

# VOYAGE TO THE END OF THE UNIVERSE

**Cosmology on the Beach 2012**

**Cancun Iberostar, 16 January**

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The background features a central point from which numerous thin, light-colored rays radiate outwards, creating a starburst or explosion effect. Two prominent bright spots are visible: a yellowish-white one on the left and a larger, more diffuse blue-white one on the right. The overall aesthetic is high-contrast and dynamic.

What you learned at mother's knee

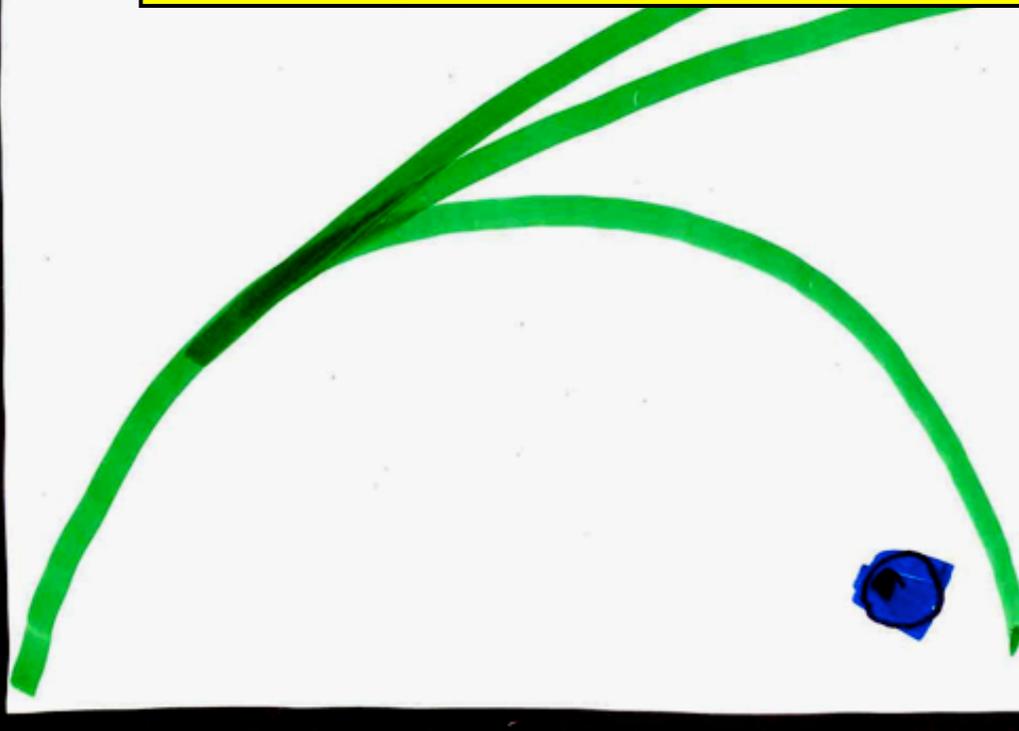
SIZE

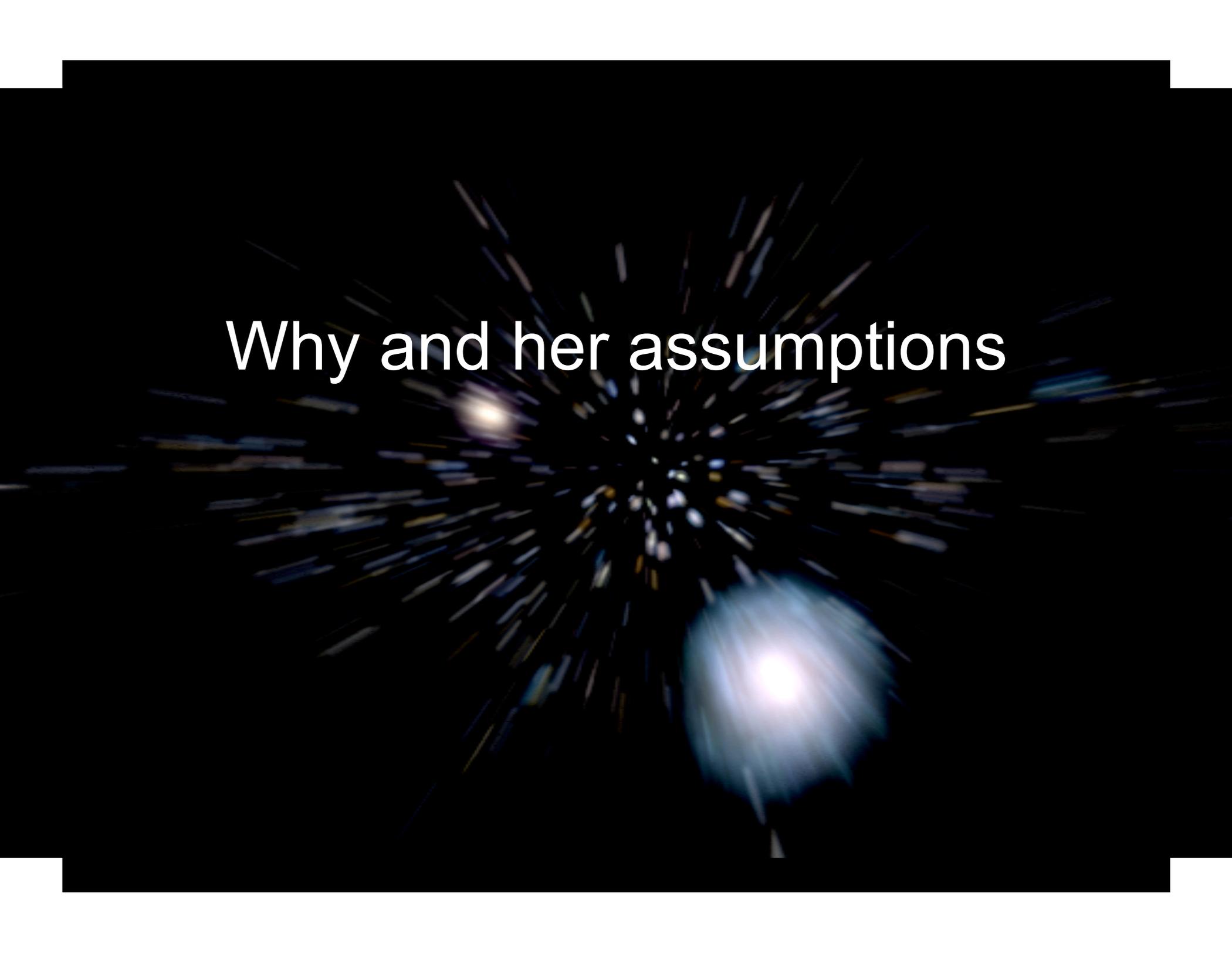
- Expansion = Scaling up
- Single “Cosmic Scale Factor”  
 $R(t)$  Summarizes Expansion
- $R = 1$  today (convention)

BIG BANG

TIME

Geometry  
is  
Destiny!





Why and her assumptions

# Mom's equations (FRW basics)

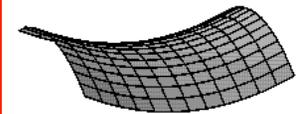
$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{R^2}$$
$$\left(\frac{\ddot{R}}{R}\right) = -\frac{4\pi G}{3}(\rho + 3p)$$

$$d(\rho R^3) = -pd(R^3)$$
$$p = w\rho \Rightarrow \rho \propto R^{-3(1+w)}$$

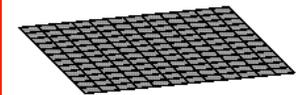
$$\Omega_i \equiv \rho_i / (3H^2 / 8\pi G) \quad \text{and} \quad \Omega_0 = \sum_i \Omega_i$$
$$\Rightarrow k/R^2 = (\Omega_0 - 1)H_0^{-2}$$

$\Omega_0$ :

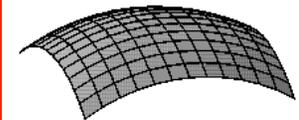
$< 0$



$= 0$



$> 0$



# Space tells matter how to move Matter tells space how to curve

$$d(\rho R^3) = -pd(R^3) \quad \text{“First Law”}$$
$$\rho \propto R^{-3(1+w)} \quad \text{for } p = w\rho$$

