Non-local infrared modifications of gravity and dark energy

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based on

M. Jaccard, MM and E. Mitsou, 1305.3034, PR D88 (2013)

MM, arXiv: 1307.3898

S. Foffa, MM and E. Mitsou, 1311.3421, 1311.3435

A. Kehagias and MM, in preparation

Y. Dirian, S. Foffa, N. Khosravi, M. Kunz, MM, in preparation

we will only give a summary of the main results.

Glad to discuss the details during this week!

- The model:

$$G_{\mu\nu} - \frac{d-1}{2d} m^2 \left(g_{\mu\nu}\Box^{-1}R\right)^{\mathrm{T}} = 8\pi G\, T_{\mu\nu}$$

where we use the fact that any symmetric tensor can be written as

$$S_{\mu\nu} = S^{\mathrm{T}}_{\mu\nu} + \frac{1}{2}(\nabla_\mu S_\nu + \nabla_\nu S_\mu)$$

where

$$\nabla^\mu S^{\mathrm{T}}_{\mu\nu} = 0$$

our initial motivation:

introduce a mass m and deform GR in the infrared in a way which is fully covariant and does not require an external reference metric

some sources of inspiration:

- Proca theory

$$S = \int d^4x \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} m_\gamma^2 A_\mu A^\mu - j_\mu A^\mu \right]$$

$$\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = j^\nu \quad \rightarrow \begin{cases} m_\gamma^2 \, \partial_\nu A^\nu = 0 \\ (\Box - m_\gamma^2) A^\mu = 0 \end{cases}$$

is equivalent to $$\left(1 - \frac{m_\gamma^2}{\Box}\right) \partial_\mu F^{\mu\nu} = j^\nu$$

(Dvali 2006; Dvali, Hofmann, Khoury 2007; review Hinterbichler 2012)

- degravitation

$$\left(1 - \frac{m^2}{\Box}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002

Cosmological consequences

- consider $$G_{\mu\nu} - \frac{d-1}{2d}\, m^2 \left(g_{\mu\nu}\Box_g^{-1} R\right)^{\mathrm{T}} = 8\pi G\, T_{\mu\nu}$$

- specialize to FRW in d=3. Define $$U = -\Box^{-1} R$$

$$S_{\mu\nu} = -U g_{\mu\nu} \qquad\qquad S_{\mu\nu} = S^{\mathrm{T}}_{\mu\nu} + (1/2)(\boldsymbol{\nabla}_\mu S_\nu + \boldsymbol{\nabla}_\nu S_\mu)$$

- in FRW $S_\mu = (S_0, 0)$, so we have 3 variables: $H(t)$, $U(t)$, $S_0(t)$

$$\begin{aligned} H^2 - \frac{m^2}{9}(U - \dot{S}_0) &= \frac{8\pi G}{3}\rho \qquad\qquad \rho = \rho_M + \rho_R \\ \ddot{U} + 3H\dot{U} &= 6\dot{H} + 12H^2 \\ \ddot{S}_0 + 3H\dot{S}_0 - 3H^2 S_0 &= \dot{U} \end{aligned}$$

- a final massaging of the eqs.

$$Y = U - \dot{S}_0 \qquad h(t) = H(t)/H_0,\ \Omega_i(t) = \rho_i(t)/\rho_c(t)$$

$$x \equiv \ln a(t),\ f' \equiv df/dx \qquad \gamma \equiv m^2/(9H_0^2)$$

and their final form is

$$h^2(x) = \Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y(x)\,,$$

$$Y'' + (3-\zeta)Y' - 3(1+\zeta)Y = 3U' - 3(1+\zeta)U\,,$$

$$U'' + (3+\zeta)U' = 6(2+\zeta)\,,$$

$$\zeta(x) \equiv \frac{h'}{h} = -\frac{3\Omega_M e^{-3x} + 4\Omega_R e^{-4x} - \gamma Y'}{2(\Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y)}\,.$$

- there is an effective DE term, with

$$\rho_{\rm DE}(x) = \rho_0 \gamma Y(x) \qquad\qquad (\text{as usual } \rho_0 = 3H_0^2/(8\pi G))$$

