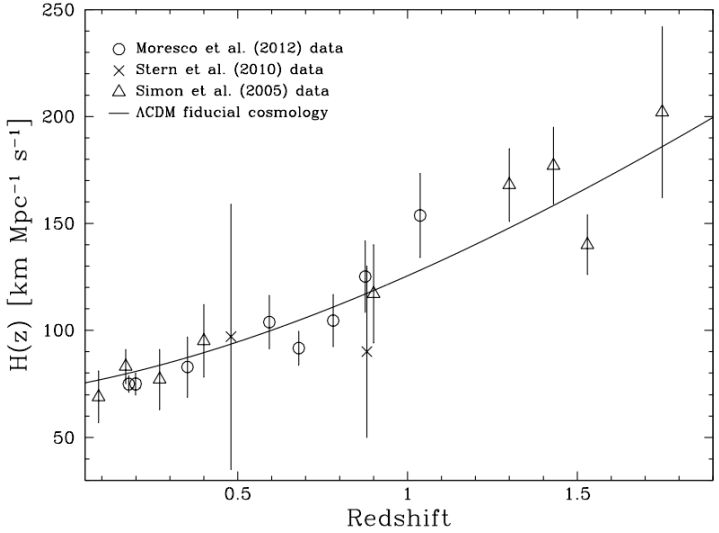
$$P(k) = \int d^3 r \xi(r) exp[-i\vec{k} \cdot \vec{r}]$$

$$C_l = \frac{1}{(2l+1)} \sum_{m=-l}^{l} |a_{lm}|^2$$



Other probes: Cosmic Chronometers

State of art



Moresco, 2012.

- expansion rate from differential evolution of massive early-type galaxies;
- bias from degenerate parameters reduced by the differential quantity;
- different methods to estimate age for galaxies.

Crossing the Phantom Divide

In DE comoving coordinate system

$$\begin{cases} \delta\rho^r = \delta\rho_e + 3\rho_e \frac{u_e}{k_H} \\ \delta p^r = \delta p_e + 3\frac{p'_e}{\rho'_e}\rho_e \frac{u_e}{k_H} \end{cases}$$

$(') = d/dln(a)$

$k_H = k/aH$

Fang, 2008.

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In DE comoving coordinate system, the conservation of momentum $(\rho_e \vec{u}_e) = T_i^0$ yelds

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$k_H = k/aH$

$A =$ grav pot a.g.

$c_s^2 = \frac{\delta p_e^r}{\delta \rho_e^r}$

$\delta_e = \frac{\delta \rho_e}{\rho_e}$

$$u'_e = 3\left(w_e + c_s^2 - \frac{p'_e}{\rho'_e} - \frac{1}{3}\right)u_e + k_H c_s^2 \delta_e + (1 + w_e)k_H A$$

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$c_s^2 = \frac{\delta p_e^r}{\delta \rho_e^r}$

$\delta_e = \frac{\delta \rho_e}{\rho_e}$

$B =$space time piece $\delta g_{\mu\nu}$ a.g.

$H_L =$space space piece $\delta g_{\mu\nu}$ a.g.

$c_K = 1 - 3K/k^2$

$$u'_e = 3\left(w_e + c_s^2 - \frac{p'_e}{\rho'_e} - \frac{1}{3}\right)u_e + k_H c_s^2 \delta_e + (1 + w_e)k_H A$$

$$\delta'_e + 3(c_s^2 - w_e)\delta_e + 9\left(c_s^2 - \frac{p'_e}{\rho'_e}\right)\frac{u_e}{k_H} = k_H u_e - (1 + w_e)(k_H B + 3H'_L)$$

$$c_K k^2 \Phi = 4\pi G a^2 \sum_i \rho_i \delta_i^r$$

Fang, 2008.

Crossing the Phantom Divide

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$c_s^2 = \frac{\delta p^r_e}{\delta \rho^r_e}$

$\delta_e = \frac{\delta \rho_e}{\rho_e}$

$B =$space time piece $\delta g_{\mu\nu}$ a.g.

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$c_K = 1 - 3K/k^2$

$$u'_e = 3\left(w_e + c_s^2 - \frac{p'_e}{\rho'_e} - \frac{1}{3}\right)u_e + k_H c_s^2 \delta_e + (1 + w_e)k_H A$$

$$\delta'_e + 3(c_s^2 - w_e)\delta_e + 9\left(c_s^2 - \frac{p'_e}{\rho'_e}\right)\frac{u_e}{k_H} = k_H u_e - (1 + w_e)(k_H B + 3H'_L)$$

$$c_K k^2 \Phi = 4\pi G a^2 \sum_i \rho_i \delta_i^r$$

$$w_e = -1 \Rightarrow \frac{p'_e}{\rho'_e} \to \infty$$

Fang, 2008.

