Non-local infrared modifications of gravity and dark energy

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based on

M. Jaccard, MM and E. Mitsou, 1305.3034, PR D88 (2013)
MM, arXiv: 1307.3898
S. Foffa, MM and E. Mitsou, 1311.3421, 1311.3435

A. Kehagias and MM, in preparation
Y. Dirian, S. Foffa, N. Khosravi, M. Kunz, MM, in preparation

we will only give a summary of the main results.
Glad to discuss the details during this week!
• The model:

\[ G_{\mu\nu} - \frac{d - 1}{2d} m^2 \left( g_{\mu\nu} \Box^{-1} R \right)^T = 8\pi G T_{\mu\nu} \]

where we use the fact that any symmetric tensor can be written as

\[ S_{\mu\nu} = S^T_{\mu\nu} + \frac{1}{2} (\nabla_\mu S_\nu + \nabla_\nu S_\mu) \]

where

\[ \nabla^\mu S^T_{\mu\nu} = 0 \]

our initial motivation:

introduce a mass m and deform GR in the infrared in a way which is fully covariant and does not require an external reference metric
some sources of inspiration:

- **Proca theory**

\[
S = \int d^4 x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_\gamma^2 A_\mu A^\mu - j_\mu A^\mu \right]
\]

\[
\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = j^\nu \rightarrow \begin{cases} m_\gamma^2 \partial_\nu A^\nu = 0 \\ (\Box - m_\gamma^2) A^\mu = 0 \end{cases}
\]

is equivalent to

\[
\left( 1 - \frac{m_\gamma^2}{\Box} \right) \partial_\mu F^{\mu\nu} = j^\nu
\]

(Dvali 2006; Dvali, Hofmann, Khoury 2007; review Hinterbichler 2012)

- **degravitation**

\[
\left( 1 - \frac{m^2}{\Box} \right) G_{\mu\nu} = 8\pi GT_{\mu\nu}
\]

Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002
Cosmological consequences

- consider \[ G_{\mu\nu} - \frac{d-1}{2d} m^2 \left( g_{\mu\nu} \Box^{-1} R \right)^T = 8\pi G T_{\mu\nu} \]

- specialize to FRW in d=3. Define \[ U = -\Box^{-1} R \]
  \[ S_{\mu\nu} = -U g_{\mu\nu} \]
  \[ S_{\mu\nu} = S^{T}_{\mu\nu} + (1/2) (\nabla_\mu S_\nu + \nabla_\nu S_\mu) \]

- in FRW \( S_\mu = (S_0, 0) \), so we have 3 variables: \( H(t), U(t), S_0(t) \)

\[
H^2 - \frac{m^2}{9} (U - \dot{S}_0) = \frac{8\pi G}{3} \rho \\
\rho = \rho_M + \rho_R \\
\ddot{U} + 3H\dot{U} = 6\dot{H} + 12H^2 \\
\dot{S}_0 + 3H\dot{S}_0 - 3H^2 S_0 = \dot{U}
\]
• a final massaging of the eqs.

\[ Y = U - \dot{S}_0 \quad \text{and} \quad h(t) = H(t)/H_0, \quad \Omega_i(t) = \rho_i(t)/\rho_c(t) \]

\[ x \equiv \ln a(t), \quad f' \equiv df/dx \quad \gamma \equiv m^2/(9H_0^2) \]

and their final form is

\[ h^2(x) = \Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y(x), \]

\[ Y'' + (3 - \zeta)Y' - 3(1 + \zeta)Y = 3U' - 3(1 + \zeta)U, \]

\[ U'' + (3 + \zeta)U' = 6(2 + \zeta), \]

\[ \zeta(x) \equiv \frac{h'}{h} = -\frac{3\Omega_M e^{-3x} + 4\Omega_R e^{-4x} - \gamma Y'}{2(\Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y)}. \]
• there is an effective DE term, with

\[ \rho_{\text{DE}}(x) = \rho_0 \gamma Y(x) \quad (\text{as usual } \rho_0 = 3H_0^2/(8\pi G)) \]

• define \( w_{\text{DE}} \) from

\[ \dot{\rho}_{\text{DE}} + 3(1 + w_{\text{DE}})H \rho_{\text{DE}} = 0 \]

then:

\[ w_{\text{DE}}(x) = -1 - \frac{Y'(x)}{3Y(x)} \]

• at the level of background evolution, the model has the same number of parameters as \( \Lambda \text{CDM} \), with \( \Omega_\Lambda \leftrightarrow \gamma \).
• results.