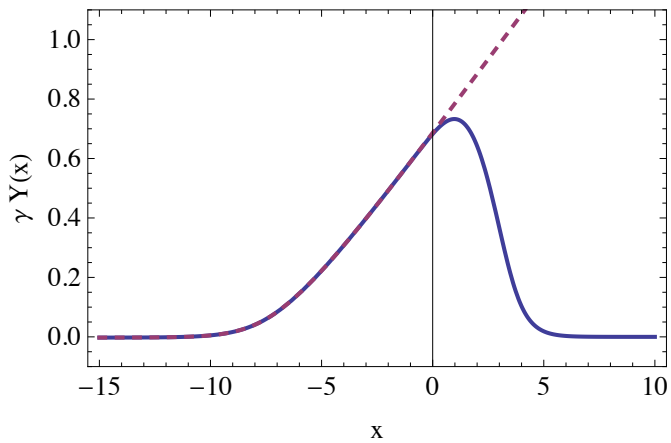
- define w_{DE} from $\dot{\rho}_{\rm DE} + 3(1+w_{\rm DE})H\rho_{\rm DE} = 0$

  then:

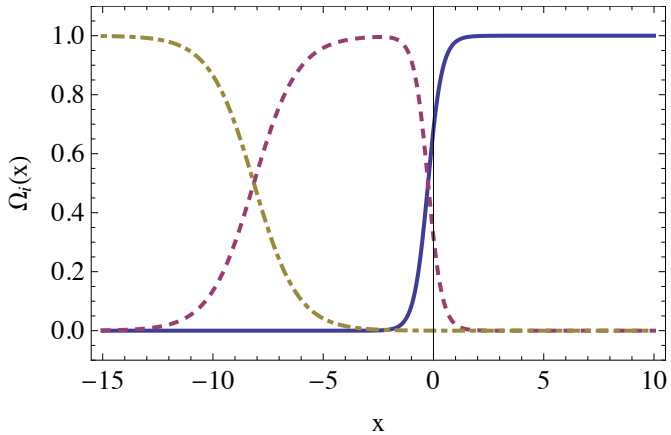
$$w_{\rm DE}(x) = -1 - \frac{Y'(x)}{3Y(x)}$$

- at the level of background evolution, the model has the same number of parameters as ΛCDM, with $\Omega_\Lambda \leftrightarrow \gamma$.

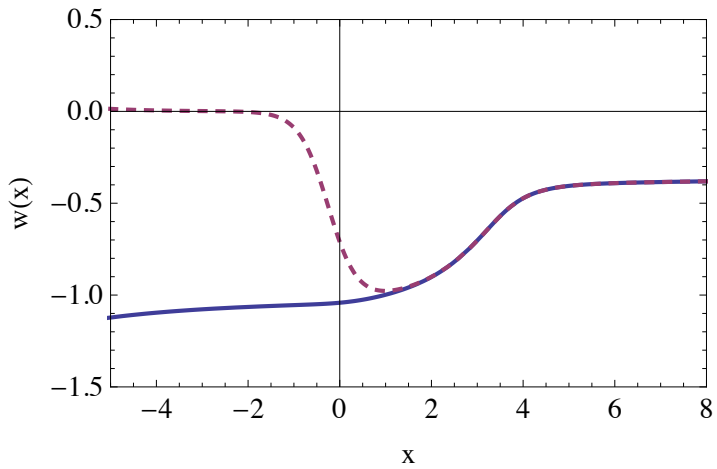
- results.



- Fixing $\gamma = 0.05..$ ($m=0.67\ H_0$) we reproduce $\Omega_{DE}=0.68$



- having fixed γ we get a pure prediction for the EOS:



fit $w(x)=w_0+(1-a)\, w_a$ in the region $-1< x <0$

best fit values: $w_0= -1.042$, $w_a=-0.020$

on the phantom side !