Crossing the Phantom Divide

In DE comoving coordinate system, the conservation of momentum $(\rho_e \vec{u}_e) = T_i^0$ yelds

$$\begin{cases} \delta\rho^r = \delta\rho_e + 3\rho_e \frac{u_e}{k_H} \\ \delta p^r = \delta p_e + 3\frac{p'_e}{\rho'_e}\rho_e \frac{u_e}{k_H} \end{cases}$$

$(') = d/dln(a)$

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$$w_e = -1 \Rightarrow \frac{p'_e}{\rho'_e} \to \infty \Rightarrow \delta_e \text{ diverges if } c_s^2 = cost$$

Fang, 2008.

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This can be solved working in the PPF framework

- momentum and density components within a unified dynamical variable
  $\Gamma = \frac{4\pi G a^2}{c_K k^2} \rho_X (\delta_X + 3u_X/k_H) - \Phi$
- first closure relation makes the anisotropic stress vanish
- second closure relation relates matter and dark energy momentum densities, making the latter be smooth under a given transition scale

Hu&Sawiki, 2007.

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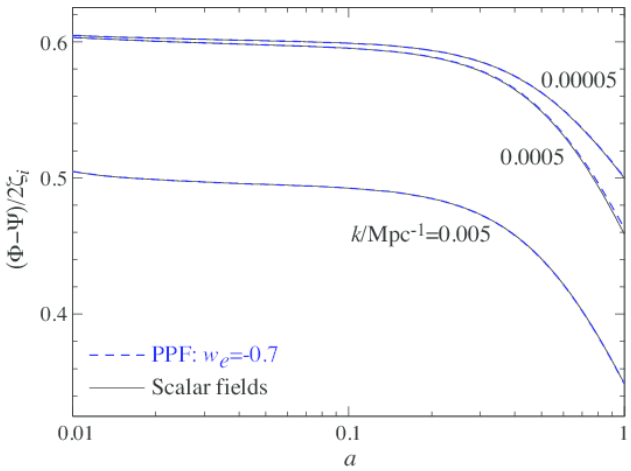

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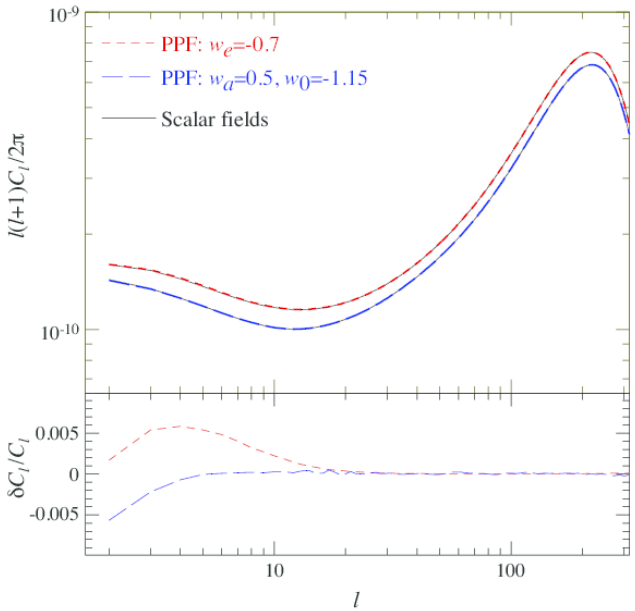

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- compute Fisher matrix $F = C^{-1}$;
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- we chose another weight matrix here, $W^T W = F$, as to say $W = F^{\frac{1}{2}}$, because it ends up with weights mostly positive, and we normalize every row to unity;
- we can compute this weight matrix as $W = O^T \Lambda^{\frac{1}{2}} O$.

CMB shift parameters

$$R = \sqrt{\Omega_m H_0^2} r(z_*)/c$$

$$l_a = \pi r(z_*)/r_s(z_*)$$

$$\theta = 100 r_s(z_*)/D_A(z_*)$$

where:

$r(z_*) \rightarrow$ comoving distance to photon-decoupling surface

$r_s(z_*) \rightarrow$ comoving sound horizon at photon-decoupling epoch

$D_A(z_*) \rightarrow$ angular diameter distance to the photon-decoupling surface

Wang & Mukherjee, 2007