• Fixing $\gamma = 0.05..$ ($m=0.67 \, H_0$) we reproduce $\Omega_{DE}=0.68$
having fixed $\gamma$ we get a pure prediction for the EOS:

$$w(x) = w_0 + (1-a) w_a$$

in the region $-1 < x < 0$

best fit values: $w_0 = -1.042, \ w_a = -0.020$

on the phantom side!

(general consequence of $\dot{\rho}_{DE} + 3(1 + w_{DE})H \rho_{DE} = 0$
together with $\rho > 0$ and $d\rho/dt > 0$)
Planck+WP+BAO gives $w_0 = -1.04 \pm 0.72$, $w_a < 1.32$

since we predict $|w_a| << 1$, we can also compare with the Planck prediction for constant $w$

**Planck+WP+SNLS:** $w_0 = -1.13 \pm 0.13$ (95% c.l.)

**Planck+WP+Union2.1** $w_0 = -1.09 \pm 0.17$ (95% c.l.)
Consistency issues

• no issue of ghost-induced vacuum decay. The non-local eqs must be understood as an effective classical eq for the in-in quantum expectation values, and not as the eq of motion of a fundamental non-local QFT
• no vDVZ discontinuity. Solar system tests passed
• No strong classical non-linearities at short distances. Corrections to the Schwarzschild solutions are $1 + O(m^2 r^2)$
• cosmological perturbations are being worked out
Conclusions

• we have an interesting IR modification of GR

• and a testable prediction for the dark energy EOS
Thank you!
A locality / gauge-invariance duality for massive gauge fields

- Example: Proca theory for massive photons

\[ S = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_\gamma^2 A_\mu A^\mu - j_\mu A^\mu \right] \]

\[ \partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = j^\nu \rightarrow \left\{ m_\gamma^2 \partial_\nu A^\nu = 0 \right\} (\Box - m_\gamma^2) A^\mu = 0 \]

- non-local formulation \hspace{1cm} (Dvali 2006; Dvali, Hofmann, Khoury 2007)

Stueckelberg trick:

\[ A_\mu \rightarrow A_\mu + \frac{1}{m_\gamma} \partial_\mu \varphi \]

add one field and gain a gauge symmetry

\[ A_\mu \rightarrow A_\mu - \partial_\mu \theta , \hspace{0.5cm} \varphi \rightarrow \varphi + m_\gamma \theta \]
\[ S = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_\gamma^2 A_\mu A^\mu - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - m_\gamma A^\mu \partial_\mu \varphi - j_\mu A^\mu \right] \]

\[ \partial_\mu F^{\mu\nu} = m_\gamma^2 A^\nu + m_\gamma \partial^\nu \varphi + j^\nu , \]

\[ \Box \varphi + m_\gamma \partial_\mu A^\mu = 0 . \]

If we choose the unitary gauge \( \phi=0 \) we get back to the original formulation of Proca theory (and loose the gauge sym because of gauge fixing).

Instead, keep the gauge sym explicit and integrate out \( \phi \) using its own equation of motion:

\[ \varphi(x) = -m_\gamma \Box^{-1} (\partial_\mu A^\mu) \]
Substituting in the eq of motion for $A^\nu$:

$$\left(1 - \frac{m^2}{\Box}\right) \partial_\mu F^{\mu \nu} = j^\nu$$

or

$$(\Box - m^2_\gamma) A^\nu = \left(1 - \frac{m^2}{\Box}\right) \partial^\nu \partial_\mu A^\mu + j^\nu$$

we have explicit gauge invariance for the massive theory, at the price non-locality