(general consequence of $\dot{\rho}_{\rm DE} + 3(1+w_{\rm DE})H\rho_{\rm DE} = 0$
together with $\rho>0$ and $d\rho/dt>0$)

- Planck+WP+BAO gives $w_0 = -1.04^{+0.72}_{-0.69}$, $w_a<1.32$

- since we predict $|w_a|<<1$, we can also compare with the Planck prediction for constant w

  Planck+WP+SNLS: $w_0 = -1.13^{+0.13}_{-0.14}$ (95% c.l.)

  Planck+WP+Union2.1 $w_0 = -1.09 \pm 0.17$ (95% c.l.)

Consistency issues

- no issue of ghost-induced vacuum decay. The non-local eqs must be understood as an effectve classical eq for the in-in quantum expectation values, and not as the eq of motion of a fundamental non-local QFT
- no vDVZ discontinuity. Solar system tests passed
- No strong classical non-linearities at short distances. Corrections to the Schwarzschild solutions are $1+O(m^2r^2)$
- cosmological perturbations are being worked out

Conclusions

- we have an interesting IR modification of GR
- and a testable prediction for the dark energy EOS

Thank you!

A locality / gauge-invariance duality for massive gauge fields

- Example: Proca theory for massive photons

$$S = \int d^4x \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} m_\gamma^2 A_\mu A^\mu - j_\mu A^\mu \right]$$

$$\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = j^\nu \quad \rightarrow \left\{ \begin{array}{r} m_\gamma^2 \, \partial_\nu A^\nu = 0 \\ (\Box - m_\gamma^2) A^\mu = 0 \end{array} \right.$$

- non-local formulation (Dvali 2006; Dvali, Hofmann, Khoury 2007)

Stueckelberg trick: $A_\mu \rightarrow A_\mu + \frac{1}{m_\gamma} \partial_\mu \varphi$

add one field and gain a gauge symmetry

$$A_\mu \rightarrow A_\mu - \partial_\mu \theta \,, \quad \varphi \rightarrow \varphi + m_\gamma \theta$$

$$S = \int d^4x \, \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} m_\gamma^2 A_\mu A^\mu - \tfrac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - m_\gamma A^\mu \partial_\mu \varphi - j_\mu A^\mu \right]$$

$$\partial_\mu F^{\mu\nu} = m_\gamma^2 A^\nu + m_\gamma \partial^\nu \varphi + j^\nu \,,$$
$$\Box \varphi + m_\gamma \partial_\mu A^\mu = 0 \,.$$

If we choose the unitary gauge $\phi=0$ we get back to the original formulation of Proca theory (and loose the gauge sym because of gauge fixing).

Instead, keep the gauge sym explicit and integrate out ϕ using its own equation of motion:

$$\varphi(x) = -m_\gamma \Box^{-1} (\partial_\mu A^\mu)$$

Substituting in the eq of motion for A^ν :

$$\left(1 - \frac{m_\gamma^2}{\Box}\right) \partial_\mu F^{\mu\nu} = j^\nu$$

or

$$(\Box - m_\gamma^2) A^\nu = \left(1 - \frac{m_\gamma^2}{\Box}\right) \partial^\nu \partial_\mu A^\mu + j^\nu$$

we have explicit gauge invariance for the massive theory,
at the price non-locality

- a sort of duality between explicit gauge-invariance and explicit locality
- we can fix the gauge $\partial_\mu A^\mu = 0$ and the non-local term disappears (and we are back to Proca eqs.)
- with hindsight, the Stueckelberg trick was not needed

the original idea: write the linearize Fierz-Pauli theory in non-local form to preserve linearized diff, and covariantize

$$S_{\rm FP} = \frac{1}{2}\int d^4x \, \left[h_{\mu\nu}\mathcal{E}^{\mu\nu,\rho\sigma}h_{\rho\sigma} - m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\right]$$

Stueckelberg trick: $$h_{\mu\nu} \to h_{\mu\nu} + \frac{1}{m}(\partial_\mu A_\nu + \partial_\nu A_\mu)\,,$$

restores inv under $h_{\mu\nu} \to h_{\mu\nu} - (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu)\,, \qquad A_\mu \to A_\mu + m\xi_\mu$

eq of motion for A_ν: $$\partial_\mu F^{\mu\nu} = -m\partial_\mu(h^{\mu\nu} - \eta^{\mu\nu}h) \equiv j^\nu$$

write $A^\nu = A^\nu_T - \partial^\nu \alpha \qquad \Box A^\nu_T = j^\nu$

Dvali, Hofmann, Khoury (2007)