Matter ( $w = 0$ ):  $\rho \propto R^{-3}$   $R \propto t^{2/3}$

Radiation ( $w = 1/3$ ):  $\rho \propto R^{-4}$   $R \propto t^{1/2}$

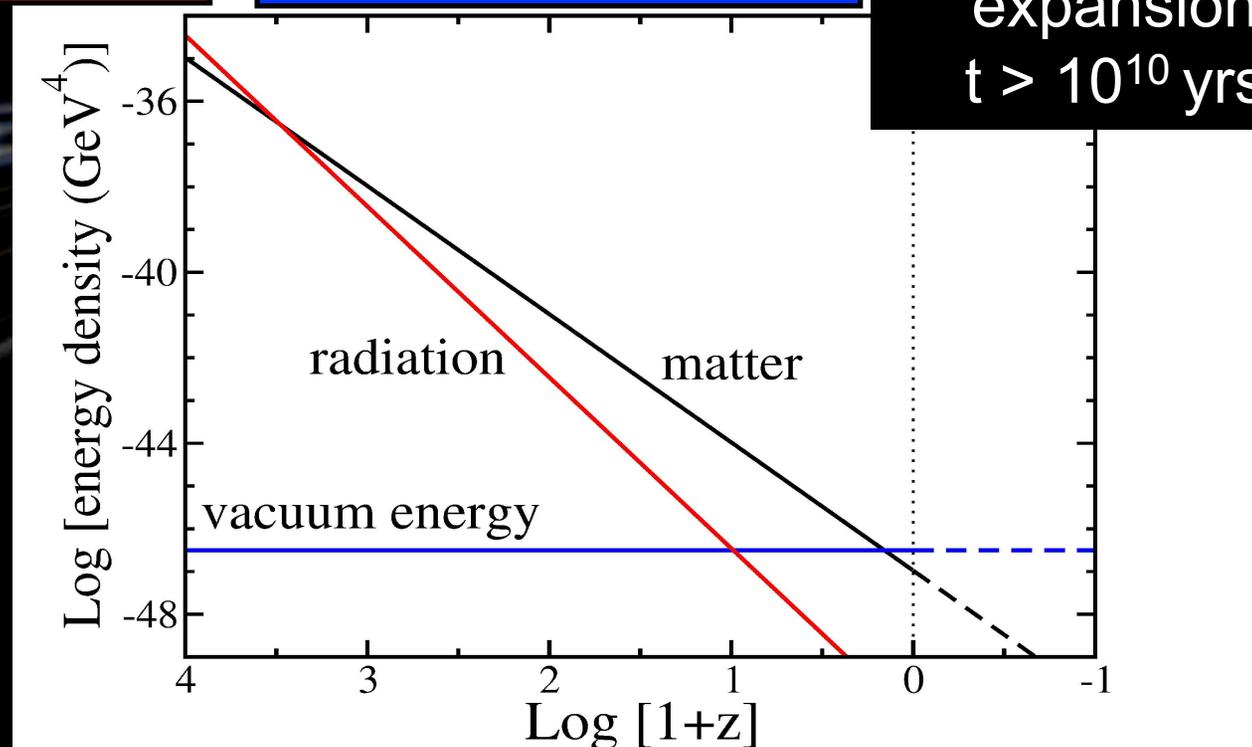
Vacuum Energy ( $w = -1$ ):  $\rho \propto \text{const}$   $R \propto \exp(Ht)$

# Three Epochs Dominated by Different Forms of Energy

1. Rad dominated  
 $R \sim t^{1/2}$  thermal bath  
 $R < 10^{-4}$ ,  $t < 10^4$  yrs

2. Matter dominated  
 $R \sim t^{2/3}$  struc. forms  
 $t \sim 10^4$  yrs –  $10^{10}$  yrs

3. Dark Energy  
 $R \sim e^{Ht}$   
accelerated expansion  
 $t > 10^{10}$  yrs



# Mom's Universe: matter only

Reformulate Friedmann 1 as particle in a one-d potential

$$R^2 \times H^2 \Rightarrow \dot{R}^2 = R^2 \frac{8\pi G \rho}{3} - k$$

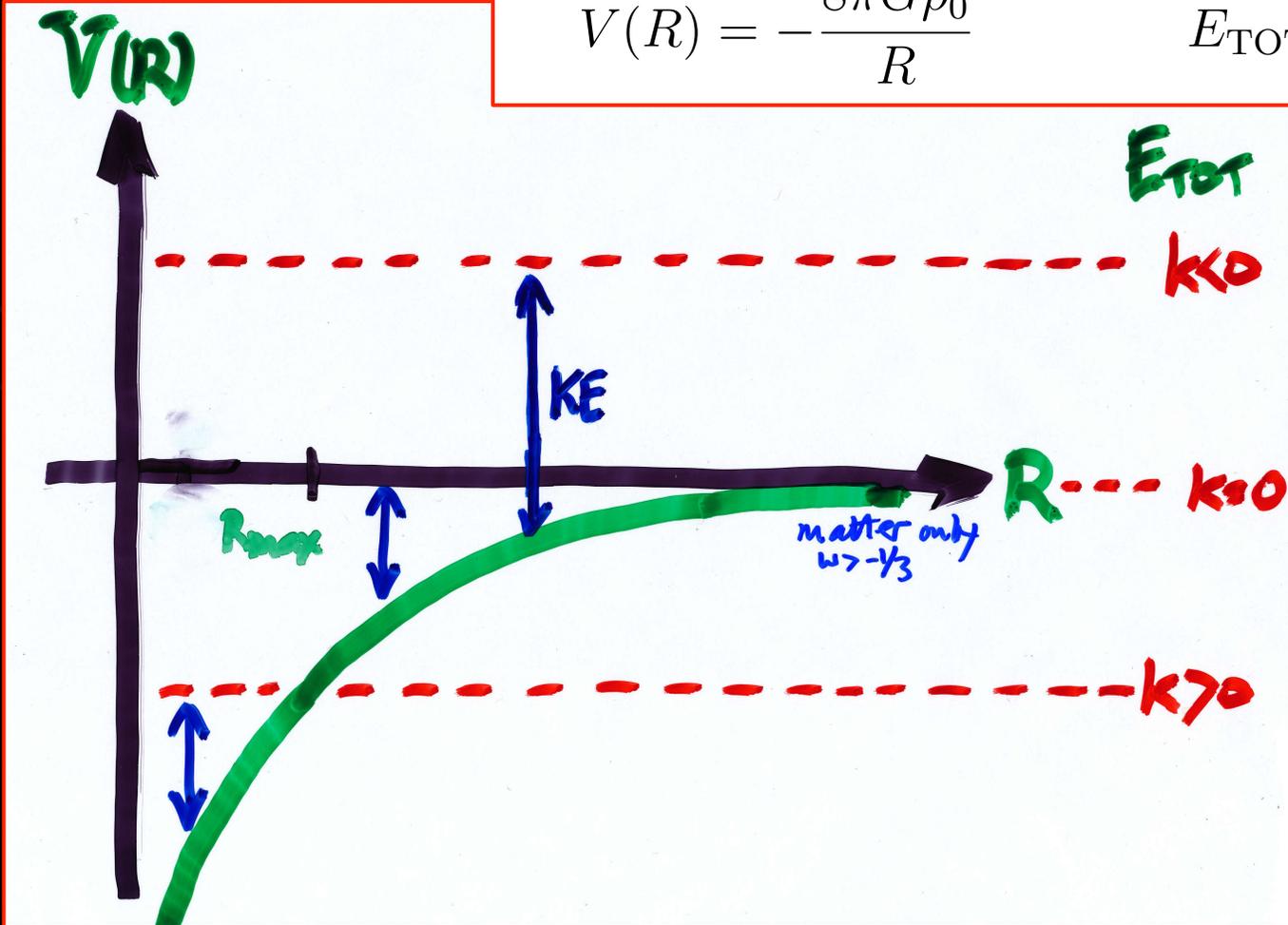
$$\dot{R}^2 - \frac{8\pi G \rho_0}{3R} = -k$$

$$KE + V(R) = E_{\text{TOT}}$$

## Matter only

Particle in a one-d potential  $V(R)$ :

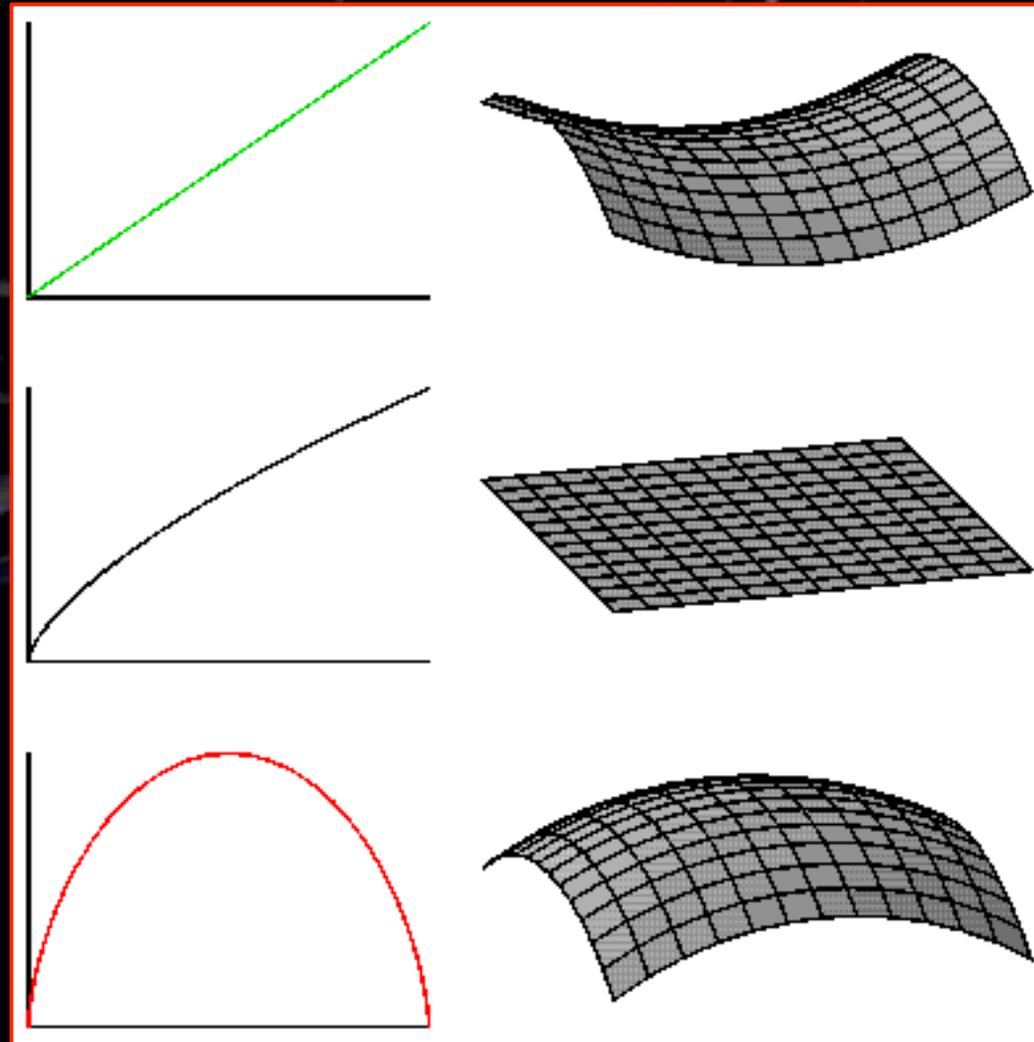
$$\dot{R}^2 + V(R) = E_{\text{TOT}}$$
$$V(R) = -\frac{8\pi G\rho_0}{R} \quad E_{\text{TOT}} = -k$$



$k > 0 \rightarrow R_{\text{max}}$  & recollapse;  $k = 0$  or  $< 0$  expand forever

# Geometry is Destiny

Just like Mom said!



