Cosmic Variance from Superhorizon Mode Coupling

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EN & S. Shandera, 1212.4550 (PRL)
M. LoVerde, EN, & S. Shandera, 1303.3549 (JCAP)
J. Bramante, J. Kumar, EN, & S. Shandera, 1307.3549 (JCAP)
EN, S. Park, S. Shandera, work in progress.

Related work:
Nurmi, Byrnes, & Tasinato, 1301.3128
Nurmi, Byrnes, Tasinato, & Wands, 1306.2370
LoVerde, 1310.5739

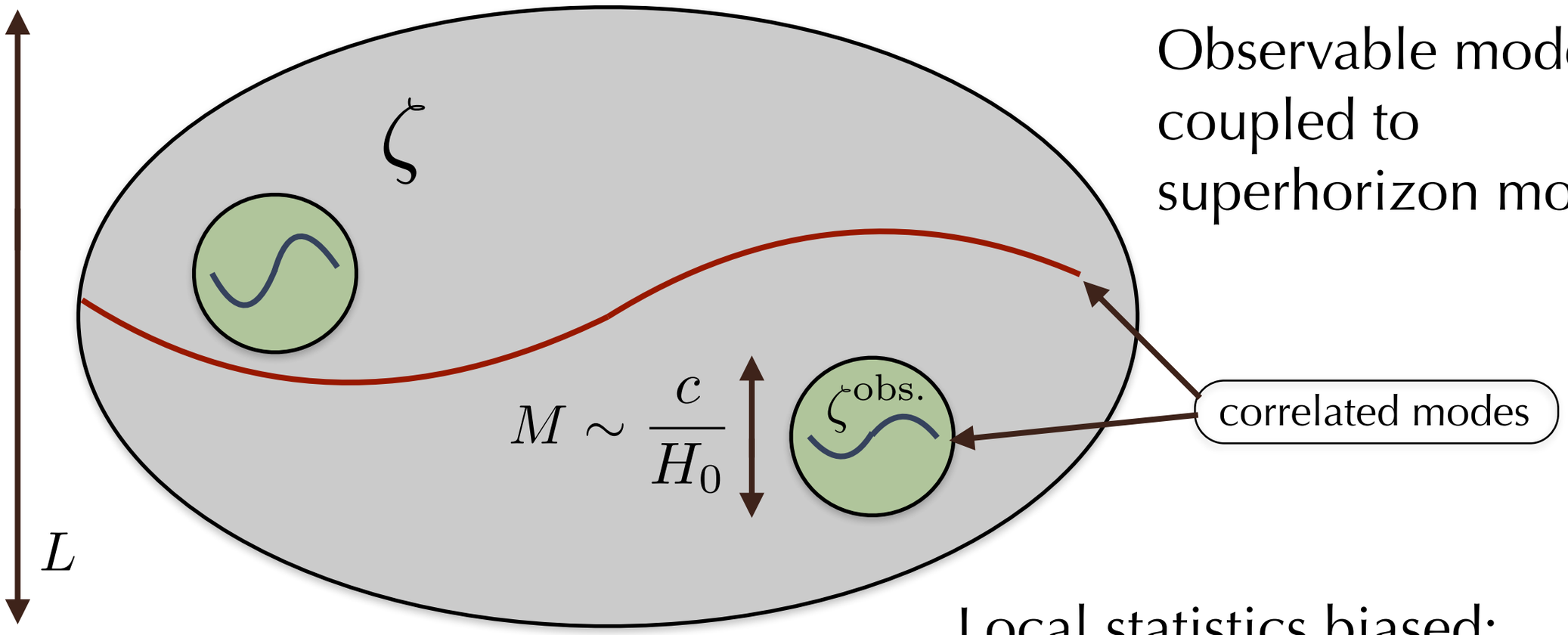
Is Our Universe a Biased Sample?

- Observable universe = finite subvolume of the larger universe
- Finite sample: volume, range of scales, number of galaxies, etc.

→ **Statistics Question: Is our Hubble volume a representative or biased sample? How limited is our access to global statistics?**

Is Our Universe a Biased Sample?

- Consider the primordial curvature perturbation ζ on a fixed spatial slice (at end of inflation): $a(\mathbf{x}) = \langle a \rangle_L (1 + \zeta(\mathbf{x}))\ , \quad x \in \mathrm{Vol}_L$



Observable modes coupled to superhorizon modes...