$\to A^\nu = \Box^{-1} j^\nu - \partial^\nu \alpha$ $\qquad$ α undetermined (peculiar of FP)

- in QED, we found that a massive deformation of the theory is obtained replacing

$$\partial_\mu F^{\mu\nu} = j^\nu \qquad \rightarrow \qquad \left(1 - \frac{m^2}{\Box}\right) \partial_\mu F^{\mu\nu} = j^\nu$$

- for gravity, a first guess for a massive deformation of GR could be

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad \rightarrow \qquad \left(1 - \frac{m^2}{\Box_g}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

however this is not correct since $\nabla^\mu (\Box_g^{-1} G_{\mu\nu}) \neq 0$

We would lose energy-momentum conservation.

eliminating A^μ and defining $N \equiv \frac{h}{2} - \frac{\Box\alpha}{m}$ we finally get

$$S_{\rm FP} = \int d^4x \, \left[\underbrace{\tfrac{1}{2} h_{\mu\nu} \left(1 - \tfrac{m^2}{\Box}\right) \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma}}_{\text{6 d.o.f: massive spin 2 + 1 scalar ghost}} - 2m^2 N \tfrac{1}{\Box} \underbrace{\partial_\mu \partial_\nu (h^{\mu\nu} - \eta^{\mu\nu} h)}_{\text{linearized Ricci scalar}} \right]$$

The field N (which is basically the longitudinal mode of A^μ) acts as a Lagrange multiplier.

At the linearized level it kills the ghost.

However, if we covariantize the theory, it imposes the condition R=0, which is not present in GR ⇒ a fully covariant vDVZ discontinuity ! (Porrati 2002)

- to understand the properties of the theory we linearize over Minkowski space: in d spatial dimensions

$$\mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{d} m^2 P^{\mu\nu} P^{\rho\sigma} h_{\rho\sigma} = -16\pi G T^{\mu\nu}$$

$$P^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\Box}$$

the corresponding Lagrangian is (only formally! see later)

$$\mathcal{L}_2 = \frac{1}{2} h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{2d} m^2 h_{\mu\nu} P^{\mu\nu} P^{\rho\sigma} h_{\rho\sigma}$$

we add a gauge fixing and we get the propagator

$$\tilde{D}^{\mu\nu\rho\sigma}(k) = \frac{-i}{2k^2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{d-1}\eta^{\mu\nu}\eta^{\rho\sigma}\right) - \frac{1}{d(d-1)}\frac{im^2}{k^2(-k^2+m^2)}\eta^{\mu\nu}\eta^{\rho\sigma}$$

- no vDVZ discontinuity!
- For m=O(H_0), solar system test easily passed. Corrections are O(m^2/k^2) =10^{-30} for k=(1 a.u)$^{-1}$.
- massless graviton + extra contribution to $\tilde{T}_{\mu\nu}(-k)\tilde{D}^{\mu\nu\rho\sigma}(k)\tilde{T}_{\rho\sigma}(k)$

$$\frac{1}{d(d-1)}\tilde{T}(-k)\left[-\frac{i}{k^2} - \frac{i}{(-k^2+m^2)}\right]\tilde{T}(k)$$

one massles scalar + one massive scalar ghost ?

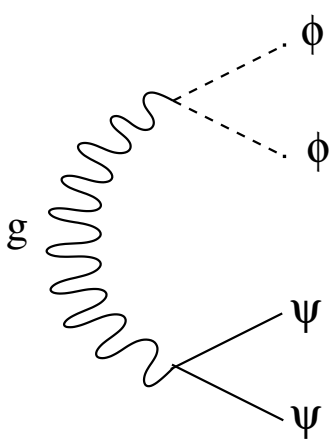
The ghost

a ghost has in principle two type of effects:

- at the classical level, it can give rise to runaway solution.

  for a ghost with m=O(H_0) this can even be welcome for explaining the acceleration of the Universe

- at the quantum level, it produces a vacuum decay rate, Γ



for a ghost interacting gravitationally with local interactions:

$$\Gamma \sim \frac{\Lambda^8}{M_{\rm Pl}^4}$$

$$\text{if } \Lambda \sim M_{\rm Pl},\ \Gamma \sim M_{\rm Pl}^4$$

Non-local QFT or classical effective equations?