# Generalized Mom

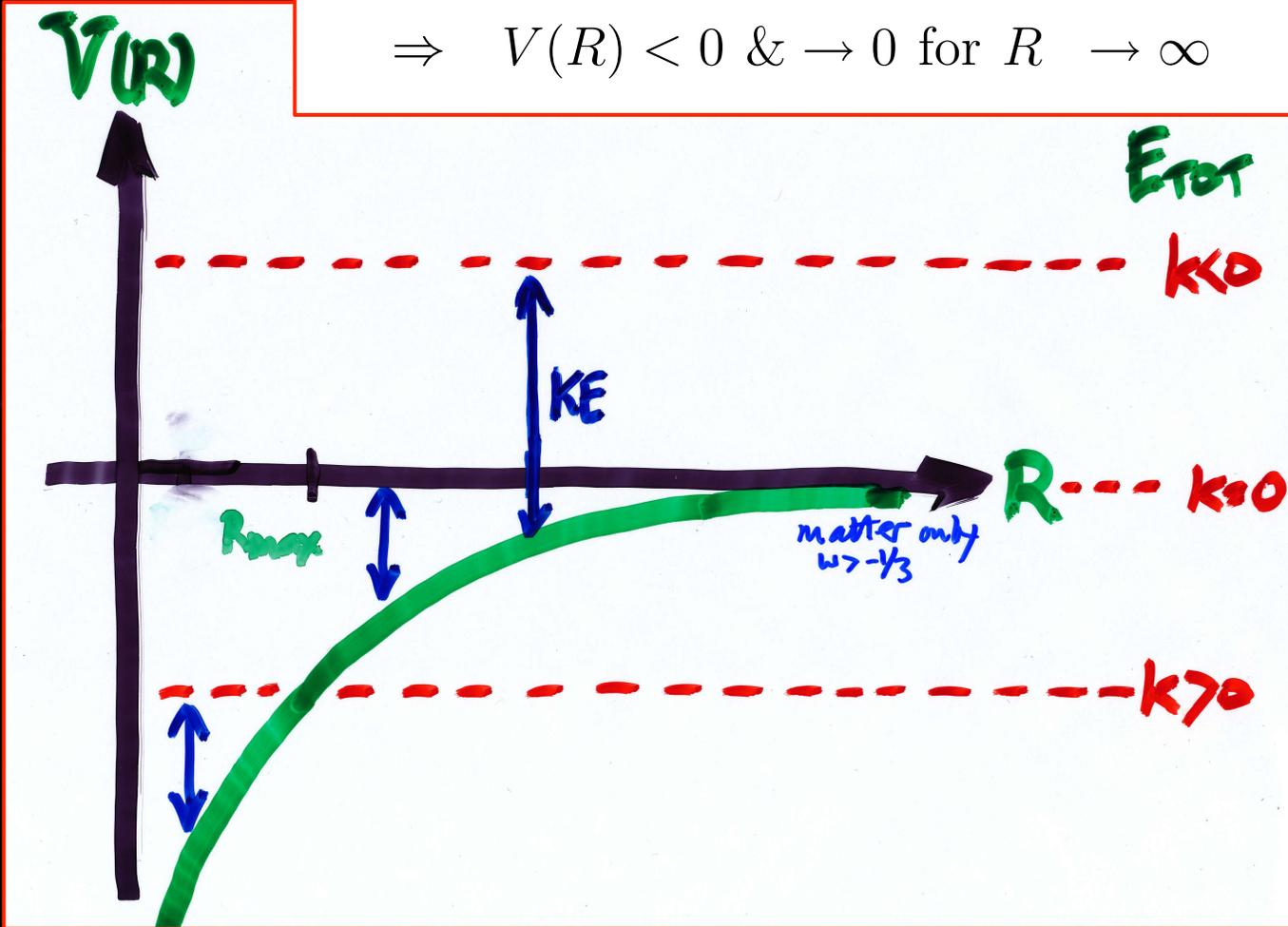
all components have  $w > -1/3$ , i.e.,  
decrease faster than  $R^{-2}$

# “Ordinary Matter”

Particle in a one-d potential  $V(R)$ :

$$V(R) \propto R^2 \rho(R) \propto R^{-1-3w}$$

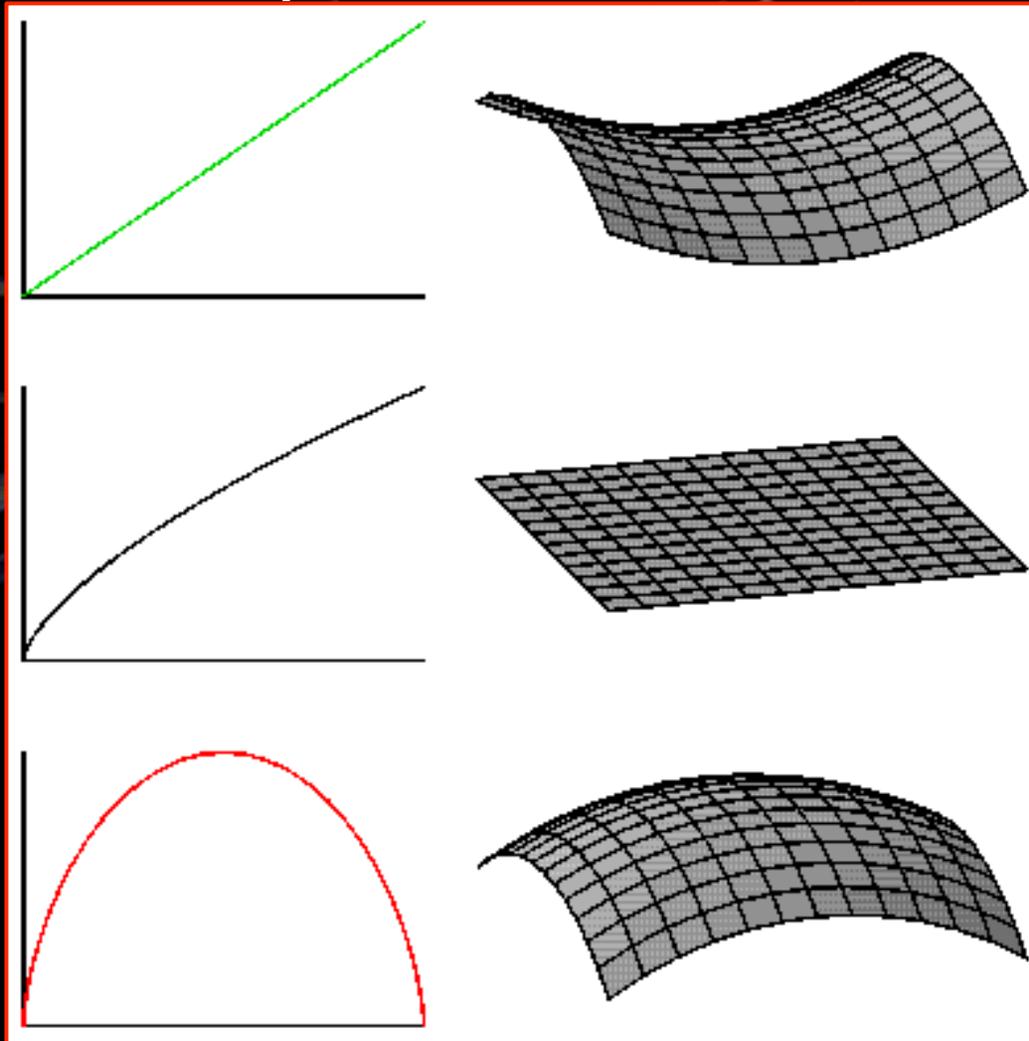
$$\Rightarrow V(R) < 0 \text{ \& } \rightarrow 0 \text{ for } R \rightarrow \infty \text{ if } w > -1/3$$