- a sort of duality between explicit gauge-invariance and explicit locality
- we can fix the gauge $\partial_\mu A^\mu = 0$ and the non-local term disappears (and we are back to Proca eqs.)
- with hindsight, the Stueckelberg trick was not needed
the original idea: write the linearize Fierz-Pauli theory in non-local form to preserve linearized diff, and covariantize

\[ S_{\text{FP}} = \frac{1}{2} \int d^4x \left[ h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - m^2(h_{\mu\nu} h^{\mu\nu} - h^2) \right] \]

Stueckelberg trick: \[ h_{\mu\nu} \rightarrow h_{\mu\nu} + \frac{1}{m}(\partial_\mu A_\nu + \partial_\nu A_\mu) , \]
restores inv under \[ h_{\mu\nu} \rightarrow h_{\mu\nu} - (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu) , \quad A_\mu \rightarrow A_\mu + m\xi_\mu \]

eq of motion for \( A_\nu \):
\[ \partial_\mu F^{\mu\nu} = -m\partial_\mu (h^{\mu\nu} - \eta^{\mu\nu} h) \equiv j^\nu \]

write \( A^\nu = A^\nu_T - \partial^\nu \alpha \)
\[ \square A^\nu_T = j^\nu \]

Dvali, Hofmann, Khoury (2007)
\[ \rightarrow A^\nu = \square^{-1} j^\nu - \partial^\nu \alpha \]
\( \alpha \) undetermined (peculiar of FP)
• in QED, we found that a massive deformation of the theory is obtained replacing

\[ \partial_\mu F^{\mu\nu} = j^\nu \rightarrow \left(1 - \frac{m^2}{\Box}\right) \partial_\mu F^{\mu\nu} = j^\nu \]

• for gravity, a first guess for a massive deformation of GR could be

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \rightarrow \left(1 - \frac{m^2}{\Box_g}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu} \]

however this is not correct since \[ \nabla^\mu (\Box_g^{-1} G_{\mu\nu}) \neq 0 \]

We would lose energy-momentum conservation.
eliminating $A^\mu$ and defining $N \equiv \frac{h}{2} - \frac{\square \alpha}{m}$ we finally get

$$S_{FP} = \int d^4x \left[ \frac{1}{2} h_{\mu\nu} \left( 1 - \frac{m^2}{\square} \right) \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - 2m^2 N \frac{1}{\square} \partial_\mu \partial_\nu (h^{\mu\nu} - \eta^{\mu\nu} h) \right]$$

6 d.o.f: massive spin 2 + 1 scalar ghost

linearized Ricci scalar

The field $N$ (which is basically the longitudinal mode of $A^\mu$) acts as a Lagrange multiplier.

At the linearized level it kills the ghost.

However, if we covariantize the theory, it imposes the condition $R=0$, which is not present in GR $\Rightarrow$ a fully covariant vDVZ discontinuity! (Porrati 2002)
to understand the properties of the theory we linearize over Minkowski space: in d spatial dimensions

\[ E^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{d} m^2 P^{\mu\nu} P^{\rho\sigma} h_{\rho\sigma} = -16\pi G T^{\mu\nu} \]

\[ P^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\Box} \]

the corresponding Lagrangian is (only formally! see later)

\[ \mathcal{L}_2 = \frac{1}{2} h_{\mu\nu} E^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{2d} m^2 h_{\mu\nu} P^{\mu\nu} P^{\rho\sigma} h_{\rho\sigma} \]

we add a gauge fixing and we get the propagator
\[ \tilde{D}^{\mu\nu\rho\sigma}(k) = \frac{-i}{2k^2} \left( \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \frac{2}{d-1} \eta^{\mu\nu} \eta^{\rho\sigma} \right) \]

\[ - \frac{1}{d(d-1)} \frac{im^2}{k^2(-k^2 + m^2)} \eta^{\mu\nu} \eta^{\rho\sigma} \]

- no vDVZ discontinuity!
- For \( m = O(H_0) \), solar system test easily passed. Corrections are \( O(m^2/k^2) = 10^{-30} \) for \( k = (1 \text{ a.u})^{-1} \).
- massless graviton + extra contribution to \( \tilde{T}_{\mu\nu}(-k) \tilde{D}^{\mu\nu\rho\sigma}(k) \tilde{T}_{\rho\sigma}(k) \)

\[ \frac{1}{d(d-1)} \tilde{T}(-k) \left[ -\frac{i}{k^2} - \frac{i}{(-k^2 + m^2)} \right] \tilde{T}(k) \]

one massles scalar + one massive scalar ghost?
The ghost

a ghost has in principle two type of effects:

- at the classical level, it can give rise to runaway solution. for a ghost with \( m = O(H_0) \) this can even be welcome for explaining the acceleration of the Universe

- at the quantum level, it produces a vacuum decay rate, \( \Gamma \)

  for a ghost interacting gravitationally with local interactions:

  \[
  \Gamma \sim \frac{\Lambda^8}{M_{P1}^4}
  \]

  if \( \Lambda \sim M_{P1} \), \( \Gamma \sim M_{P1}^4 \)
Non-local QFT or classical effective equations?

• we have $\square^{-1}_{\text{ret}}$ directly in the EoM (rather than in the solution). This EoM cannot come from the variation of a Lagrangian. E.g.

$$
\frac{\delta}{\delta \phi(x)} \int dx' \phi(x')(\square^{-1} \phi)(x') = \frac{\delta}{\delta \phi(x)} \int dx'dx'' \phi(x')G(x', x'')\phi(x'')
$$

$$
= \int dx' [G(x, x') + G(x', x)]\phi(x')
$$

• we can replace $\square^{-1} \rightarrow \square^{-1}_{\text{ret}}$ after the variation, as a formal trick to get the EoM from a Lagrangian. 

However, any connection to the QFT described by this Lagrangian is lost.
Taking
\[ \mathcal{L}_2 = \frac{1}{2} h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{2d} m^2 h_{\mu\nu} P^{\mu\nu} P^{\rho\sigma} h_{\rho\sigma} \]
as the quantum Lagrangian of our problem introduces spurious propagating degrees of freedom. In
\[ \tilde{D}^{\mu\nu\rho\sigma}(k) = \frac{-i}{2k^2} \left( \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \frac{2}{d-1} \eta^{\mu\nu} \eta^{\rho\sigma} \right) \]
the apparent ghost-like pole in the propagator already comes with a retarded prescription. It is not a Feynman propagator and does not describe a propagating dof!

Our non-local EoM is not the classical EoM of a non-local QFT!
EoMs involving $\Box^{-1}_{\text{ret}}$ emerge from a classical or a quantum averaging of a more fundamental (local) QFT

- **Classically**, when separating long and short wavelength and integrating out the short wavelength (e.g., cosmological perturbation theory, or GWs)

- **In QFT**, when computing the effective action that includes the effect of radiative corrections. This provides effective non-local field eqs for $\langle 0|\hat{\phi}|0\rangle$, $\langle 0|\hat{g}_{\mu\nu}|0\rangle$

- The in-in matrix elements satisfy non-local and retarded equations

  Jordan 1986, Calzetta-Hu 1987
• So, we interpret our non-local eqs as a classical, effective equation, derived from a more fundamental local theory by a classical or quantum averaging.

• any problem of quantum vacuum stability can only be addressed in this fundamental theory.

• in any case, the apparent ghost that we found has nothing to do with quantum vacuum decay in our model. It can however trigger classical cosmological instabilities.
A fake ghost in massless GR

\[ S_{EH}^{(2)} = \frac{1}{2} \int d^{d+1} x \ h_{\mu \nu} \mathcal{E}^{\mu \nu, \rho \sigma} h_{\rho \sigma} \]

\[ h_{\mu \nu} = h_{\mu \nu}^{TT} + \frac{1}{2} (\partial_{\mu} \epsilon_{\nu} + \partial_{\nu} \epsilon_{\mu}) + \frac{1}{d} \eta_{\mu \nu} s \]

\[ S_{EH}^{(2)} = \frac{1}{2} \int d^{d+1} x \left[ h_{\mu \nu}^{TT} \Box (h_{\mu \nu}^{\mu \nu})^{TT} - \frac{d - 1}{d} s \Box s \right] \]

\[ S_{\text{int}} = \frac{\kappa}{2} \int d^{d+1} x \ h_{\mu \nu} T^{\mu \nu} = \frac{\kappa}{2} \int d^{d+1} x \left[ h_{\mu \nu}^{TT} (T_{\mu \nu}^{TT})^{TT} + \frac{1}{d} s T \right] \]

\[ \Box h_{\mu \nu}^{TT} = -\frac{\kappa}{2} T_{\mu \nu}^{TT}, \quad \Box s = \frac{\kappa}{2(d - 1)} T \]

