Relativistic Contribution to LSS in ΛCDM

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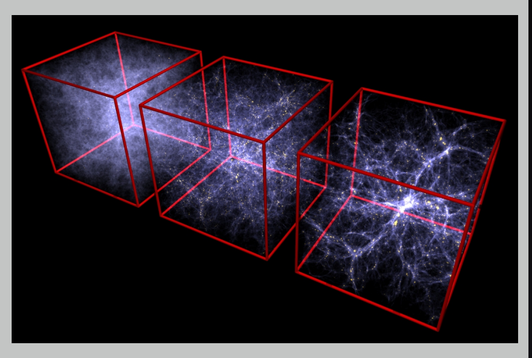
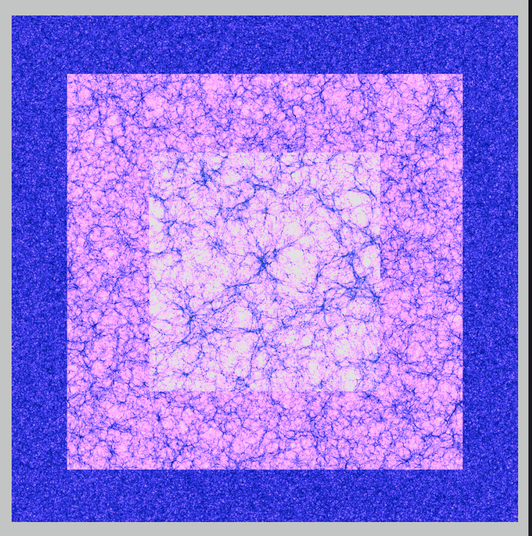
Introduction: Newtonian simulations in Relativistic regimes





- N-body simulations of Large Scale Structure (LSS) assume Newtonian gravitational interactions reaching great acuracy
- Poisson Equation is the only Field Eqn.

$$\nabla^2\Phi_N = 4\pi G a^2 \bar{\rho}\delta_N$$

- Simulation volumes are reaching the Hubble scale, where GR effects are expected:
  When $v/c \lesssim 1 \Rightarrow r/t \lesssim 1 \Rightarrow$
  $\nabla^2/\mathcal{H}^2 \lesssim 1$
- **How can we make N-body simulations compatible with General Relativity?**

Newtonian Framework (Fluid Approximation)

- Newtonian Quantities:
  Matter density $\rho_N := \bar{\rho}\,[1+\delta_N(x,\tau)]$,
  Particle velocity $u^i_N(x,t) := x^i\mathcal{H} + \partial^i v_N$,
  Total Energy $E_N := \bar{E}_N + \delta E_N(x)$,
- Newtonian Equations:
  - Continuity Eqn. $\frac{d}{d\tau}(\delta_N) = -(1+\delta_N)\nabla^2 v_N$,
  - Euler Eqn. $\frac{d}{d\tau}(\nabla^2 v_N) + \mathcal{H}\nabla^2 v_N + \partial^i\partial_k v_N \partial^k\partial_i v_N + \nabla^2\Phi_N = 0$.
- $+$ Poisson Eqn. $\Rightarrow$ Evolution Equation

$$\delta_N'' + \mathcal{H}\delta_N' - \frac{3}{2}\mathcal{H}^2\Omega_m\delta_N = (\delta_N\nabla^2 v_N)' - 2\mathcal{H}\delta\nabla^2 v_N + 2\partial^i\partial_j v_N\partial^j\partial_i v_N$$

And an Energy Constraint, preserved in time:

$$\nabla^2 E_N(x) = \frac{1}{2}\nabla^2 u_N^2 - 4\pi G a^2\rho_N\,. \tag{1}$$

Relativistic Description of dust in FRW

- Metric (Synchronous-Comoving):

$$ds^2 = a^2(\tau)\left(-d\tau^2 + \{\exp[-2\mathcal{R}_c]\delta_{ij} + \chi_{ij}\}\,dx^i dx^j\right), \tag{2}$$

- Comoving Density $\rho_c := \bar{\rho}\,[1+\delta_c(x,\tau)]$,
- Deformation $\vartheta^\mu_\nu := a(\tau)\partial^\mu u_\nu - \mathcal{H}\delta^\mu_\nu$,
- Ricci Curvature $\mathbb{R} := \exp(2\mathcal{R}_c)\left[4\nabla^2\mathcal{R}_c - 2(\nabla\mathcal{R}_c)^2\right]$.

Relativistic Equations:

- Continuity Eqn. $\frac{d}{d\tau}(\delta_c) = -(1+\delta_c)\vartheta$,
- Raichaudhuri Eqn. $\vartheta' + \mathcal{H}\vartheta + \vartheta^i_j\vartheta^j_i + 4\pi G a^2\bar{\rho}\delta_c = 0$.

$\Rightarrow$ Evolution Equation

Same as the Newtonian evolution

Provided we use the correspondence [1,2]:

- Newtonian speed with Deformation: $\partial^i\partial_k v_N = \vartheta^i_k$
- Newtonian density with Comoving density: $\delta_N = \delta_c$
- Newtonian energy with Spatial curvature: $4\nabla^2\delta E_N(x) = -\mathbb{R}$

Where is the difference in a ΛCDM Universe?