...Local statistics biased:

$$\langle \zeta_{\mathbf{k}_1} \times \cdots \zeta_{\mathbf{k}_N} \rangle |_{\zeta_L} \neq \langle \zeta_{\mathbf{k}_1} \times \cdots \zeta_{\mathbf{k}_N} \rangle |_0$$

average over observable universe

single roll of the dice

global statistics

Case Study: Local Non-Gaussianity

- Nearly Gaussian: $\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL}(\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2 \rangle) + \frac{9}{25} g_{\rm NL} \zeta_G^3(\mathbf{x}) + \cdots$
- Long-short wavelength split:

$$\zeta_G \equiv \zeta_{Gs} + \zeta_{Gl} = \int_{L^{-1}}^{M^{-1}} \frac{d^3k}{(2\pi)^3} \zeta_G(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \int_{M^{-1}}^{k_{\rm max}} \frac{d^3k}{(2\pi)^3} \zeta_G(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

- Perturbations in subvolume:

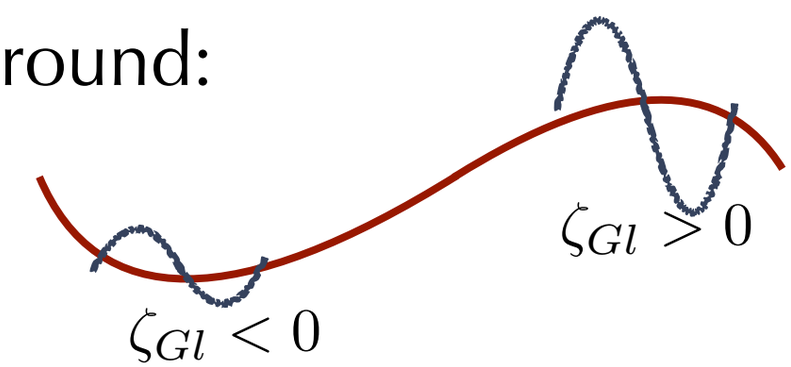
$$\zeta^{\rm obs}(\mathbf{x}) = \zeta_G^{\rm obs}(\mathbf{x}) + \frac{3}{5} f_{\rm NL}^{\rm obs}[\zeta_G^{\rm obs}(\mathbf{x})^2 - \langle \zeta_G^{\rm obs}(\mathbf{x})^2 \rangle] + \frac{9}{25} g_{\rm NL}^{\rm obs} \zeta_G^{\rm obs}(\mathbf{x})^3 + \ldots$$

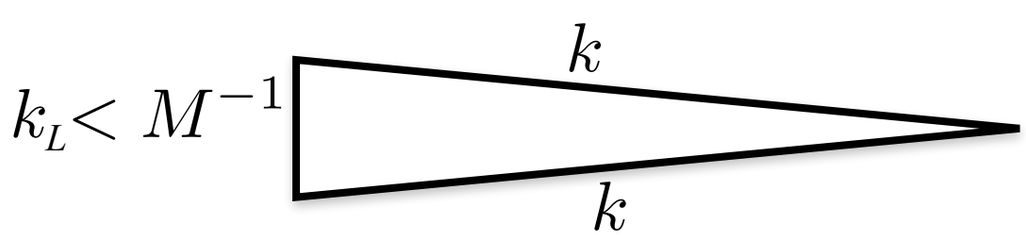
$$(1 + \frac{6}{5} f_{\rm NL} \zeta_{Gl}) \zeta_{Gs} + \cdots$$

- Local statistics modulated by the local background:

$$P_\zeta^{\rm obs}(k) = \left[1 + \frac{12}{5} f_{NL} \zeta_{Gl} + \ldots\right] P_G(k)$$

$$f_{\rm NL}^{\rm obs} = f_{\rm NL} - \frac{12}{5} f_{\rm NL}^2 \zeta_{Gl} + \frac{9}{5} g_{\rm NL} \zeta_{Gl} + \ldots$$





[M. LoVerde, EN, & S. Shandera, 1303.3549]
[Nurmi, Byrnes, & Tasinato, 1301.3128]

Case Study: Local Non-Gaussianity

- Nearly Gaussian: $\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL}(\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2 \rangle) + \frac{9}{25} g_{\rm NL} \zeta_G^3(\mathbf{x}) + \cdots$
- Long-short wavelength split:

