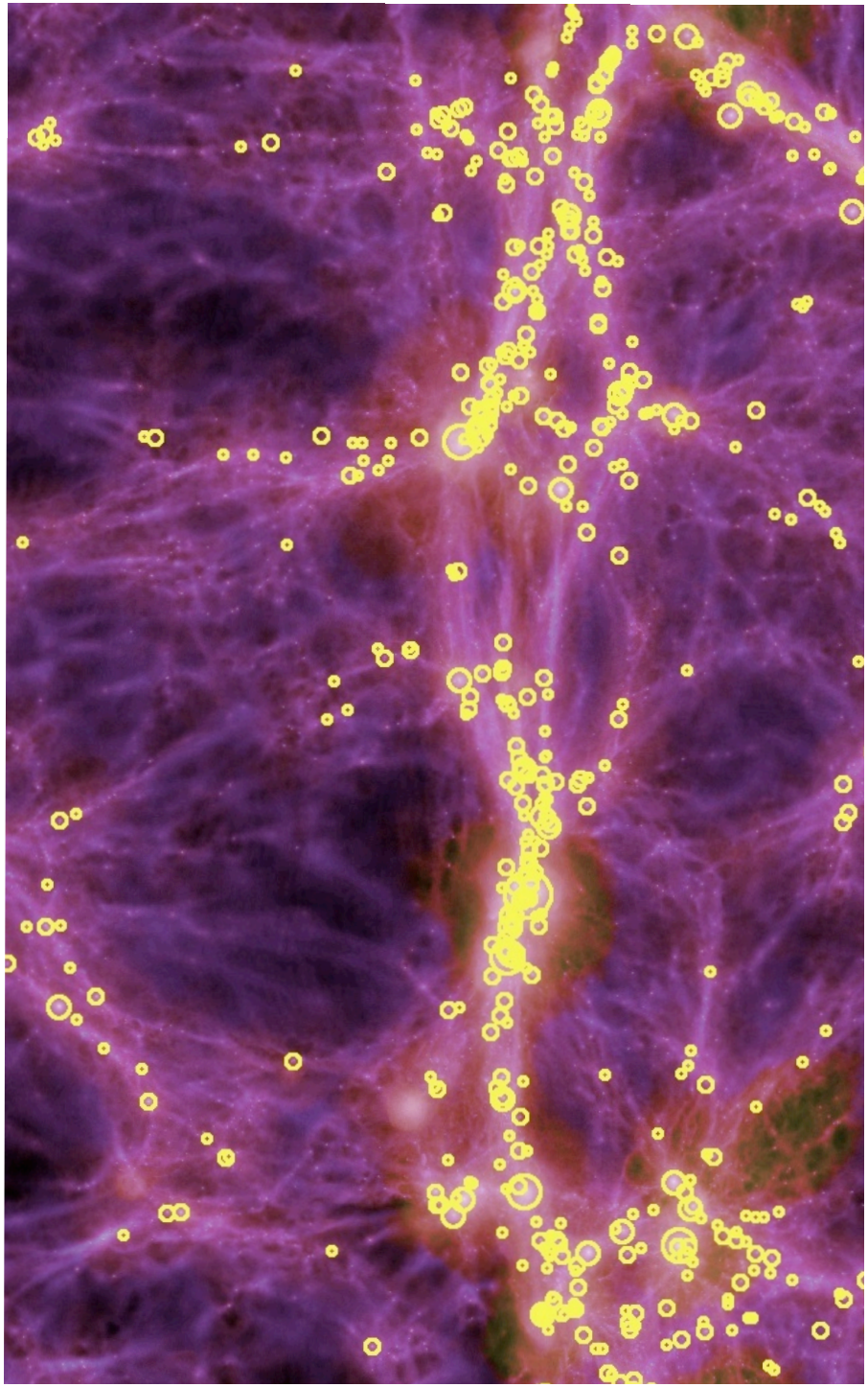
From Black holes to the Universe Simulations

Tiziana Di Matteo

BRUCE AND ASTRID MCWILLIAMS
Center for Cosmology
Carnegie Mellon



Lectures plan

intro

- 'Classical' Cosmology and 'Modern' Cosmology and Structure in the Universe (what do we simulate?).
- The initial conditions (why cosmological simulations are unique)
- Physics and algorithms

Lecture I

(1) Multiscale simulations of black holes and galaxy formation

Lecture II

(2) Cosmological Simulations: The first massive black holes and quasars

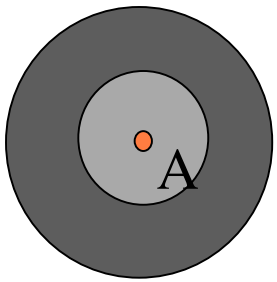
Lecture III

(3) The IGM and Lyman-α (FEATURING: R. Croft!)