- we have $\Box_{\rm ret}^{-1}$ directly in the EoM (rather than in the solution). This EoM cannot come from the variation of a Lagrangian. E.g.

$$\frac{\delta}{\delta\phi(x)}\int dx'\phi(x')(\Box^{-1}\phi)(x') = \frac{\delta}{\delta\phi(x)}\int dx'dx''\phi(x')G(x',x'')\phi(x'')$$
$$= \int dx'[G(x,x')+G(x',x)]\phi(x')$$

- we can repalce $\Box^{-1} \to \Box_{\rm ret}^{-1}$ after the variation, as a formal trick to get the EoM from a Lagrangian.

Deser-Waldron 2007,
Barvinski 2012

However, any connection to the QFT described by this Lagrangian is lost.

Taking

$$\mathcal{L}_2 = \frac{1}{2} h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{2d} m^2 h_{\mu\nu} P^{\mu\nu} P^{\rho\sigma} h_{\rho\sigma}$$

as the quantum Lagrangian of our problem introduces spurious propagating degrees of freedom. In

$$\begin{aligned} \tilde{D}^{\mu\nu\rho\sigma}(k) &= \frac{-i}{2k^2} \left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{d-1}\eta^{\mu\nu}\eta^{\rho\sigma} \right) \\ &\quad - \frac{1}{d(d-1)} \frac{im^2}{k^2(-k^2+m^2)} \eta^{\mu\nu}\eta^{\rho\sigma} \end{aligned}$$

the apparent ghost-like pole in the propagator already comes with a retarded prescription. It is not a Feynman propagator and does not describe a propagating dof!

Our non-local EoM is not the classical EoM of a non-local QFT!

EoMs involving $\Box_{\rm ret}^{-1}$ emerge from a classical or a quantum averaging of a more fundamental (local) QFT

- classically, when separating long and short wavelength and integrating out the short wave-length

  (e.g cosmological perturbation theory, or GWs)

- in QFT, when computing the effective action that includes the effect of radiative corrections. This provides effective non-local field eqs for $\langle 0|\hat{\phi}|0\rangle, \langle 0|\hat{g}_{\mu\nu}|0\rangle$

- the in-in matrix elements satisfy non-local and retarded equations

Jordan 1986, Calzetta-Hu 1987

- So, we interpret our non-local eqs as a classical, effective equation, derived from a more fundamental local theory by a classical or quantum averaging

- any problem of quantum vacuum stability can only be addressed in this fundamental theory

- in any case, the apparent ghost that we found has nothing to do with quantum vacuum decay in our model.

  It can however trigger classical cosmological instabilities

A fake ghost in massless GR

$$S_{\rm EH}^{(2)} = \frac{1}{2}\int d^{d+1}x\, h_{\mu\nu}\mathcal{E}^{\mu\nu,\rho\sigma}h_{\rho\sigma}$$

$$h_{\mu\nu} = h_{\mu\nu}^{\rm TT} + \frac{1}{2}(\partial_\mu\epsilon_\nu + \partial_\nu\epsilon_\mu) + \frac{1}{d}\eta_{\mu\nu}s$$

$$S_{\rm EH}^{(2)} = \frac{1}{2}\int d^{d+1}x\, \left[h_{\mu\nu}^{\rm TT}\Box(h^{\mu\nu})^{\rm TT} - \frac{d-1}{d}\, s\Box s\right]$$

$$S_{\rm int} = \frac{\kappa}{2}\int d^{d+1}x\, h_{\mu\nu}T^{\mu\nu} = \frac{\kappa}{2}\int d^{d+1}x\, \left[h_{\mu\nu}^{\rm TT}(T^{\mu\nu})^{\rm TT} + \frac{1}{d}sT\right]$$

$$\Box h_{\mu\nu}^{\rm TT} = -\frac{\kappa}{2}T_{\mu\nu}^{\rm TT}\,, \quad \Box s = \frac{\kappa}{2(d-1)}T$$

It looks as if there are many more propagating d.o.f
Furthermore s seems a ghost !