$\zeta_G \equiv \zeta_G$ … $\int^{M^{-1}} d^3k$ … $\int^{k_{\rm max}} \frac{d^3k}{\ldots \pi)^3} \zeta_G(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$

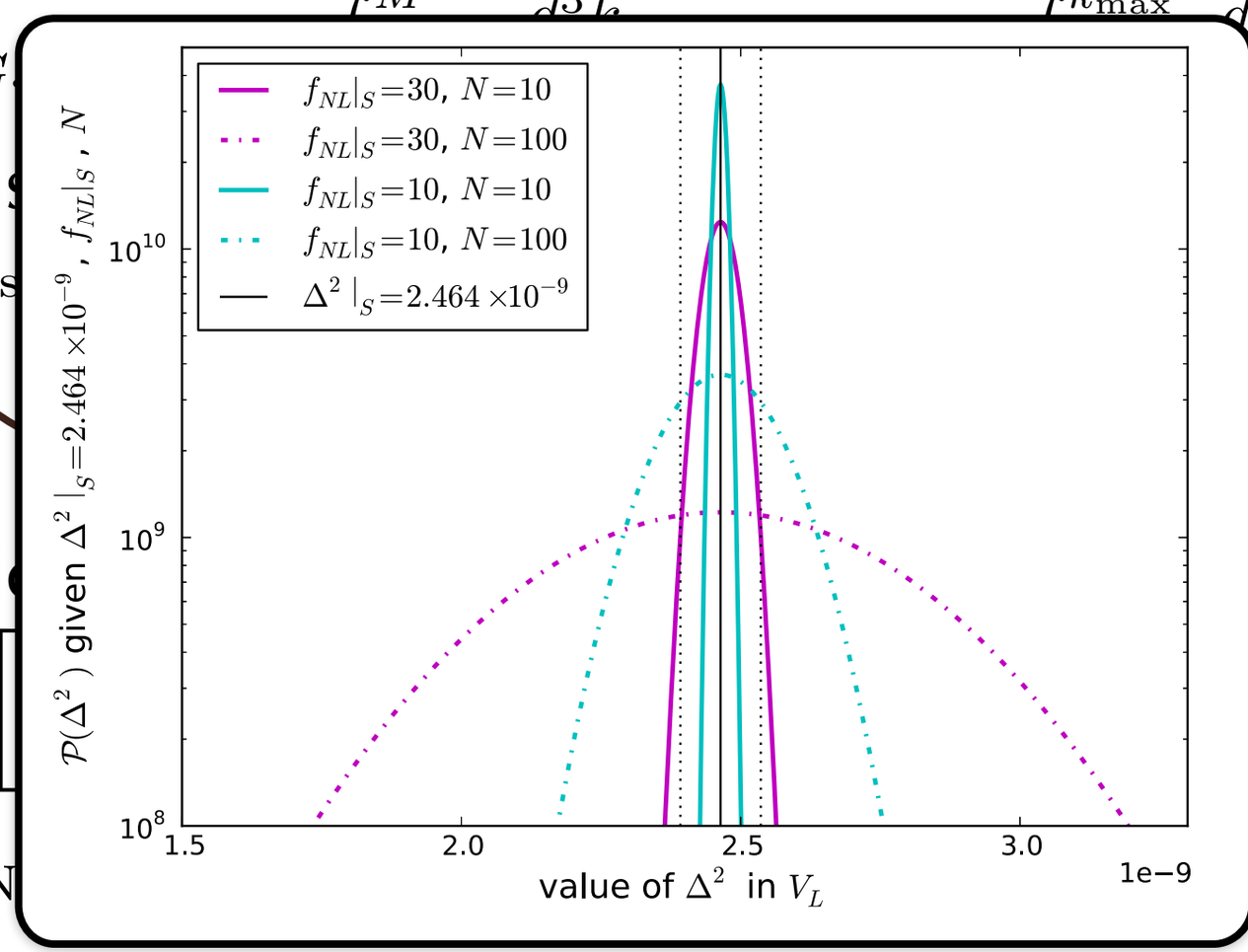
- Perturbations

