Superluminality in scalar-tensor theories

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Talk's content

Motivation (Vainshtein mechanism)

Horndeski's Lagrangian

Effective theory for the

Perturbations around spherically symmetric solutions

Superluminality and other instabilities

Motivation

$$\Lambda \quad \text{Vs} \quad \phi(x^\mu)$$

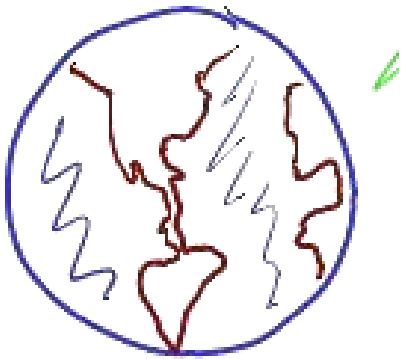
$$f(\phi)\mathcal{L}_m \ ?$$

Thus, unless symmetries forbid these couplings, one should screen ϕ's fifth force near the Earth to match observations!

Vainshtein mechanism

Vainshtein, 1972 (see Babichev and Deffayet, 2013)

In the context of massive gravity, proposed a screening mechanism



Asymptotically flat or dS

Solutions differ from those of GR

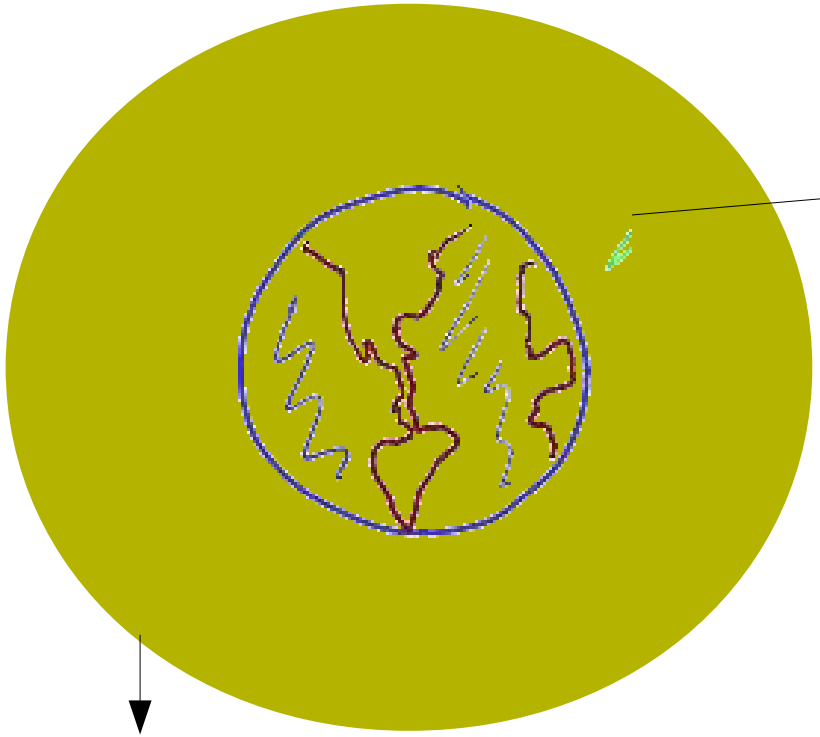
ϕ is weakly coupled

$$\mathcal{L} \sim (\partial\phi)^2 \quad +m^2\phi^2$$

Vainshtein mechanism

Vainshtein, 1972 (see Babichev, 2013)

In the context of massive gravity, proposed a screening mechanism



ϕ is strongly coupled

$$\mathcal{L} \sim (\partial\phi)^2(\Box\phi + \dots)$$

Higher order derivative interactions!

Vainshtein radius

Recovers GR solutions

Horndeski's Lagrangian

Must general scalar-tensor theory which gives second order differential equations (Horndeski, 70's)

$$\mathcal{L}_H = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5$$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5X}(\phi, X)\Big[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\Big]$$

$$X := -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2 \qquad A_B := \partial A/\partial B$$

“Vainshtein” effective theory

Would like a minimal description which captures the Vainshtein effect

(Tasinato et al)

Thus, truncate the theory $\mathcal{L} = \mathcal{L}_H + \mathcal{L}_m$ so that

1) It is an expansion around flat space

$$\phi = \phi_0 + \pi, \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

2) Leading order in $h_{\mu\nu}$ and $\pi, \partial\pi$

3) Keep all second order derivatives of π, $(i.e.\ \partial^2\pi)$

with an associated scale Λ

“Vainshtein” effective theory

Remarks

- It is equivalent to demand the Galilean symmetry (c.f. Nicolis et al)

$$\pi \to \pi + c + b_\mu x^\mu$$

- Reduces to well know theories:

  dRGT massive gravity, DGP , (restricted) Galileons, etc.