It looks as if there are many more propagating d.o.f
Furthermore s seems a ghost!
• the origin of the problem is that $s$ is a non-local function of $h_{\mu\nu}$:

$$s = \left( \eta_{\mu\nu} - \frac{1}{\Box} \partial^{\mu} \partial^{\nu} \right) h_{\mu\nu} = P^{\mu\nu} h_{\mu\nu}$$

• example: $\nabla^2 \phi = \rho$

$$\tilde{\phi} \equiv \Box^{-1} \phi \quad \square \tilde{\phi} = \nabla^{-2} \rho \equiv \tilde{\rho}$$

it looks as if we have generated a dynamical dof!

However, the solution of the homogeneous eq are spurious!

the same happens for $s$: $s$ is non-radiative, and we must discard the solutions of the homogeneous eq $\Box s = 0$

• at the quantum level, no annihilation/creation operators associated to it; $s$ cannot be put on the external lines (otherwise, the vacuum in GR would decay!)
the same happens in our non-local theory. The extra term in

\[ \mathcal{L}_2 = \frac{1}{2} h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d - 1}{2d} m^2 (P^{\mu\nu} h_{\mu\nu})^2 \]

\[ = \frac{1}{2} \left[ h_{\mu\nu}^{TT} \Box (h_{\mu\nu}^{TT}) - \frac{d - 1}{d} s (\Box + m^2) s \right] \]

is just a mass term for \( s \). However, it remains a non-radiative field, as in GR, and we must discard the plane-wave solutions of

\[ (\Box + m^2) s = \frac{\kappa}{2(d - 1)} T, \]

again, no propagating dof associated to \( s \), and no issue of quantum vacuum decay!
• the local formulation introduces a spurious degrees of freedom, given by $U_{\text{hom}}(x)$. We must be aware that, given a definition of the original non-local model, $U_{\text{hom}}(x)$ is fixed (so also the initial conditions on $U$ are fixed).

• in flat space, this spurious degree of freedom gives rise to plane waves, but there are no creation/annihilation operators associated to it

  (it is another way to see that the apparent ghost is non-dynamical)

• the stability problem is different in the original non-local formulation and in the local one
• a degravitation mechanism

\[ G_{\mu\nu} + \frac{d-1}{2d} m^2 (U g_{\mu\nu})^T = 8\pi G T_{\mu\nu} , \]

\[ -\Box U = R \]

the local formulation of the model is invariant under

\[ g_{\mu\nu} \rightarrow g_{\mu\nu} , \quad U \rightarrow U + c_\Lambda , \quad T_{\mu\nu} \rightarrow T_{\mu\nu} + \frac{\Lambda}{8\pi G} g_{\mu\nu} \]

with \( c_\Lambda \equiv [2d/(d - 1)](\Lambda/m^2) \)

it is a sort of degravitation mechanism where a cosmological constant can be traded for the initial condition \( u_0 \). However, this transformation connect different non-local theories
A more general class of models

- when we write \( (g_{\mu \nu} \Box_{\text{ret}}^{-1} R)^T \) we must define \( \Box_{\text{ret}}^{-1} \) and \((...)^T\)
  