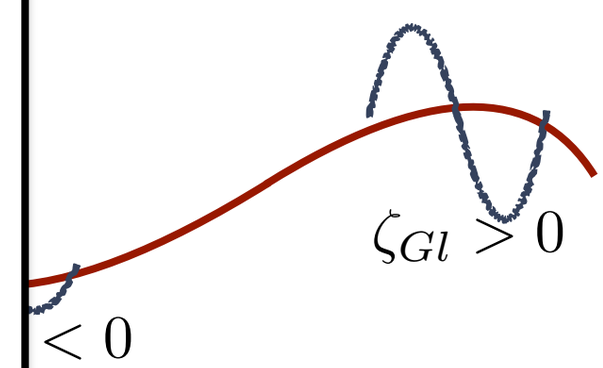
$\zeta^{\rm obs}(\mathbf{x}) = \zeta_G^{\rm obs}$ … $\ldots {}^{\rm bs}_{\rm NL} \zeta_G^{\rm obs}(\mathbf{x})^3 + \ldots$

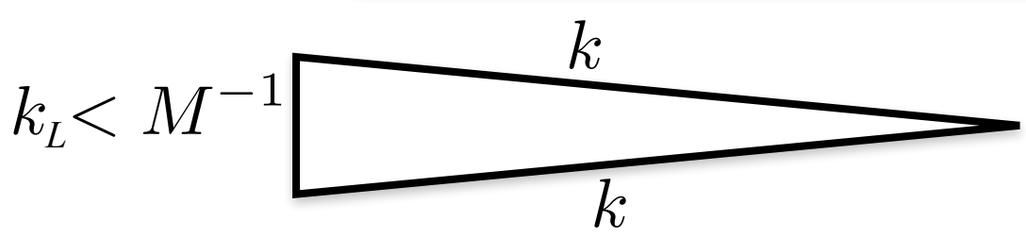
- Local statisti…