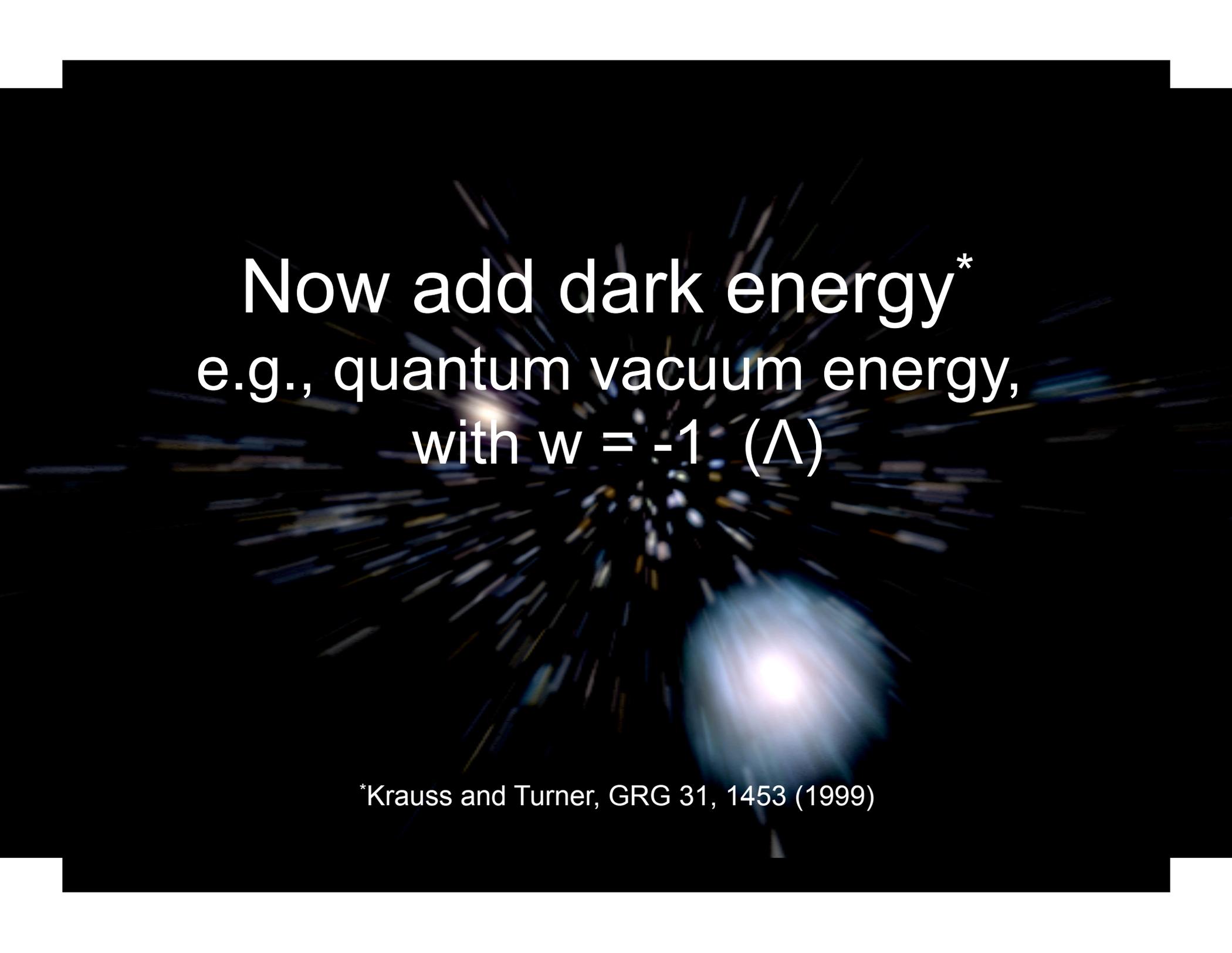


$k > 0 \rightarrow R_{max}$  & recollapse;  $k = 0$  or  $< 0$  expand forever

# Geometry is Destiny!

(if all components have  $w > -1/3$ )





Now add dark energy\*  
e.g., quantum vacuum energy,  
with  $w = -1$  ( $\Lambda$ )

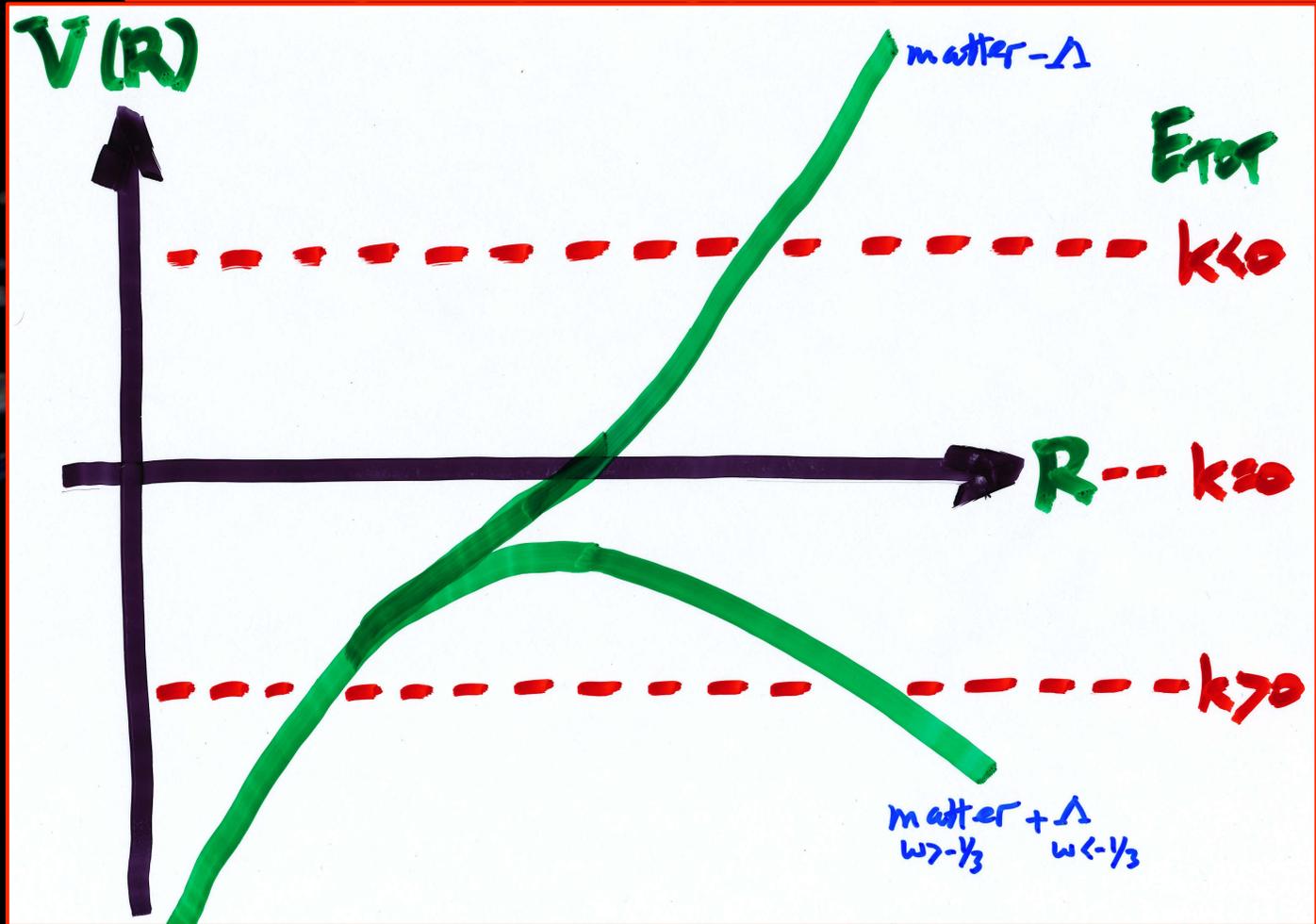
\*Krauss and Turner, GRG 31, 1453 (1999)

“Matter + DE”