- the origin of the problem is that s is a non-local function of $h_{\mu\nu}$:

$$s = \left(\eta^{\mu\nu} - \frac{1}{\Box}\partial^\mu\partial^\nu\right) h_{\mu\nu} = P^{\mu\nu} h_{\mu\nu}$$

- example: $\boldsymbol{\nabla}^2\phi = \rho$

$$\tilde{\phi} \equiv \Box^{-1}\phi \qquad\qquad \Box\tilde{\phi} = \boldsymbol{\nabla}^{-2}\rho \equiv \tilde{\rho}$$

it looks as if we have generated a dynamical dof!

However, the solution of the homogeneous eq are spurious!

the same happens for s: s is non-radiative, and we must discard the solutions of the homogeneous eq $\Box s = 0$

- at the quantum level, no annihilation/creation operators associated to it; s cannot be put on the external lines (otherwise, the vacuum in GR would decay!)

- the same happens in our non-local theory. The extra term in

$$\begin{aligned}\mathcal{L}_2 &= \frac{1}{2} h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{2d} m^2 (P^{\mu\nu} h_{\mu\nu})^2 \\ &= \frac{1}{2} \left[h^{\mathrm{TT}}_{\mu\nu} \Box (h^{\mu\nu})^{\mathrm{TT}} - \frac{d-1}{d} s(\Box + m^2) s \right]\end{aligned}$$

is just a mass term for s ! However, it remains a non-radiative field, as in GR, and we must discard the plane-wave solutions of

$$(\Box + m^2) s = \tfrac{\kappa}{2(d-1)} T \,,$$

again, no propagating dof associated to s, and no issue of quantum vacuum decay !

- the local formulation introduces a spurious degrees of freedom, given by $U_{hom}(x)$. We must be aware that, given a definition of the original non-local model, $U_{hom}(x)$ is fixed (so also the initial conditions on U are fixed)

- in flat space, this spurious degree of freedom gives rise to plane waves, but there are no creation/annihilation operators associated to it

  (it is another way to see that the apparent ghost is non-dynamical)

- the stability problem is different in the original non-local formulation and in the local one

- a degravitation mechanism

$$
\begin{aligned}
G_{\mu\nu} + \frac{d-1}{2d}\, m^2 (U g_{\mu\nu})^{\mathrm{T}} &= 8\pi G\, T_{\mu\nu}\,, \\
-\Box U &= R
\end{aligned}
$$

the localformulation of the model is invariant under

$$
g_{\mu\nu} \to g_{\mu\nu}\,, \qquad U \to U + c_\Lambda\,, \qquad T_{\mu\nu} \to T_{\mu\nu} + \tfrac{\Lambda}{8\pi G} g_{\mu\nu}
$$

with $c_\Lambda \equiv [2d/(d-1)](\Lambda/m^2)$

it is a sort of degravitation mechanism where a cosmological constant can be traded for the initial condition u_0. However, this transformation connect different non-local theories

A more general class of models

- when we write $\left(g_{\mu\nu}\Box_{\rm ret}^{-1}R\right)^{\rm T}$ we must define $\Box_{\rm ret}^{-1}$ and $(...)^{\rm T}$

  In FRW me must invert

$$
\begin{aligned}
-\Box U &= R, \qquad & -\Box &= \partial_0^2 + dH\partial_0 \\
\mathcal{D}S_0 &= \dot{U}, \qquad & \mathcal{D} &= \partial_0^2 + dH\partial_0 - dH^2
\end{aligned}
$$

even after choosing the retarded Green's function we have the freedom of specifying the solutions of

$$\Box U = 0, \qquad \mathcal{D}S_0 = 0$$

the possible definitions of the non-local model are in one-to-one correspondence with the initial conditions on U, S_0