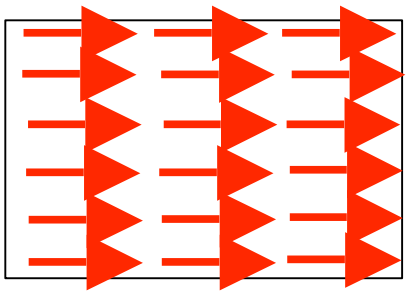
Einstein's field Equation and The fate of Universe

- The Cosmological Principle dictates that the Universe on large scales is both homogeneous and isotropic about every point:
  - -homogenous: the same at every point
  - -isotropic: the same in every direction

homogeneous

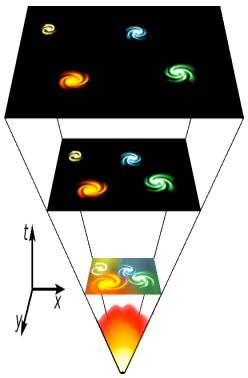


isotropic about point A

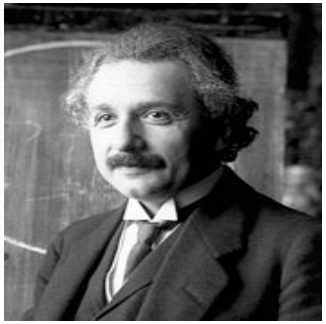


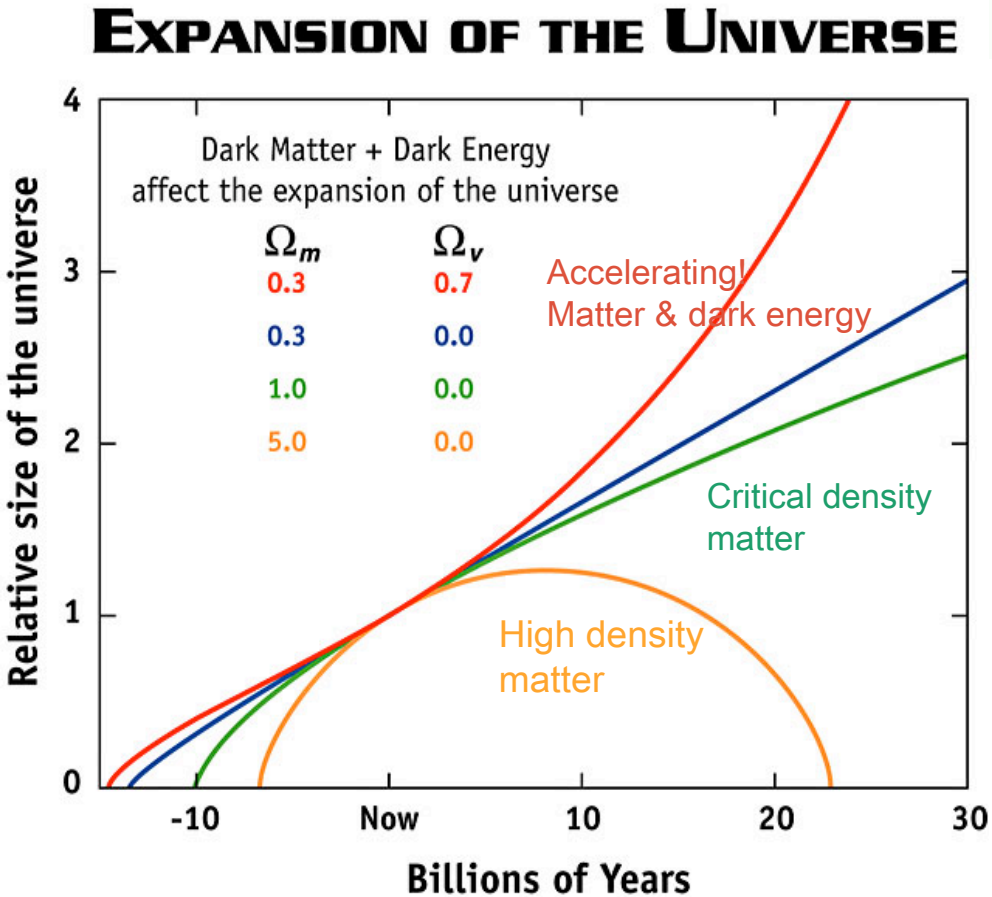
homogenous but not isotropic

Einstein's field Equation and The fate of Universe



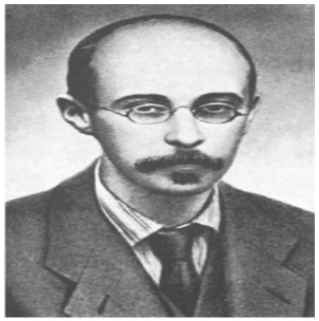
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$





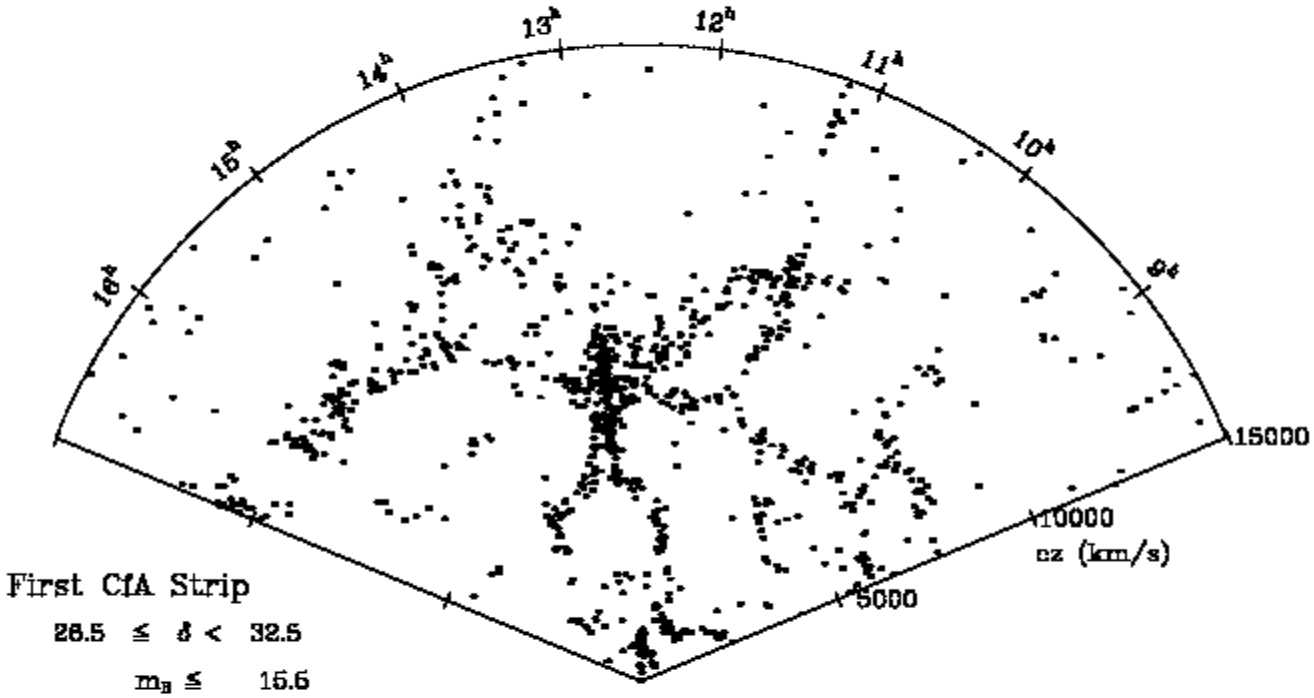
$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G \rho}{3}$$