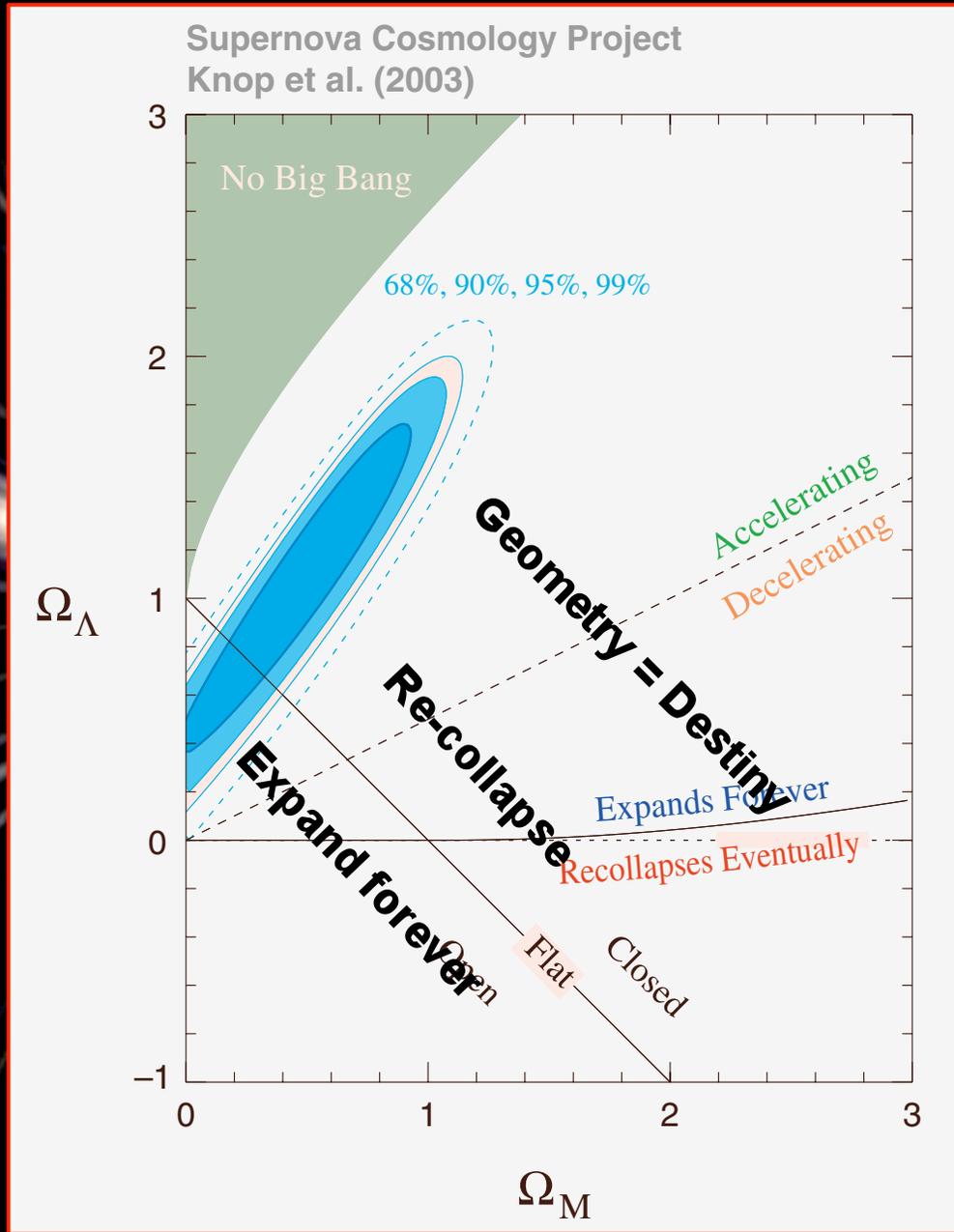
Homework Problem

Particle in a one-d potential  $V(R)$ :

$$\dot{R}^2 + V(R) = E_{\text{TOT}}$$
$$V(R) = -\frac{8\pi G\rho_0}{R} - \frac{8\pi G\rho_{\text{VAC}}}{3}R^2 \quad E_{\text{TOT}} = -k$$



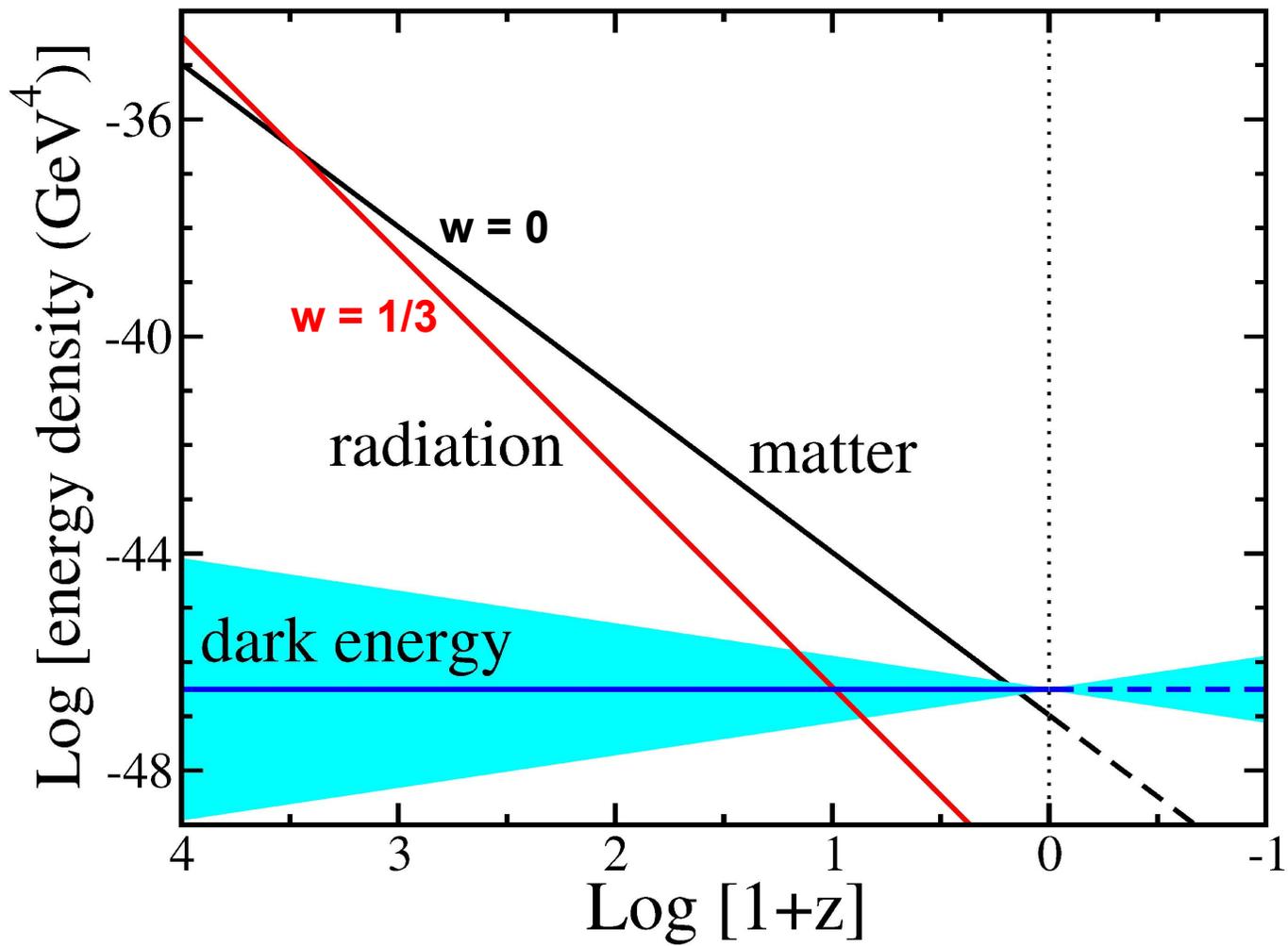
# Answer to the HW problem



# Duality and the Big Rip\*

if  $w < -1$ , dark energy density increases with time leading to a future singularity and the disassembly of ALL structures in the Universe

\*Caldwell et al, PRL 91, 071301 (2003)



$w < -1$   
 $w = -1$   
 $w > -1$

# Re-formulate the problem:

$$x = 1/R, t \rightarrow -t, \text{ \& } w \rightarrow w'$$

Recall  $\rho_{\text{DE}} \propto R^{-3(1+w)}$

if  $w > -1 \Rightarrow R \propto t^{2/3(1+w)}$

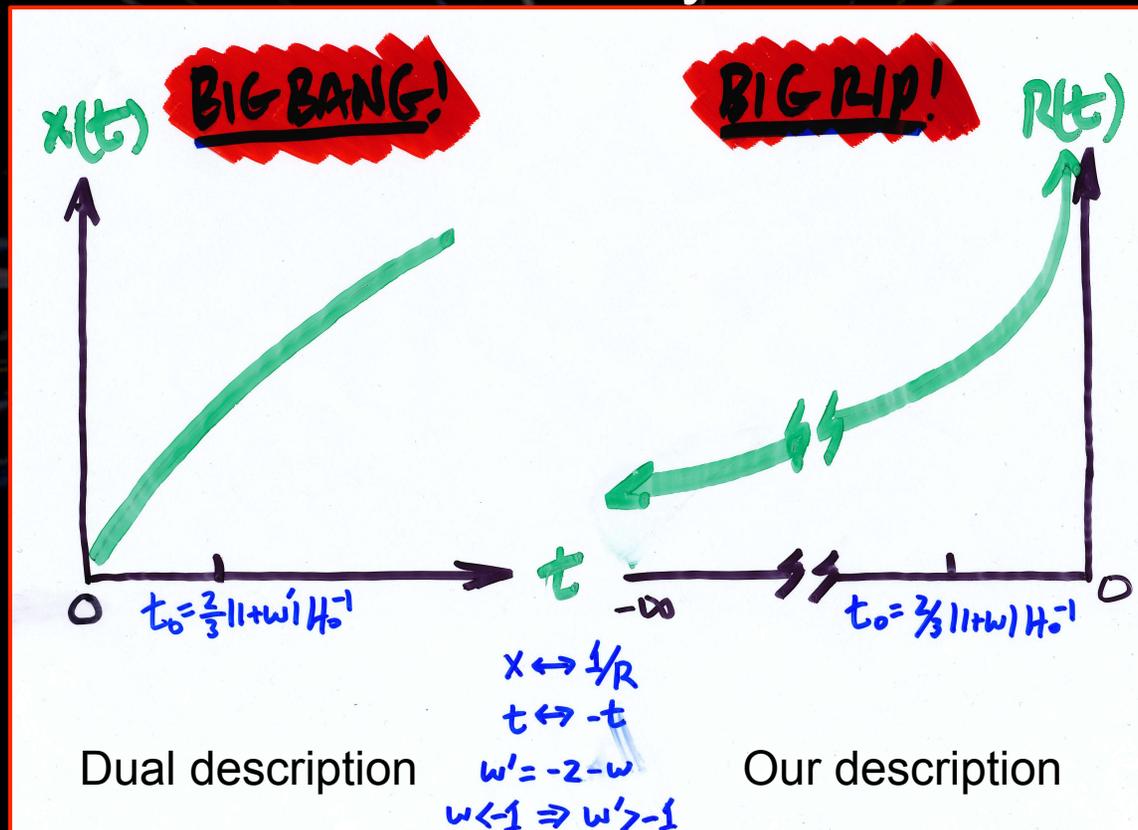
**→ Big bang, etc**

if  $w < -1$  then  $\rho_{\text{DE}} \propto R^n$  with  $n > 0$  and ??