- the initial conditions on U,S_0 are fixed by the definition of the non-local operators. In general, the solution of $\Box f = j$ is

$$f(x) = (\Box^{-1} j)(x) \equiv f_{\rm hom}(x) + \int d^{d+1}x' \sqrt{-g(x')}\, G(x;x') j(x')$$

we chose G=G_{ret} but we must also specify f_{hom}

In FRW, $\Box f = -a^{-d}\partial_0(a^d \partial_0 f)$ and we define

$$U(t) \equiv -(\Box_{\rm ret}^{-1} R)(t) \equiv -\int_{t_*}^{t} dt' \, \frac{1}{a^d(t')} \int_{t_*}^{t'} dt'' \, a^d(t'') R(t'')$$

Deser-Waldron 2007

(with t_* in RD), i.e. we choose U_{hom}=0. Then

$$U(t_*) = \dot{U}(t_*) = 0, \qquad S_0(t_*) = \dot{S}_0(t_*) = 0$$

Later we will study the most general definition

- the space of initial conditions:

recall that
$$Y'' + (3-\zeta)Y' - 3(1+\zeta)Y = 3U' - 3(1+\zeta)U,$$
$$U'' + (3+\zeta)U' = 6(2+\zeta)$$

where
$$\zeta(x) = -\frac{3\Omega_M e^{-3x} + 4\Omega_R e^{-4x} - \gamma Y'}{2(\Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y)}$$

in the early Universe we can neglect the contribution of Y to ζ

and write $\zeta(x) \simeq \zeta_0$ (-2 in RD, -3/2 in MD)

$$U(x) = \frac{6(2+\zeta_0)}{3+\zeta_0}x + u_0 + u_1 e^{-(3+\zeta_0)x}$$

$$Y(x) = -\frac{2(2+\zeta_0)\zeta_0}{(3+\zeta_0)(1+\zeta_0)} + \frac{6(2+\zeta_0)}{3+\zeta_0}x + u_0 - \frac{6(2+\zeta_0)u_1}{2\zeta_0^2 + 3\zeta_0 - 3}e^{-(3+\zeta_0)x}$$
$$+a_1 e^{\alpha_+ x} + a_2 e^{\alpha_- x}, \qquad \alpha_\pm < 0 \text{ in RD, MD}$$

trade $\{U,U',Y,Y'\}(x_{in})$ for $\{u_0,u_1,a_1,a_2\}$

u_0= marginal direction u_1,a_1,a_2 =decaying directions

- we can forget about u_1, a_1,a_2 and the most general modification of the model amounts to including u_0.

This means that we now define

$$U(t) \equiv -(\Box_{\rm ret}^{-1} R)(t) \equiv u_0 - \int_{t_*}^{t} dt' \, \frac{1}{a^d(t')} \int_{t_*}^{t'} dt'' \, a^d(t'') R(t'')$$

inserting this into

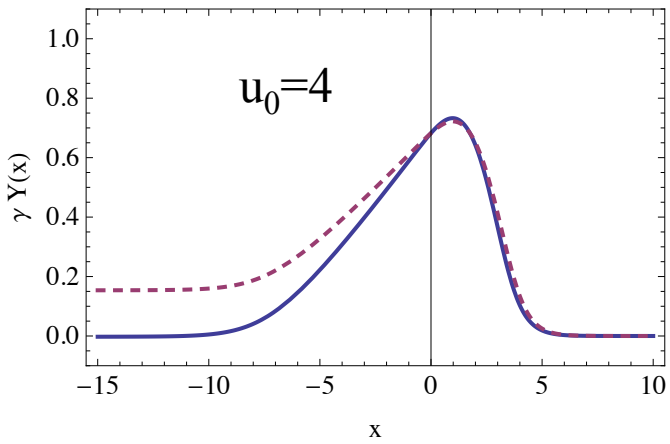
$$G_{\mu\nu} - \tfrac{d-1}{2d} \, m^2 \left(g_{\mu\nu} \Box_g^{-1} R \right)^{\rm T} = 8\pi G \, T_{\mu\nu}$$