$P_\zeta^{\rm obs}(k) = \Big[$

$f_{\rm NL}^{\rm obs} = f_{\rm N}$







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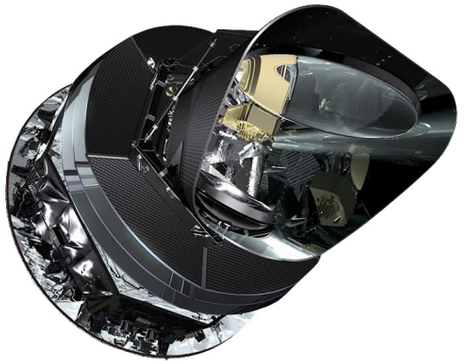
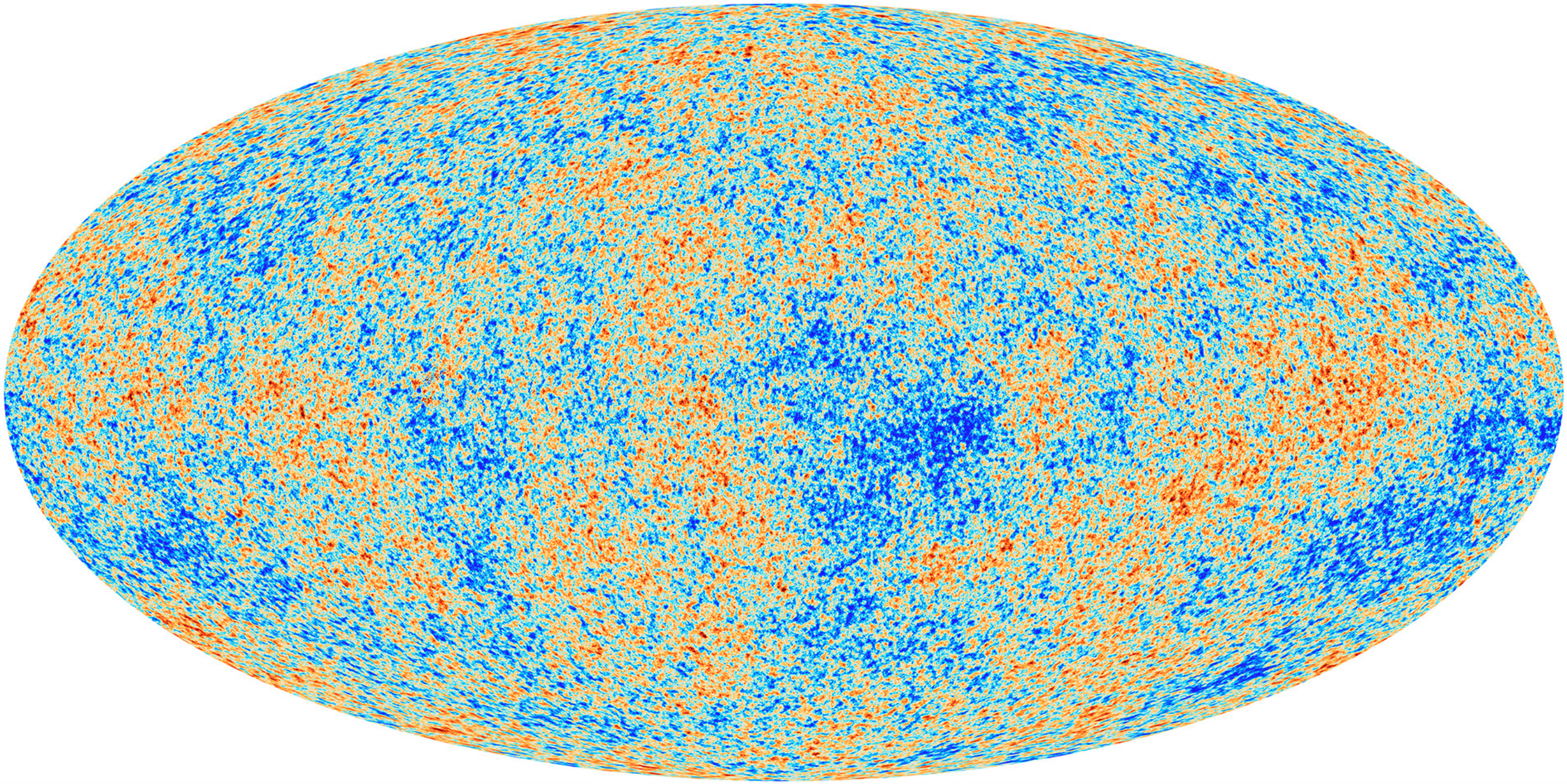
Planck CMB Measurements



