High Precision Gravitational Hubble Diagram with Pulsar Timing Arrays

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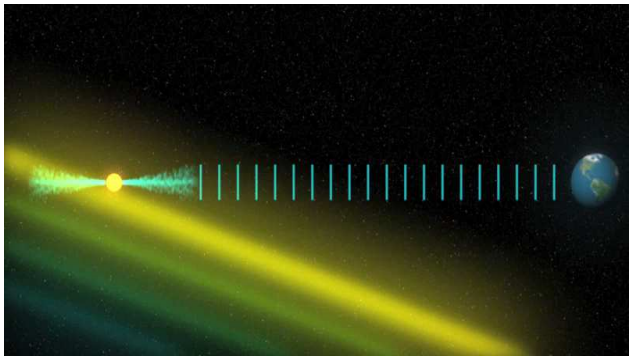
Outline

- Brief introduction on gravitational wave detection through pulsar timing arrays
- An important application on cosmology
- High precision measurement on Hubble constant and dark energy equation of state
- Conclusion and propect

Brief Introduction

The passage of gravitational waves through the Pulsar-Earth line of sight changes the distance that pulses must propagate, and correspondingly changes their arrival time.



The timing residual is

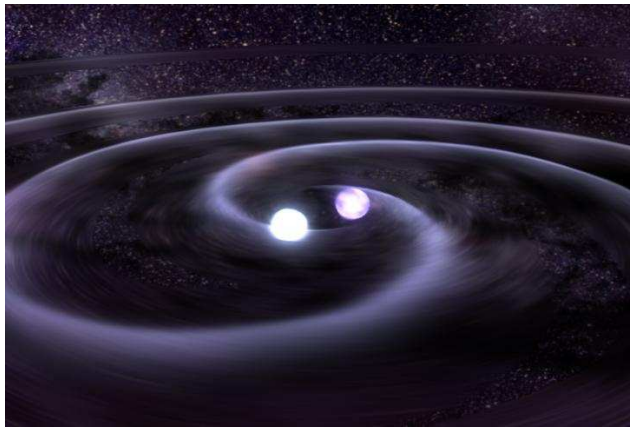
$$\tau_{GW} \sim \frac{h}{\omega} \sim 10\mathrm{ns}\left(\frac{h}{10^{-14}}\right)\left(\frac{4\times 10^{-7}s^{-1}}{\omega}\right)$$

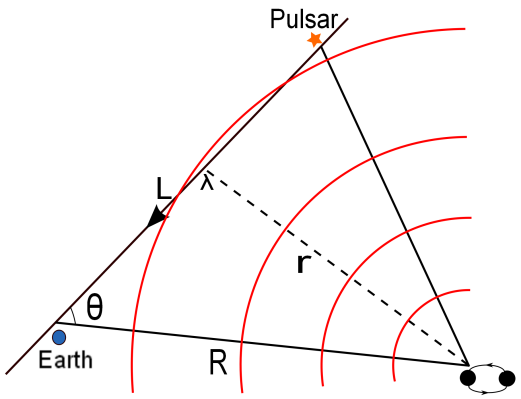
Where h is the GW strain amplitude and ω is its angular frequency. The timing residual for a black hole binary system with equal mass $10^9\mathrm{M}_\odot$ and a 1 year period is about 10ns if it is 100Mpc away.

Current and Future Radio Telescope

Application to Cosmology

During the pulse propagation from the pulsar to the Earth, the black hole binaries will evolve driven by gravitational wave emission (chirp)



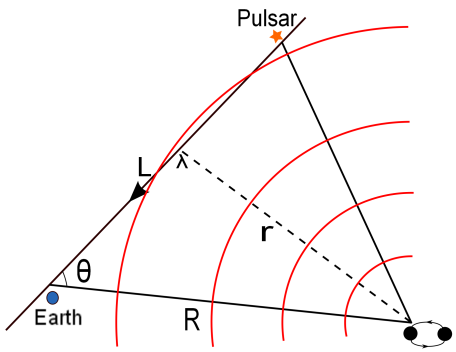


Luminosity distance to the binary can be determined by

$$h = \frac{M_c^{5/3}\omega^{2/3}}{d_L} \qquad \dot{\omega} = \frac{12}{5}2^{1/3}M_c^{5/3}\omega^{11/3}$$

Gravitational Wave Timing Signal

Consider gravitational wave evolution, gravitational wave timing signal



$$\tau_{GW} = \frac{h(1+\cos\theta)}{\omega}\sin\left(\omega t + \phi\right) - \frac{h(1+\cos\theta)}{\omega}\sin\left(\omega t + \phi - \omega L(1-\cos\theta) + \frac{3}{5}2^{4/3}M_c^{5/3}\omega^{11/3}L^2(1-\cos\theta)^2\right)$$

The signal tells us the chirp mass $M_c(=(m_1 m_2)^{3/5}/(m_1+m_2)^{1/5})$ and GW amplitude $h = M_c^{5/3}\omega^{2/3}/d_L$, and thus determines the luminosity distance d_L.