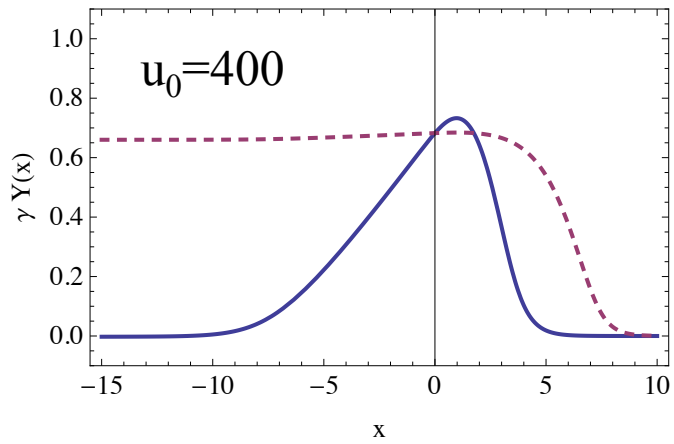
we get a cosmological constant!!

$$\Lambda = [(d-1)/2d] m^2 u_0$$

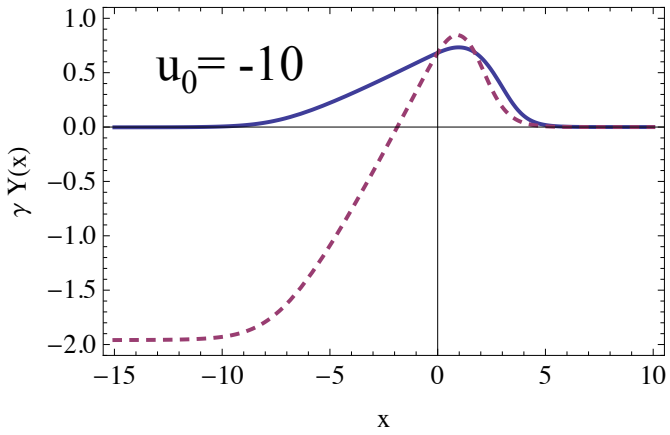
in d=3, $\Omega_\Lambda = \gamma u_0$

- effect of u_0 on the evolution:



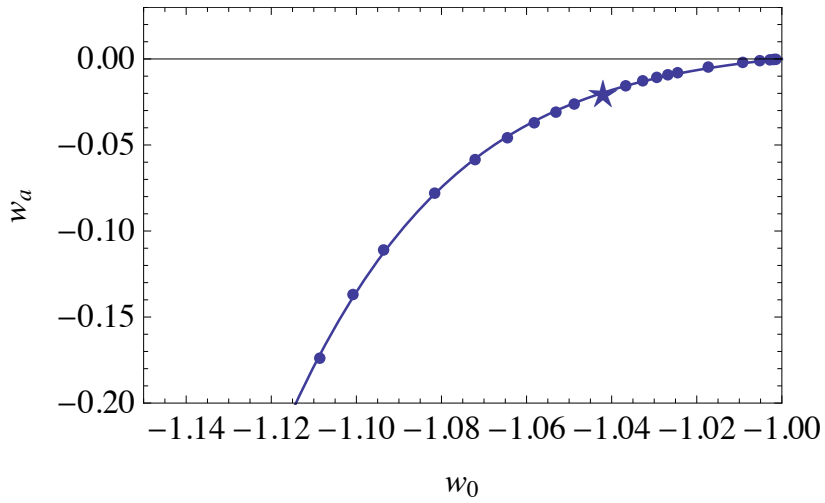


(w_0,w_a) shift toward their ΛCDM values (-1,0), but w_0 is always on the phantom side



for $u_0 < u_c \simeq -12$, no solution with $\gamma\, Y(0) = 0.68$

- the results for this class of models can be summarized by:



- w0 always on the phantom side: $-1.33 < w_0 < -1$
- prediction of a precise relation $w_a = w_a(w_0)$

- EUCLID nominal target: $\Delta w_0 = 0.01$, $\Delta w_a = 0.1$

future directions

- understanding what fundamental theory is behind these effective classical eqs

- cosmological perturbations

- structure formation