Planck satellite measurements:

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$$ [Nearly Gaussian perturbations]

$$n_s = 0.9603 \pm 0.0073$$ [Red tilt]



Gaussian Patches in an NG Universe

$$\zeta(\mathbf{x}) = \zeta_G^p(\mathbf{x}) - \langle \zeta_G^p \rangle$$

Strongly NG field can appear nearly Gaussian:

$$\zeta^{\rm obs}(\mathbf{x}) = p\zeta_{Gl}^{p-1}\zeta_{Gs}(\mathbf{x}) + \frac{p!}{2!(p-2)!}\zeta_{Gl}^{p-2}(\zeta_{Gs}^2(\mathbf{x}) - \langle \zeta_{Gs}^2 \rangle) + \ldots$$

$$\zeta_{Gl} \gg \sqrt{\langle \zeta_{Gs}^2 \rangle} \leftrightarrow \frac{L}{M} \gg k_{\rm max} M \leftrightarrow N_{\rm superhorizon} \gg N_{\rm subhorizon}$$

$\rightarrow$ Recover statistically natural, nearly Gaussian series, in typical subvolumes; series controlled by $\langle \zeta_{Gs}^2 \rangle^{1/2}/\zeta_{Gl}$

Minimum level of NG in subvolumes:

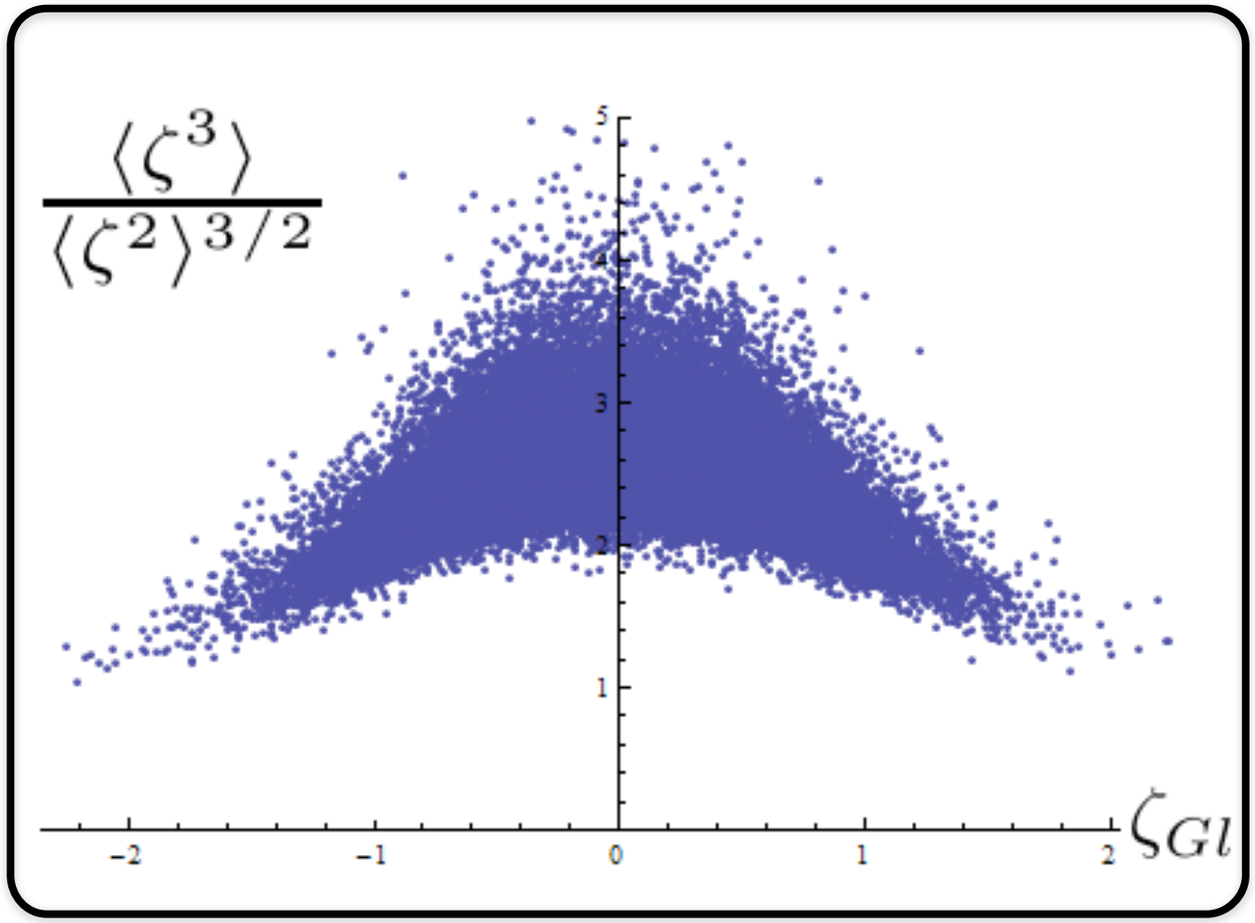
$$\frac{3}{5} f_{\rm NL}^{\rm obs} \approx \frac{p-1}{2p\zeta_{Gl}^p} \sim \frac{1}{\zeta_l}$$

$\rightarrow$ Constrain superhorizon cosmic variance by tightening $f_{\rm NL}^{\rm obs}$ bounds