**Try dual**  $x \equiv 1/R$   $t \rightarrow -t$   $H_x = H_R$  with  $w' = -2 - w$

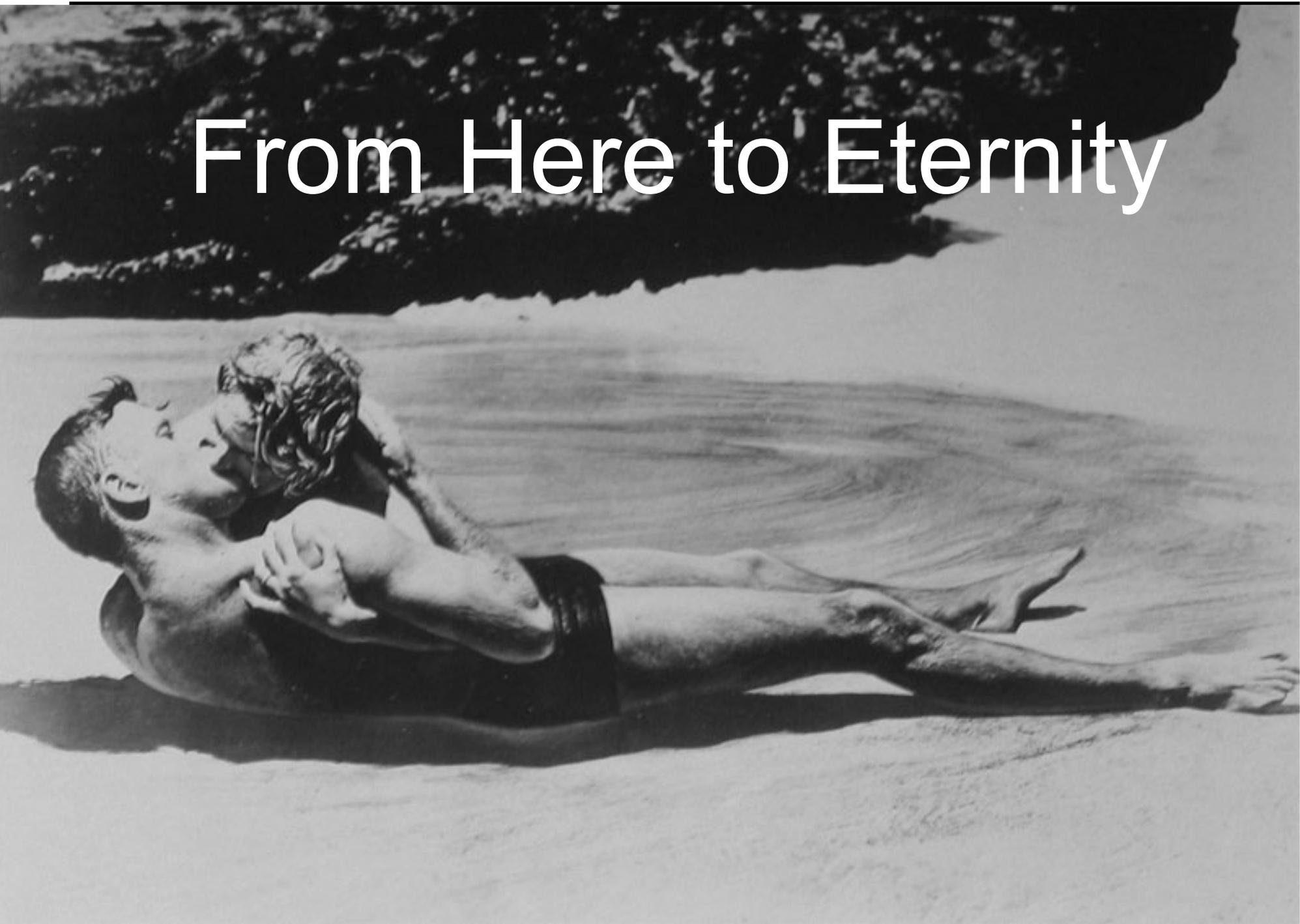
$H_x^2 = \frac{8\pi G}{3} x^{-3(1+w')}$  for  $w < -1$  we have  $w' > -1$  and big bang solution

Dual picture: standard big bang (finite time back to bang, expansion forever)  $\rightarrow$   
 Our picture: infinite time back to  $R = 0$  and finite time to infinite density &  $R = \infty$

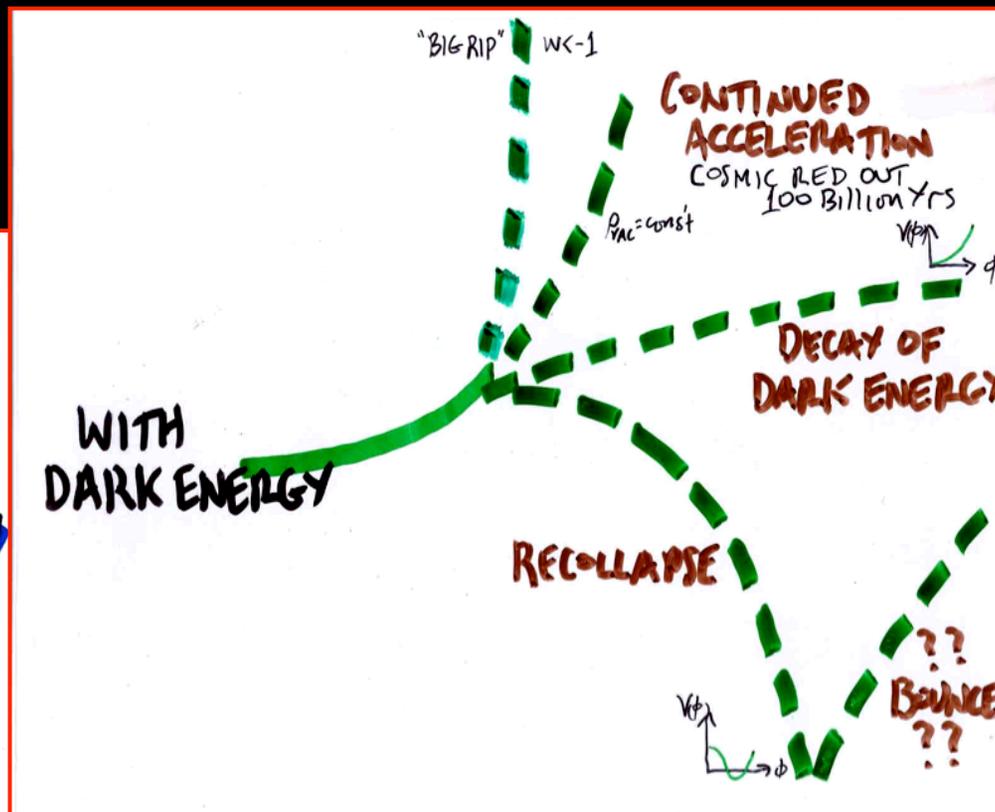
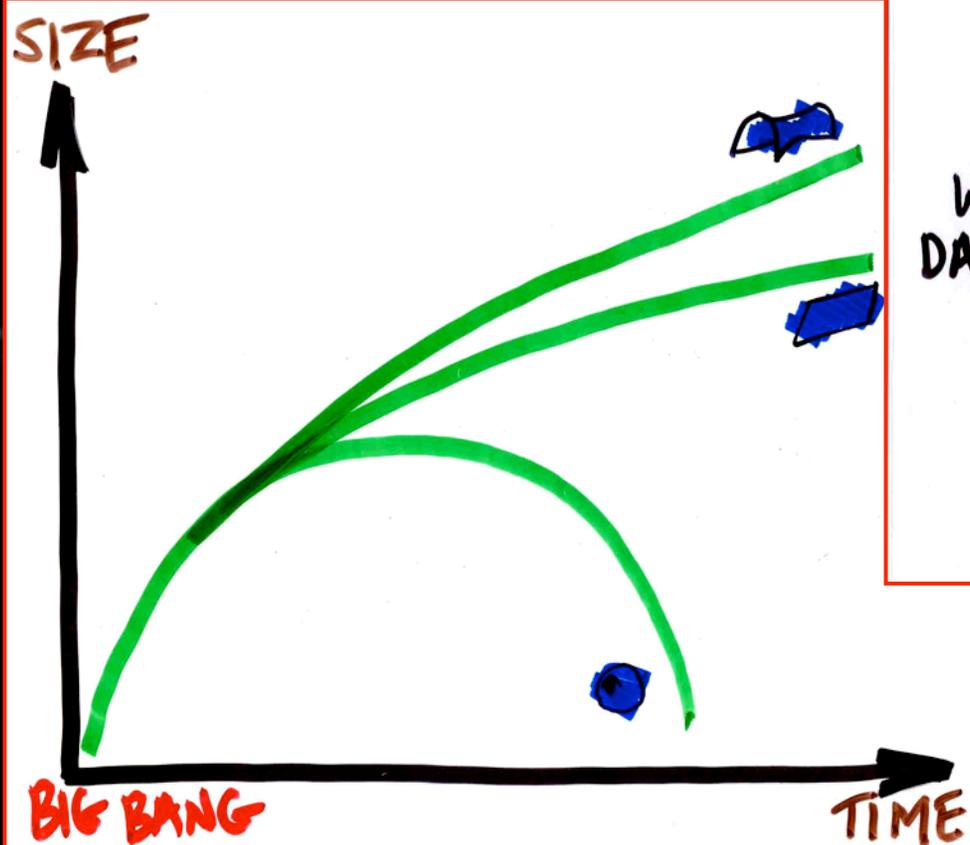


$$R(t) \propto (-t)^{2/3(1+w)} \quad \text{today : } t = -\frac{2}{3|1+w|} H_0^{-1}$$

# From Here to Eternity



In the Presence of Dark Energy, a Flat Universe Can Expand Forever, Re-collapse, or Even Experience a Big Rip!

