Unified description for quintessence fields

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(arXiv:1108.4712)
**Quintessence (Dark Energy)**

- **Accelerating solutions** in a Friedmann-Robertson-Walker universe from scalar fields are well known from inflationary theories.

- **All we need is an appropriate scalar potential**, and good conditions for late-time domination, after the required transitions: radiation -> matter -> dark energy

- **Proposal**: The physically relevant solutions are represented by heteroclinic lines connecting critical points on the phase-space of the field variables.
Dynamical structure

Full equations of motion in a FRW universe with a matter perfect fluid + quintessence field:

\[
\begin{align*}
\dot{H} &= \frac{8\pi G}{2} \left( \gamma \rho_\gamma + \dot{\phi}^2 \right) \\
\dot{\rho}_\gamma &= -3H \gamma \rho_\gamma \\
\ddot{\phi} &= 3H \dot{\phi} - \partial_\phi V \\
H^2 &= \frac{8\pi G}{3} \left( \rho_\gamma + \frac{1}{2} \dot{\phi}^2 + V \right) \\
p_\gamma &= (\gamma - 1) \rho_\gamma \\
\end{align*}
\]

Dynamical variables:

\[
\begin{align*}
x &\equiv \frac{\kappa \dot{\phi}}{\sqrt{6H}}, \quad y \equiv \frac{\kappa \sqrt{V}}{\sqrt{3H}}, \quad \lambda = -\partial_\phi V/V, \quad N = \ln(a)
\end{align*}
\]

Quintessence physical parameters:

\[
\begin{align*}
\gamma_\phi &= \frac{p_\phi + \rho_\phi}{\rho_\phi} = \frac{2x^2}{x^2 + y^2}, \quad \Omega_\phi = \frac{\kappa^2 \rho_\phi}{3H^2} = x^2 + y^2 \\
\text{Density parameter}
\end{align*}
\]
Dynamical structure

* New equations of motion:

\[
\begin{align*}
x' &= -3x + \lambda \sqrt{\frac{3}{2}y^2 + \frac{3}{2}x [2x^2 + \gamma(1 - x^2 - y^2)]}, \\
y' &= -\lambda \sqrt{\frac{3}{2}xy + \frac{3}{2}y [2x^2 + \gamma(1 - x^2 - y^2)]}, \\
\lambda' &= -\sqrt{6x\lambda^2[V\partial\phi V/(\partial\phi V)^2 - 1]}
\end{align*}
\]

* Phase-space structure

**Critical points**: points in phase space at which the phase velocity vanishes;

**Heteroclinic lines**: trajectories in the phase-space that connect two critical points.
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Copeland, Liddle, Wands
**Dynamical structure (Exponential case)**

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PRD 57, 4686 (1998);
U-L, arXiv:1108.4712
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Because of the long matter-dominated period, appropriate initial conditions correspond to phase-space points nearby the matter-dominated point A; late-dynamics of the quintessence field is influenced by this fact and its evolution must not start arbitrarily.

Flow parameter
(Cahn, de Putter, and Linder
JCAP 0811, 015, 2008)

\[ F = \frac{\gamma \phi}{\Omega \phi \lambda^2} \simeq \text{const} \]
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**Thawing constraint**

Because of matter-domination, the quintessence field follows the phase-space trajectory:

\[ x(y) = \frac{2}{2 + \gamma \sqrt{6}} y^2 \]

In terms of the flow parameter:

\[ F = \frac{4}{3(2 + \gamma)^2} \]
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Large initial values of the roll parameter are not allowed by observations.
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\[ \lambda = 2 \]

*Projection onto the 2D phase space*
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**Heteroclinic trajectories under different initial conditions**
Conclusions

The dynamics of dark energy scalar fields, for physically relevant solutions, is fully described by critical points and heteroclinic trajectories on the phase-space.

The viability of a given model is also easily determined by looking at its heteroclinic trajectories departing from the critical point of matter domination.

The phase space of any quintessence field is topologically similar to that of the exponential potential.

That physically relevant trajectories are also heteroclinic must be true for other models of dark energy (in terms of properly chosen phase variables) (conjecture).