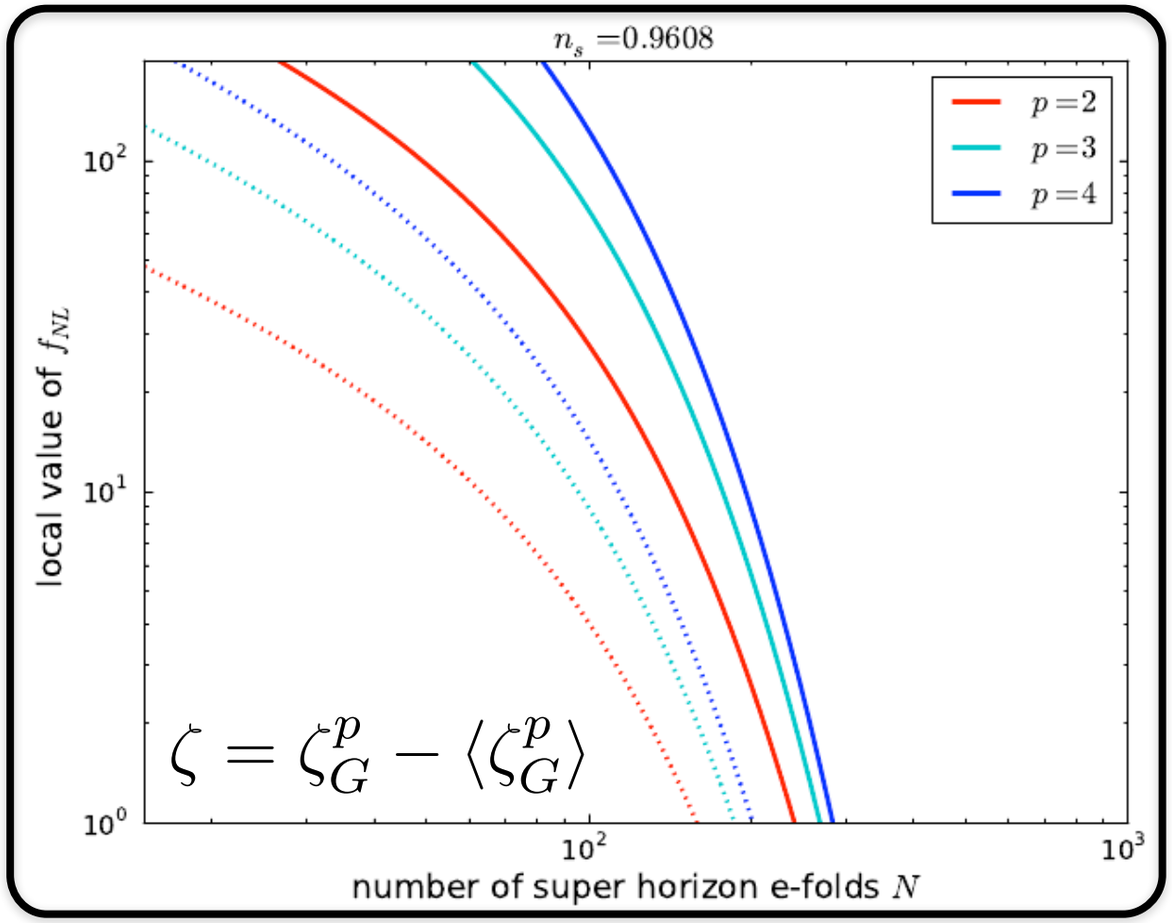
[EN, S. Shandera, 1212.4550; LoVerde et. al., 1303.3549]

Gaussian Patches in an NG Universe

Numerical Realizations:



Suppression of $f_{\rm NL}^{\rm obs}$:



Biased Scale Dependence: $n_s^{\rm obs.} \neq n_s$

- Generalized Local Ansatz:

$$\zeta(\mathbf{k}) = \phi_G(\mathbf{k}) + \sigma_G(\mathbf{k}) + \frac{3}{5} f_{\rm NL}(k)[\sigma_G^2](\mathbf{k}) + \frac{9}{25} g_{\rm NL}(k)[\sigma_G^3](\mathbf{k}) + \ldots$$

Weak scale-dependence:

$$f_{\rm NL}(k) = f_{\rm NL}(k_p)\left(\frac{k}{k_p}\right)^{n_f}$$

- Biased power spectrum:

$$P_\zeta^{\rm obs}(k) = P_\zeta(k)\left[1 + \frac{12}{5}\xi_m(k)\frac{f_{\rm NL}(k)\sigma_{Gl} + \frac{3}{5}f_{\rm NL}^2(k)(\sigma_{Gl}^2 - \langle\sigma_{Gl}^2\rangle)}{1 + \frac{36}{25}f_{\rm NL}^2(k)\langle\sigma_{Gl}^2\rangle}\right]$$

- Running in global bispectrum leaks into observed power spectrum

$$\Delta n_s(k) \approx \frac{12}{5}\xi_m(k) f_{\rm NL}(k)\sigma_{Gl}(n_f + n_f^{(m)}), \quad \frac{6}{5} f_{\rm NL}(k)\sigma_{Gl} \ll 1$$

→ Observed red tilt $n_s^{\rm obs} < 1$

...consistent with globally flat or blue tilt $n_s > 1$

Summary

- Local statistics can be biased by background modes
  - Constraints on global statistics ($P_\zeta,\ f_{\rm NL},\ n_s,\ n_t,\ n_{\rm sq.}$) are probabilistic
  - Parameters from models need only render our observations typical among Hubble volumes
  - Bias grows with size of universe ~ length of inflation
  - Local and global statistics can be qualitatively different
- Nearly Gaussian Hubble volumes in a NG universe
  - Statistical naturalness of weakly NG local ansatz
- Observations could tell us if this uncertainty is relevant to our interpretation of data
- Ongoing work: bias from models of inflation, subsampling with nonlocal NG