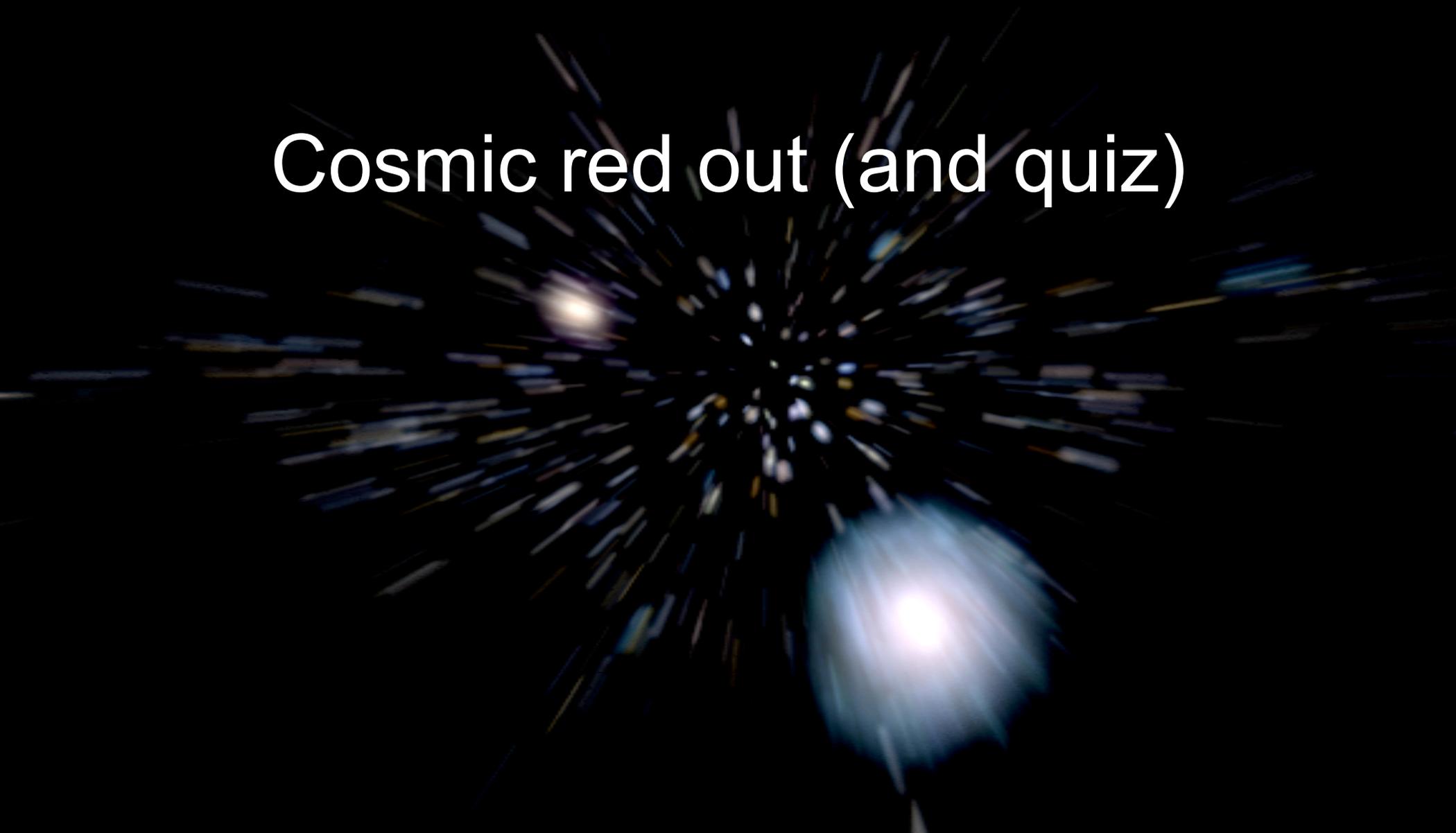


Cannot Understand Our Cosmic Destiny Until We Understand What Dark Energy Is!

# All destinies possible

- Expand forever
  - $\Lambda$ CDM
  - Rolling scalar field + CDM
- Re-collapse
  - $k > 0$  + rolling scalar field
  - $k = 0$  + rolling scalar field  $-\Lambda$
- Goldilocks (EdS)
  - $k = 0$  + rolling scalar field

# Cosmic red out (and quiz)



# Quiz

1. Which model Universe (dS or EdS) has more galaxies within the horizon (assuming same comoving galaxy density)? **dS or EdS (class says: 1 for dS, 80 for EdS)**
2. Which model Universe (dS or EdS) has more galaxies w/redshift less than  $O(10)$ ? **dS or EdS (class says: 6 for dS & 12 for EdS)**

# Answer

1. Which model Universe (dS or EdS) has more galaxies within the horizon (assuming same comoving galaxy density)? **de Sitter**

$$N_{\text{H}} = \frac{4\pi}{3} n_{\text{GAL}} r_{\text{H}}^3$$

$$r_{\text{H}} = \int dt' / R(t') = \int dz / H(z) = \int_0^{R(t)} dR' / R'^2 H(R')$$

$$r_{\text{H}}(\text{dS}) = \infty \quad \Rightarrow \quad N_{\text{H}}(\text{dS}) \rightarrow \infty$$

$$r_{\text{H}}(\text{EdS}) = 2H_0^{-1}(t/t_0)^{1/3} \quad \Rightarrow \quad N_{\text{H}}(\text{EdS}) = \frac{32\pi}{3} H_0^{-3} (t/t_0)$$

# Answer

2. Which model Universe (dS or EdS) has more galaxies w/redshift less than  $O(10)$ ?:

**Einstein deSitter**

$$N_{z_*} = \frac{4\pi}{3} n_{\text{GAL}} r_{z_*}^3$$

$$r_{z_*} = \int dt' / R(t') = \int dz / H(z) = \int_{R(t)/(1+z_*)}^{R(t)} dR' / R'^2 H(R')$$

$$r_{z_*}(\text{dS}) = H_0^{-1} z_* / R(t) \quad \text{and} \quad r_{z_*}(\text{EdS}) = 2H_0^{-1} (t/t_0)^{1/3} [1 - 1/(1+z_*)^{1/2}]$$

$$\Rightarrow N_{z_*}(\text{dS}) = \frac{4\pi}{3} n_G z_*^3 R^{-3}(t) \rightarrow 0 \quad \text{as} \quad R \rightarrow \infty$$

$$\Rightarrow N_{z_*}(\text{EdS}) = \frac{32\pi}{3} H_0^{-3} (t/t_0) [1 - 1/(1+z_*)^{1/2}]^3 \propto t$$

# Footnote to Q1:

If one truncates the horizon integral at redshift of 1000 (CMB), because galaxies couldn't have formed before, then  $r_H = 1000/H_0$  and  $N_H(\text{dS}) \rightarrow 0$  as  $t \rightarrow$  infinity and  $N_H(\text{EdS})$  is larger.

$$N_{z_*} = \frac{4\pi}{3} n_{\text{GAL}} r_{z_*}^3$$

$$r_{z_*} = \int dt' / R(t') = \int dz / H(z) = \int_{R(t)/(1+z_*)}^{R(t)} dR' / R'^2 H(R')$$

$$r_{z_*}(\text{dS}) = H_0^{-1} z_* / R(t) \quad \text{and} \quad r_{z_*}(\text{EdS}) = 2H_0^{-1} (t/t_0)^{1/3} [1 - 1/(1+z_*)^{1/2}]$$

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# The end of cosmology

The background of the slide is a dark, cosmic scene. At the center is a dense cluster of stars, appearing as a bright, multi-colored point of light. Radiating from this center are numerous thin, elongated streaks of light in shades of blue, white, and yellow, creating a starburst or explosion effect. In the lower right quadrant, there is a large, bright, glowing blue-white object with a soft, ethereal aura. In the upper left quadrant, there is a smaller, bright yellow-white object with a similar glow. The overall composition is centered and symmetrical, with the text 'The end of cosmology' overlaid in the middle.