  (de Rham et al) (Dvali et al) (Berezhiani et al, Nicolis et al)

- Does not include other models from Horndeski which may present the Vainshtein mechanism,

  eg. the Fab Four (Charmousis al, Kaloper et al)

“Vainshtein” effective theory

Constants and normalisation

$$\mathcal{L}_2 = K(\phi, X) \qquad \mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$G_4(\phi_0) = M_{pl} \qquad \bar{h}_{\mu\nu} = M_{pl}h_{\mu\nu}$$

$$\mathcal{L}_5 = G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5X}(\phi, X)\big[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\big]$$

“Vainshtein” effective theory

$$\mathcal{L}^{\rm eff} = -\frac{1}{4}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}{}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{\eta}{2}\pi\Box\pi \;\; + \frac{1}{2M_{pl}}\bar{h}^{\mu\nu}T_{\mu\nu}$$

$$+\frac{\mu}{\Lambda^3}\mathcal{L}_3^{\rm gal} + \frac{\nu}{\Lambda^6}\mathcal{L}_4^{\rm gal} + \frac{\varpi}{\Lambda^9}\mathcal{L}_5^{\rm gal}$$

$$-\xi\bar{h}^{\mu\nu}X^{(1)}_{\mu\nu} - \frac{1}{\Lambda^3}\alpha\bar{h}^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{1}{2\Lambda^6}\beta\bar{h}^{\mu\nu}X^{(3)}_{\mu\nu}$$

“Vainshtein” effective theory

$$\mathcal{L}^{\rm eff} = -\frac{1}{4}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}{}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{\eta}{2}\pi\Box\pi + \frac{1}{2M_{pl}}\bar{h}^{\mu\nu}T_{\mu\nu}$$

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$$G_{4\phi} = M_{pl}\xi,\ \ K_X - 2G_{3\phi} = \eta,\ \ -G_{3X} + 3G_{4\phi X} = \frac{\mu}{\Lambda^3},\ \ G_{5X} = -\frac{3M_{pl}}{\Lambda^6}\beta$$

$$G_{4X} - G_{5\phi} = \frac{M_{pl}}{\Lambda^3}\alpha, \quad G_{4XX} - \frac{2}{3}G_{5\phi X} = \frac{\nu}{\Lambda^6}, \quad G_{5XX} = -\frac{3\varpi}{\Lambda^9},$$

Quantities evaluated at the background $\phi_0,\ X = 0$

“Vainshtein” effective theory

$$\mathcal{L}^{\rm eff} = -\frac{1}{4}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}{}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{\eta}{2}\pi\Box\pi + \frac{1}{2M_{pl}}\bar{h}^{\mu\nu}T_{\mu\nu}$$

Hilbert Einstein piece expanded to second order

$$+\frac{\mu}{\Lambda^3}\mathcal{L}_3^{\rm gal} + \frac{\nu}{\Lambda^6}\mathcal{L}_4^{\rm gal} + \frac{\varpi}{\Lambda^9}\mathcal{L}_5^{\rm gal}$$

$$-\xi\bar{h}^{\mu\nu}X^{(1)}_{\mu\nu} - \frac{1}{\Lambda^3}\alpha\bar{h}^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{1}{2\Lambda^6}\beta\bar{h}^{\mu\nu}X^{(3)}_{\mu\nu}$$

“Vainshtein” effective theory

$$\mathcal{L}^{\rm eff} = -\frac{1}{4}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}{}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{\eta}{2}\pi\Box\pi + \frac{1}{2M_{pl}}\bar{h}^{\mu\nu}T_{\mu\nu}$$

$$+\frac{\mu}{\Lambda^3}\mathcal{L}_3^{\rm gal} + \frac{\nu}{\Lambda^6}\mathcal{L}_4^{\rm gal} + \frac{\varpi}{\Lambda^9}\mathcal{L}_5^{\rm gal}$$

$$\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$$

$$\mathcal{L}_3^{\rm gal} = -\frac{1}{2}(\partial\pi)^2[\Pi]$$

$$\mathcal{L}_4^{\rm gal} = -\frac{1}{2}(\partial\pi)^2([\Pi]^2 - [\Pi^2])$$

$$\mathcal{L}_5^{\rm gal} = -\frac{1}{4}(\partial\pi)^2([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3])$$

$$-\xi\bar{h}^{\mu\nu}X_{\mu\nu}^{(1)} - \frac{1}{\Lambda^3}\alpha\bar{h}^{\mu\nu}X_{\mu\nu}^{(2)} + \frac{1}{2\Lambda^6}\beta\bar{h}^{\mu\nu}X_{\mu\nu}^{(3)}$$