  In FRW we must invert

\[
-\Box U = R, \quad -\Box = \partial_0^2 + dH \partial_0 \\
\mathcal{D} S_0 = \dot{U}, \quad \mathcal{D} = \partial_0^2 + dH \partial_0 - dH^2
\]

even after choosing the retarded Green's function we have the freedom of specifying the solutions of

\[
\Box U = 0, \quad \mathcal{D} S_0 = 0
\]

the possible definitions of the non-local model are in one-to-one correspondence with the initial conditions on \( U, S_0 \)
the initial conditions on $U,S_0$ are fixed by the definition of the non-local operators. In general, the solution of $\Box f = j$ is

$$f(x) = (\Box^{-1} j)(x) \equiv f_{\text{hom}}(x) + \int d^{d+1}x' \sqrt{-g(x')} G(x;x')j(x')$$

we chose $G=G_{\text{ret}}$ but we must also specify $f_{\text{hom}}$. In FRW, $\Box f = -a^{-d}\partial_0 (a^d\partial_0 f)$ and we define

$$U(t) \equiv - (\Box_{\text{ret}}^{-1} R)(t) \equiv - \int_{t_*}^t dt' \frac{1}{a^d(t')} \int_{t_*}^{t'} dt'' a^d(t'') R(t'')$$

Deser-Waldron 2007

(with $t_*$ in RD), i.e. we choose $U_{\text{hom}}=0$. Then

$$U(t_*) = \dot{U}(t_*) = 0, \quad S_0(t_*) = \dot{S}_0(t_*) = 0$$

Later we will study the most general definition
the space of initial conditions:

recall that

\[ Y'' + (3 - \zeta)Y' - 3(1 + \zeta)Y = 3U' - 3(1 + \zeta)U, \]

\[ U'' + (3 + \zeta)U' = 6(2 + \zeta) \]

where

\[ \zeta(x) = - \frac{3\Omega_M e^{-3x} + 4\Omega_R e^{-4x} - \gamma Y'}{2(\Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y)} \]

in the early Universe we can neglect the contribution of \( Y \) to \( \zeta \)
and write \( \zeta(x) \approx \zeta_0 \) \((-2 \text{ in RD, } -3/2 \text{ in MD})\)

\[ U(x) = \frac{6(2+\zeta_0)}{3+\zeta_0} x + u_0 + u_1 e^{-(3+\zeta_0)x} \]

\[ Y(x) = - \frac{2(2 + \zeta_0)\zeta_0}{(3 + \zeta_0)(1 + \zeta_0)} + \frac{6(2 + \zeta_0)}{3 + \zeta_0} x + u_0 - \frac{6(2 + \zeta_0)u_1}{2\zeta_0^2 + 3\zeta_0 - 3} e^{-(3+\zeta_0)x} + a_1 e^{\alpha x} + a_2 e^{\alpha - x}, \quad \alpha_\pm < 0 \text{ in RD, MD} \]

trade \( \{U,U',Y,Y'\}(x_{in}) \) for \( \{u_0,u_1,a_1,a_2\} \)

\( u_0 = \text{marginal direction} \quad u_1,a_1,a_2 = \text{decaying directions} \)
• we can forget about $u_1$, $a_1,a_2$ and the most general modification of the model amounts to including $u_0$.

This means that we now define

$$U(t) \equiv -\left(\square^{-1}_{\text{ret}} R\right)(t) \equiv u_0 - \int_{t_*}^{t} dt' \frac{1}{a^d(t')} \int_{t_*}^{t'} dt'' a^d(t'') R(t'')$$

inserting this into

$$G_{\mu\nu} - \frac{d-1}{2d} m^2 \left(g_{\mu\nu} \square^{-1} R\right)^T = 8\pi G T_{\mu\nu}$$

we get a cosmological constant!!

$$\Lambda = \left[(d - 1)/2d\right] m^2 u_0$$

in $d=3$, $$\Omega_\Lambda = \gamma u_0$$
• effect of $u_0$ on the evolution:

$(w_0, w_a)$ shift toward their $\Lambda$CDM values (-1,0), but $w_0$ is always on the phantom side

for $u_0 < u_c \approx -12$, no solution with $\gamma Y(0) = 0.68$
• the results for this class of models can be summarized by:

• $w_0$ always on the phantom side: $-1.33 < w_0 < -1$
• prediction of a precise relation $w_a = w_a(w_0)$
• EUCLID nominal target: $\Delta w_0 = 0.01$, $\Delta w_a = 0.1$
future directions

• understanding what fundamental theory is behind these effective classical eqs

• cosmological perturbations

• structure formation