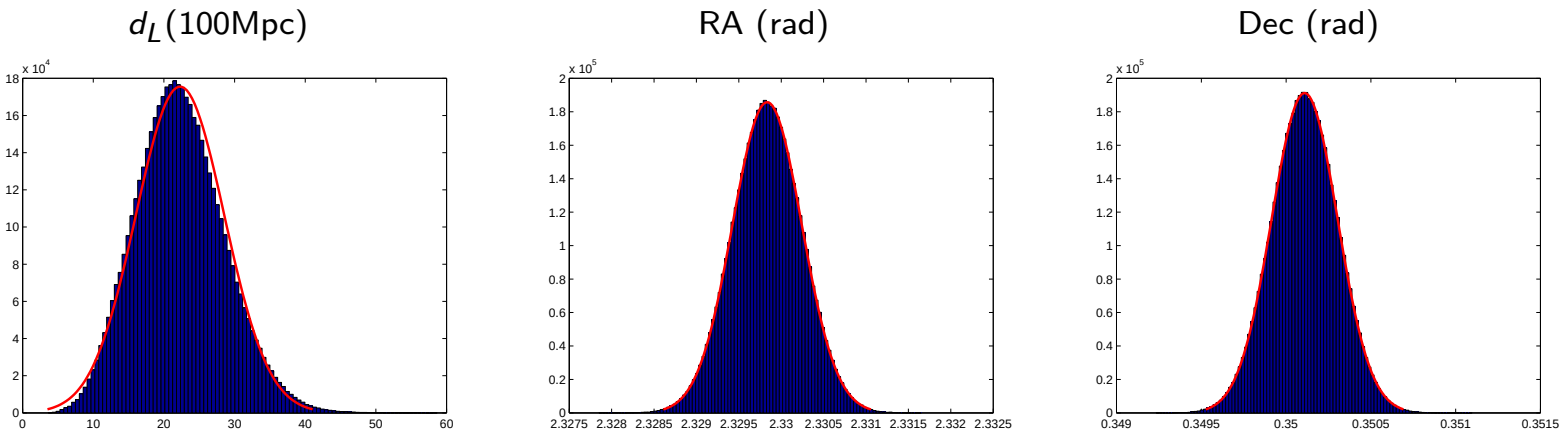
It also localizes the source within arcmin because $\delta\theta \lesssim \lambda_{GW}(\sim \mathrm{lyr})/L(\sim \mathrm{Kpc})$, and helps find the host galaxy with corresponding redshift.

Accuracy and Precision of Our Technique

Using 8 pulsars to detect and characterize a supermassive black hole binary with chirp mass $10^9 \mathrm{M}_\odot$ located at redshift 0.4 (2 Gpc).

The signal-to-noise ratio is 10, just above the detection threshold.

Through MCMC, the accuracies and precisions of the luminosity distance and sky location are

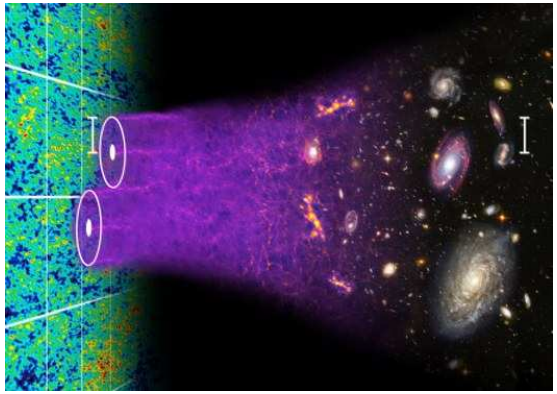


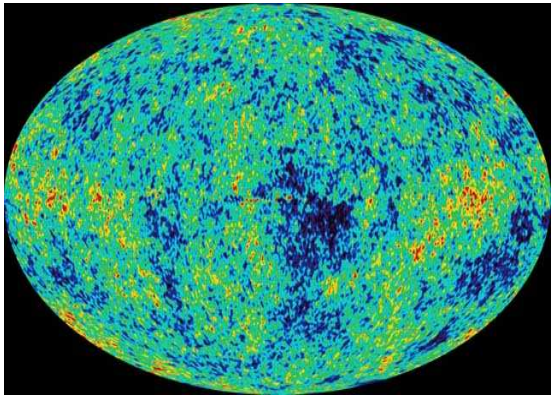
Pulsar Timing Arrays can determine the luminosity distance to the gravitational wave source within 25% and localize it within arcmin

High Precision Hubble Diagram

Pulsar timing arrays are expected to detect 20-200 individual supermassive black hole binaries from redshift 0.1-0.5(Sesana *et al* 2009, Sesana *et al* 2011), with their luminosity distances and redshifts from the host galaxies, we can construct a Hubble diagram.

If we detect 25 GW sources, we determine the Hubble constant within 5% few **systematic errors**!





Combining CMB and BAO data, we can determine the dark energy equation of state coefficient w within 10% ($p_\Lambda = w\rho_\Lambda$).

Conclusion and Prospect

- Pulsar Timing Arrays can directly determine the luminosity distances to supermassive black hole binaries and localize their host galaxies with corresponding redshifts.
- Detecting and characterizing 25 GW sources will construct a high precision gravitational Hubble diagram, which can pin-point Hubble constant within 5% without systematic errors and help measure dark energy equation of state within 10%.
- This multi-messenger technique will have a strong impact in next generation cosmology.