Probing nonlinear electromagnetic cosmological models with GRBs.

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The model
Outline

1 Introduction

2 The model

3 Calibrating GRBs
Outline

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4. Data Analysis and Results
5. Conclusions
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- Novello et al. (2004)

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Instead to consider an EM field with a volumetric average, we study a model proposed by Dyadichev et al. (2002) that contains homogeneous and isotropic solutions supported by the SU(2) gauge field governed by the Born-Infeld Lagrangian.

The model starts with the action

\[ S = -\frac{1}{4\pi} \int \left\{ \frac{1}{4G} R + \beta^2 (\mathcal{R} - 1) \right\} \sqrt{-g} \, d^4 x, \]  

(3)

where \( R \) is the scalar curvature, \( \beta \) is the BI critical field strength, and the quantity

\[ \mathcal{R} = \sqrt{1 + \frac{1}{2\beta^2} F_{\mu\nu}^a F_{\nu\mu}^a - \frac{1}{16\beta^4} (\tilde{F}_{\mu\nu}^a F_{\nu\mu}^a)^2}, \]

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The model

The model deals with the homogeneous and isotropic configurations of the SU(2) Yang-Mills field and considers RW geometry.

We shall consider the case of spatially flat geometry for the RW cosmology

\[ ds^2 = dt^2 - a^2 \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] . \]  

(4)

In Dyadichev et al. (2002) it is obtained the reduced action

\[ S_1 = \frac{1}{4\pi G \beta} \int dt \left\{ \frac{3}{2} a \dot{a}^2 - g a^3 \left[ \sqrt{(1 - K^2)(1 + V^2)} - 1 \right] \right\} , \]

(5)

where

\[ K = \frac{\sqrt{3w}}{a} , \quad V = -\frac{\sqrt{3w^2}}{a^2} \]

(6)

and \( g = \beta G \) is a dimensionless coupling constant.
Friedmann equations

\[ \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho, \]  
(7)

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p), \]  
(8)

with

\[ p = \frac{1}{3} \rho_c (3 - \mathcal{P} - 2\mathcal{P}^{-1}) \quad \text{and} \quad \rho = \rho_c (\mathcal{P} - 1), \]  
(9)

\[ p = \frac{\rho (\rho_c - \rho)}{3 (\rho_c + \rho)} \]  
(10)

where \( \rho_c = \beta/4\pi \) plays a role of the BI critical energy density and \( \mathcal{P} \) is given by

\[ \mathcal{P} = \sqrt{\frac{1 + V^2}{1 - K^2}}. \]  
(11)
On the other hand, the gauge field influences the metric only through the quantity $\mathcal{P}$, related to the energy density. This quantity obeys the differential equation:

\[
\dot{\mathcal{P}} = 2\frac{\dot{a}}{a} \left( \frac{1}{\mathcal{P}} - \mathcal{P} \right),
\]  \hspace{1cm} (12)

and this last equation can be integrated once, giving $\mathcal{P}$ as a function of $a$:

\[
\mathcal{P} = \sqrt{1 - \kappa \left( \frac{a_0}{a} \right)^4},
\]  \hspace{1cm} (13)

where $\kappa$ is an integration constant.

The **Friedmann equation** turns out to be

\[
H^2 = \frac{2g}{3} \left[ \sqrt{1 - \kappa(1 + z)^4} - 1 \right]
\]  \hspace{1cm} (14)
Since the earliest evidence of tight correlations in gamma-ray bursts spectral properties, the possibility arose of using GRBs as standard candles. Being so GRBs may open a window in redshift as far as $z \sim 8$, extending then the attainable range provided by SNe Ia observations.

Amati relation

We shall apply the empirical relation $E_p - E_{iso}$ derived by Amati (2008), that connects

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We calibrated the Amati relation for 50 GRBs low-redshift \((z < 1.4)\) using the 557 Union2 SNe Ia data following the cosmology-independent calibration method proposed by Liang (2008):

\[
\log \frac{E_{iso}}{\text{erg}} = \lambda + b \log \frac{E_p}{300\text{keV}}. \tag{15}
\]

With

\[
\lambda = 52,7636 \pm 0,0626, \quad b = 1,6283 \pm 0,1059. \tag{16}
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Extrapolating the calibrated Amati relation to 59 high-redshift GRBs, we obtained the distance moduli \(\mu\) for the extended sample of 59 GRBs at \(z > 1.4\) using

\[
E_{iso} = 4\pi d_L^2 S_{bol}(1 + z)^{-1} \quad \text{and} \quad \mu = 5 \log \frac{d_L}{\text{Mpc}} + 25. \tag{17}
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The theoretical distance modulus is defined by

\[ \mu_{th}(z; a_1, \ldots, a_n) = 5 \log \frac{d_{L}^{th}(z; a_1, \ldots, a_n)}{\text{Mpc}} + 25. \]  

On the other hand, given a parametrization \( H(z; a_1, \ldots, a_n) \) depending on \( n \) parameters \( a_i \), the corresponding Hubble free luminosity distance in a flat cosmology is

\[ d_{L}^{th}(z; a_1, \ldots, a_n) = c(1 + z) \int_{0}^{z} \frac{1}{H(z'; a_1, \ldots, a_n)} dz'. \]  

Using the maximum likelihood technique we can find the goodness of fit for the corresponding observed \( d_{L}^{obs}(z_i) \).
The best-fit model parameters are determined by minimizing $\chi^2_{\mu}(g, \kappa)$:

$$\chi^2_{\mu}(g, \kappa) = \sum_i \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i, g, \kappa)]^2}{\sigma^2_{\mu_{obs}}(z_i)}.$$  (18)
## Results

The best-fit value for the nonlinear electromagnetic cosmological model parameters \((g, \kappa)\) and the \(\chi^2_{d.o.f.}\) using SNe Ia and SNe Ia + GRBs are summarized in the following table:

<table>
<thead>
<tr>
<th></th>
<th>SNe Ia</th>
<th>SNe Ia + GRBs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g)</td>
<td>5999.9993 +59,9983</td>
<td>5999.9994 +59,5110</td>
</tr>
<tr>
<td>(\kappa)</td>
<td>-3.0742 +0,0535</td>
<td>-3.0732 +0,0535</td>
</tr>
<tr>
<td>(\chi^2_{d.o.f.})</td>
<td>1.5647</td>
<td>1.6242</td>
</tr>
</tbody>
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\[ H_0 = 63.83 \text{ km s}^{-1} \text{ Mpc}^{-1} \]
We show the joint confidence regions in the \((g, \kappa)\) plane for the nonlinear electromagnetic cosmological model. The contours correspond to \(1\sigma-4\sigma\) confidence regions using SNe Ia (left panel) and SNe Ia + GRBs (right panel).
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\begin{align*}
 g &= 5999.9993^{+59,9983}_{-49,9993} \\
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