# Suppose dark energy is quantum vacuum energy

- In  $\sim 200$  Billion years (15 e-folds):
  - $N_{\text{GAL}} \sim 10^{-20} N_{\text{GAL}} (\text{today}) \ll 1$
  - $T_{\text{CMB}} \sim 10^{-7} \text{ 3K}$
- No expansion, no CMB  $\rightarrow$  one galaxy (one cluster) + static Universe and no evidence of the big bang (Helium abundance?) – “Cosmic Amnesia”\*

\*Krauss and Scherrer, GRG 39, 1545 (2007)

# Fall of cosmology!

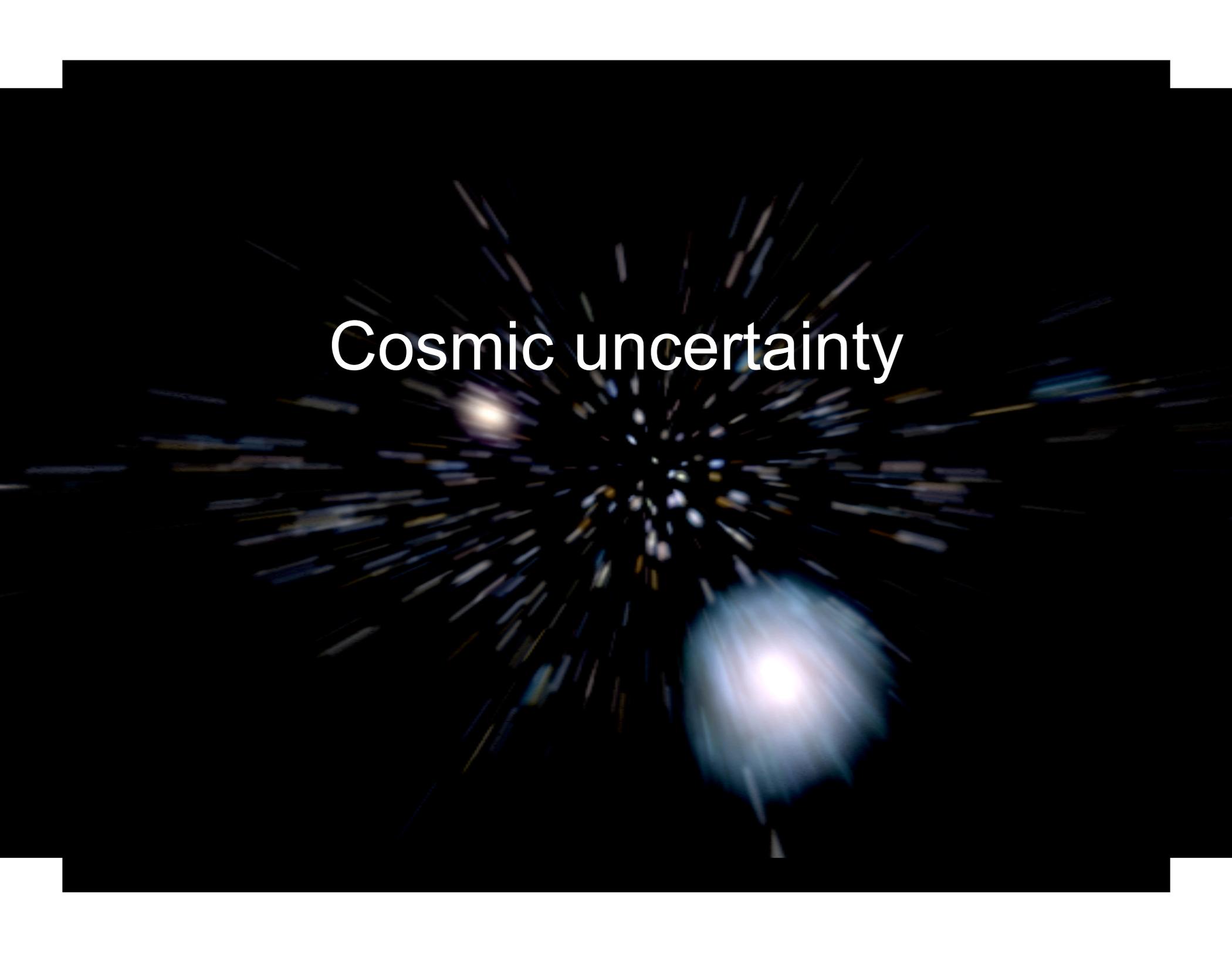


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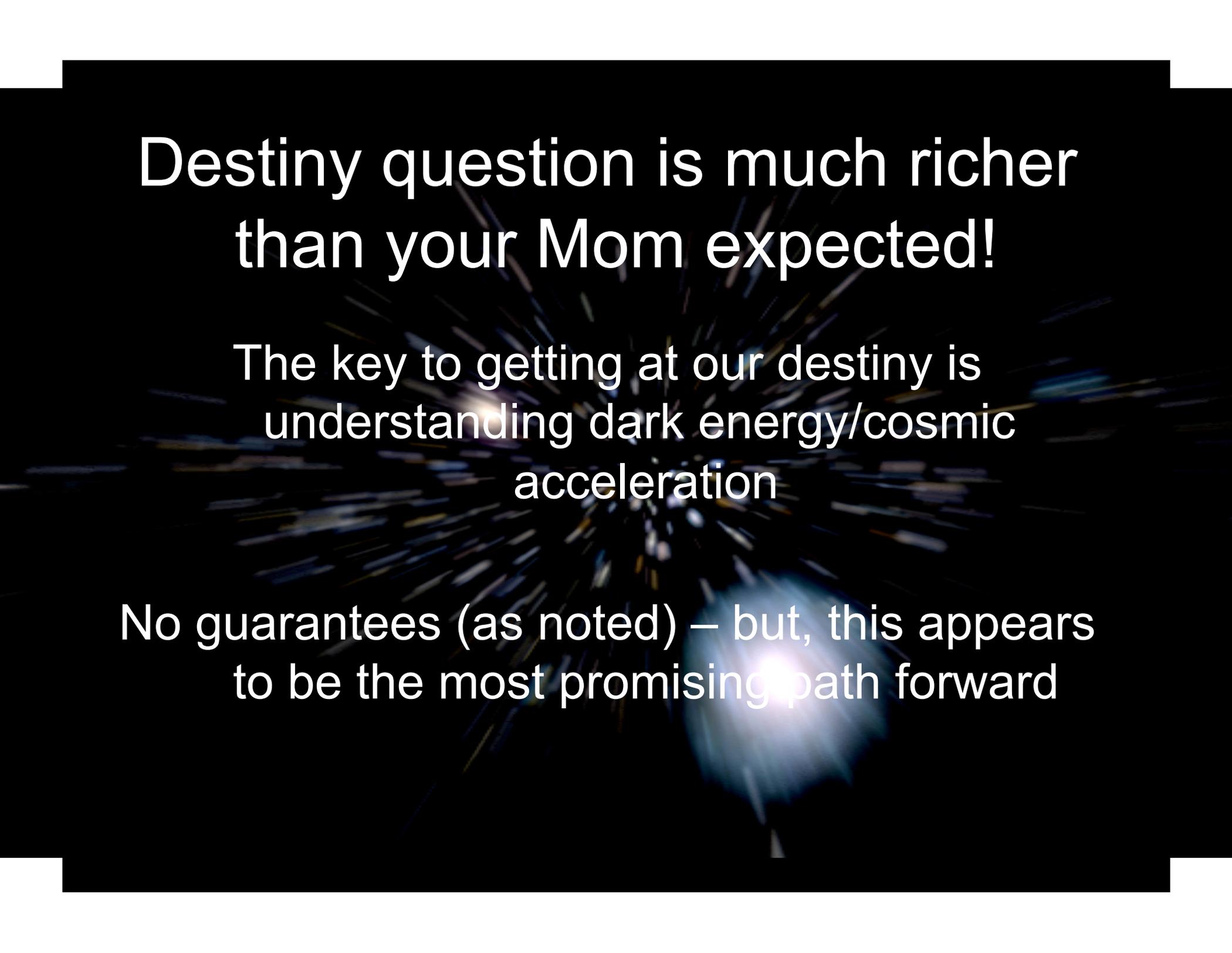
# Cosmic uncertainty

A visualization of cosmic uncertainty. The background is black, filled with a dense field of small, multi-colored streaks (blue, yellow, white) radiating from a central point, creating a starburst or explosion effect. Two prominent, larger, bright blue-white spots are visible, one in the lower right and one in the upper left, suggesting specific points of interest or high-energy events within the field.

# How well can we determine our cosmic destiny?\* Suppose:

- Planck measures  $\Omega_0 = 1.1$ , DE = rolling scalar field (w/ undetected  $\Omega_{\text{VAC}} = 0.001$ )
  - Conclude that Universe re-collapses, but actually due to an undetectable QVE it will expand forever!
- Planck measures  $\Omega_0 = 1.00$ , DE = rolling scalar field (w/undetected  $\Omega_{\text{VAC}} = -0.001$ )
  - Conclude that Universe expands forever, but actually due to an undetectable small QVE it will re-collapse!

\*Krauss and Turner, GRG 31, 1453 (1999)



Destiny question is much richer  
than your Mom expected!

The key to getting at our destiny is  
understanding dark energy/cosmic  
acceleration

No guarantees (as noted) – but, this appears  
to be the most promising path forward

# What could we learn?

- $\Omega_0$  to 1%, discovery that DE = rolling scalar field  $\rightarrow$  end of accelerated expansion
- New theoretical understanding QVE (and proof that  $QVE = 0$ )
- Surprises: crack in GR, ...

**DARK ENERGY**

**MAY BE THE MOST**

**PROFOUND PROBLEM**

**IN ALL OF SCIENCE TODAY**

# Lessons from this story

- Humility in face of the vastness of Universe, which is often beyond the reach of our ideas and instruments is a good idea
- It is remarkable how far we have come with such simple ideas (isotropy, homogeneity, dark matter, inflation, ...), cf. galaxy formation and evolution, but don't be taken in. ("For every complex natural phenomenon there is a simple, elegant, compelling, wrong explanation." - T. Gold)
- Be bold but be flexible and humble