- Newtonian Energy Constraint:

$$4\mathcal{H}\nabla^2 v_N - 16\pi G a^2\bar{\rho}\delta_N + \mathbb{R}(x) = 0,$$

- Relativistic Energy Constraint (Synchronous-Comoving):

$$\vartheta^2 - \vartheta^i_j\vartheta^j_i - 16\pi G a^2\bar{\rho}\delta_c + 4\mathcal{H}\vartheta + \mathbb{R} = 0\,.$$

Exact correspondence at linear order

We choose a perturbative expansion, valid for $\delta_c \ll 1$. Every quantity is expanded as

$$A(x,\tau) = \bar{A}(\tau) + \sum_{n=1}^{\infty}\frac{1}{n!}A^{(n)}(x,\tau), \tag{3}$$

- Curvature sources density via the growing mode D_+, also $d/d\tau(\mathcal{R}_c^{(1)}) = 0$.
- At first order both Newtonian and Relativistic constraints are equivalent[3].
  - At early times τ_{IN} In dust-dominated Universe $\Omega_m(\tau_{IN}) = 1$ and

$$\delta^{(1)}(x,\tau) = \frac{2}{3}\frac{\nabla^2\Phi_N}{\mathcal{H}^2} = \frac{2}{5}\frac{\nabla^2\mathcal{R}_c^{(1)}(x)}{\mathcal{H}^2(\tau)} = \frac{1}{10}\frac{\mathbb{R}^{(1)}}{\mathcal{H}^2(\tau)}. \tag{4}$$

  - At late times we define $f_1 = d/d\tau\,\ln(D_+)\mathcal{H}^{-1}$ and both theories dictate

$$\delta^{(1)}(\tau,x) = \nabla^2\mathcal{R}_c^{(1)}(x)\left[\frac{3}{2} + \frac{f_1(\Omega_{mIN})}{\Omega_{mIN}}\right]^{-1}\frac{D_+(\tau)}{\mathcal{H}^2_{IN}D_{+IN}}$$

Higher order GR corrections initial conditions (Our work!)

The full GR solution initial conditions $\neq$ to Newtonian. Most importantly, at large scales, gradients are small and we can compute the curvature $\mathbb{R}$ from only the conformal part of the spatial metric in (2), that is:

- The Conformal curvature can be computed at any order[4]:

$$\mathbb{R} = 4\nabla^2\mathcal{R}_c - \sum_{m=0}^{\infty}\frac{(2)^{m+1}}{m!}\left[(\nabla\mathcal{R}_c)^2 + \frac{4}{m+1}\mathcal{R}_c\nabla^2\mathcal{R}_c\right](\mathcal{R}_c)^m$$

- The solution at large scales (lowest order in gradient expansion) takes the same form as the linear solution (4), i.e., at early times and large scales [5,6]

$$\delta^{(n)}(x,\tau) = \frac{1}{10}\frac{\mathbb{R}^{(n)}}{\mathcal{H}^2(\tau)}\,. \tag{5}$$

This yields, at any order in perturbative expansion, the dominant GR contribution to the matter density fluctuation.

Results: GR induces non-Gaussianity in LSS

Curvature perturbations may have been sourced at non-linear order from inflation. Non-linearity implies interaction between different (Fourier) modes which breaks the Gaussian hypothesis. In the local limit (squeezed configurations), the non-Gaussian parameters f_{NL} and g_{NL} quantify the mild non-linearity of the curvature perturbation as (c.f. Eq. (3)):

$$\mathcal{R}_c = \mathcal{R}_c^{(1)} + \frac{1}{2}\mathcal{R}_c^{(2)} + \frac{1}{6}\mathcal{R}_c^{(3)} = \mathcal{R}_c^{(1)} + \frac{3}{5}f_{NL}(\mathcal{R}_c^{(1)})^2 + \frac{9}{25}g_{NL}(\mathcal{R}_c^{(1)})^3$$

Our result for the non-linear $\delta^{(n)}$ in Eq. (5) shows that non-linear correlations in the matter density field carry an effective non-Gaussianity from General Relativity. In short, GR induces the following effective non-Gaussianities in the local limit [5,6],

$$f_{NL}^{GR} = -\frac{5}{3} \qquad g_{NL}^{GR} = \frac{50}{27} - \frac{10}{3}f_{NL}$$

Summary and Future work

- We found non-linear correlations (Non-Gaussianities) induced by General Relativity in the matter distribution of a ΛCDM Universe.
- We propose to include these results in the non-linear initial conditions of N-body simulations and also take them on account in the measurement of non-Gaussianities from the LSS.
  - The featured results modify the implementation of the Zel'dovich approximation and the second order lagrangian fluctuations (2LPT). *Work in progress*
- The evolution of inhomogeneities is a function of the initial geometry [5].
  - The correspondence of perturbations with inhomogeneous cosmologies with spherical and non-spherical symmetries quantify the difference in the non-linear regime. *Work in progress*

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