“Vainshtein” effective theory

$$\mathcal{L}^{\rm eff} = -\frac{1}{4}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}{}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{\eta}{2}\pi\Box\pi + \frac{1}{2M_{pl}}\bar{h}^{\mu\nu}T_{\mu\nu}$$

$$+\frac{\mu}{\Lambda^3}\mathcal{L}_3^{\rm gal} + \frac{\nu}{\Lambda^6}\mathcal{L}_4^{\rm gal} + \frac{\varpi}{\Lambda^9}\mathcal{L}_5^{\rm gal}$$

$$-\xi\bar{h}^{\mu\nu}X^{(1)}_{\mu\nu} - \frac{1}{\Lambda^3}\alpha\bar{h}^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{1}{2\Lambda^6}\beta\bar{h}^{\mu\nu}X^{(3)}_{\mu\nu}$$

$$\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$$

$$X^{(1)}_{\mu\nu} = \eta_{\mu\nu}[\Pi] - \Pi_{\mu\nu}$$

$$X^{(2)}_{\mu\nu} = \Pi^2_{\mu\nu} - [\Pi]\Pi_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}([\Pi]^2 - [\Pi^2]),$$

$$X^{(3)}_{\mu\nu} = 6\Pi^3_{\mu\nu} - 6\Pi^2_{\mu\nu}[\Pi] + 3\Pi_{\mu\nu}([\Pi]^2 - [\Pi^2])$$
$$-\eta_{\mu\nu}([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3])$$

“Vainshtein” effective theory

$$\mathcal{L}^{\rm eff} = -\frac{1}{4}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}{}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{\eta}{2}\pi\Box\pi + \frac{1}{2M_{pl}}\bar{h}^{\mu\nu}T_{\mu\nu}$$

$$+\frac{\mu}{\Lambda^3}\mathcal{L}_3^{\rm gal} + \frac{\nu}{\Lambda^6}\mathcal{L}_4^{\rm gal} + \frac{\varpi}{\Lambda^9}\mathcal{L}_5^{\rm gal}$$

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Can be eliminated by a field redefinition

$$\bar{h}_{\mu\nu} = \hat{h}_{\mu\nu} - 2\xi\pi\eta_{\mu\nu} + \frac{2\alpha}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi$$

which introduces new couplings $-\frac{2\xi}{M_{pl}}\pi T + \frac{2\alpha}{M_{pl}\Lambda^3}\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}$

Spherically symmetric solutions

Consider the following setup

$$\hat{h}_{tt} = -2\Phi(r), \quad \hat{h}_{ij} = -2\Psi(r)\delta_{ij}, \quad \phi = \phi_0 + \pi(r)$$

Field equations become

1 $$P(x, A) \equiv \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2\right) x + (\mu + 6\alpha\xi - 3\beta A(r))\, x^2 + \left(\nu + 2\alpha^2 + 4\beta\xi\right) x^3 - 3\beta^2 x^5 = 0$$

"Mass" inside object
Related to Vainshtein radius outside

$$x(r) = \frac{1}{\Lambda^3}\frac{\pi'}{r}$$

2 $$\frac{M_{pl}}{\Lambda^3}\frac{\Phi'}{r} = \frac{M_{pl}}{\Lambda^3}\frac{\Psi'}{r} = \beta x^3 + A(r)$$

Vainshtein solutions + perturbations

Solutions which show up the Vainshtein effect do exist

Koyama et al, Sbisa et al, Chkareuli et al, Sjors et al, Kaloper et al (2), Kimura et al, Berezhiani et al, Babichev et al (2), Li et al, Hiramatsu et al, de Rham et al, Renaux-Petel, ...

Scalar perturbations around these lead to

$$S_\varphi = \frac{1}{2}\int d^4x \Big[K_t(r)(\partial_t\varphi)^2 - K_r(r)(\partial_r\varphi)^2 - K_\Omega(r)(\partial_\Omega\varphi)^2 \Big]$$

Where all the $K's$ are functions of the effective theory parameters, particularly of $\alpha,\ \beta$

which tune the terms $\alpha\ \partial_\mu\pi\partial_\nu\pi T^{\mu\nu}, \quad \beta\ \bar{h}^{\mu\nu}X^{(3)}_{\mu\nu}$

Perturbations around Vainshtein sols

3 Cases (Conclusions)

1. $\alpha = \beta = 0$ Original Galileon, studied by Nicolis et al

   Superluminal propagation in radial modes

   Extremely subluminal in radial piece!

2. $\alpha \neq 0, \ \beta = 0$ Disformal coupling $\alpha \ \partial_\mu \pi \partial_\nu \pi T^{\mu\nu}$

   Leads to Restricted Galileon (Berezhiani et al)

   $\alpha < 0$ has a ghost mode

   There are no asymptotically flat Vainshtein solutions

Perturbations around Vainshtein sols

3 Cases (Conclusions)

$$S_\varphi \sim \int d^4x \Big[K_t (\partial_t \varphi)^2 - K_r (\partial_r \varphi)^2 - K_\Omega (\partial_\Omega \varphi)^2 \Big]$$

❸ $\alpha \neq 0, \ \beta \neq 0$

Inside the Vainshtein radius and outside the source, the leading terms are

$$K_t \sim 0, \quad K_r \sim -12\beta A(r) x_0, \quad K_\Omega \sim 6\beta A(r) x_0,$$

Speed of fluctuations is always superluminal, and **unstable**.

- The instability can be avoided inside the source
- Need to look beyond static & spherical symmetry!