Mass/energy density



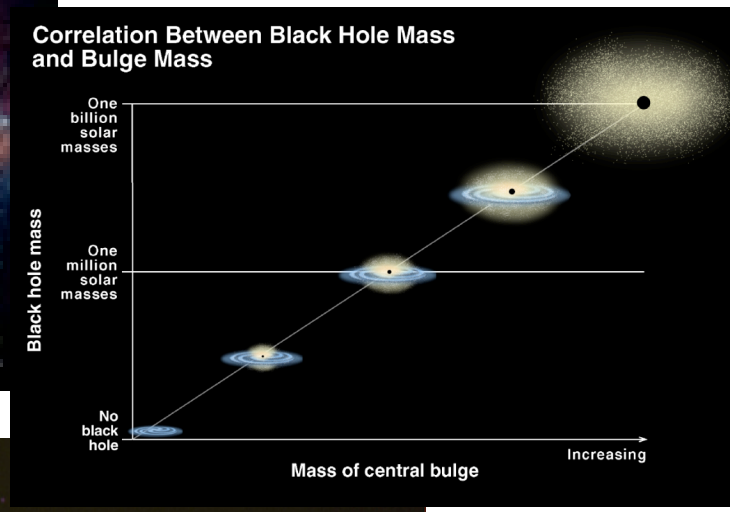
But..

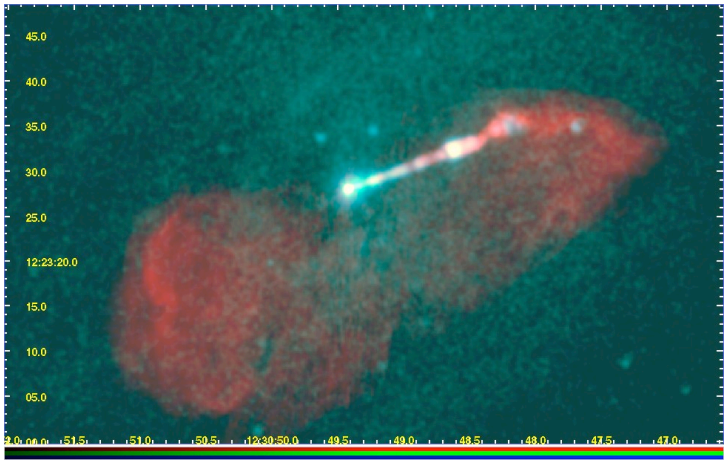
How does structure arise?

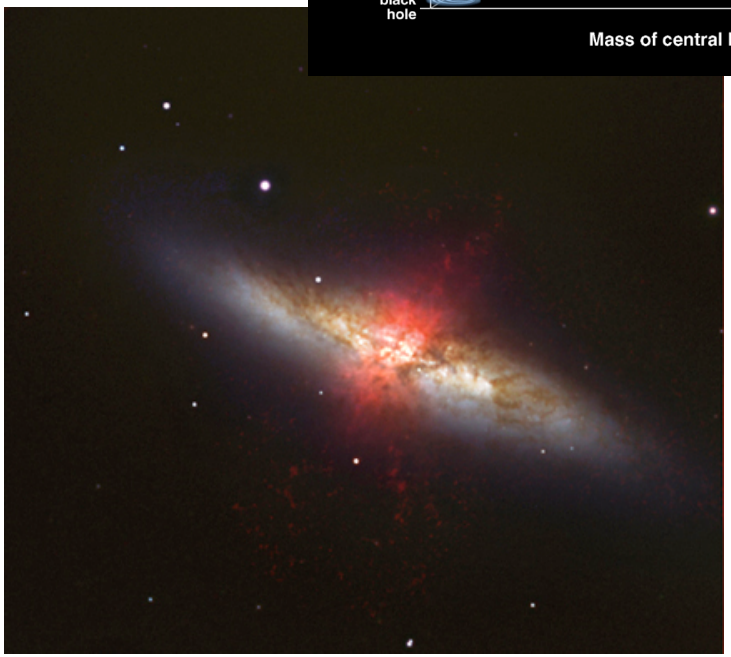


Copyright SAO 1998

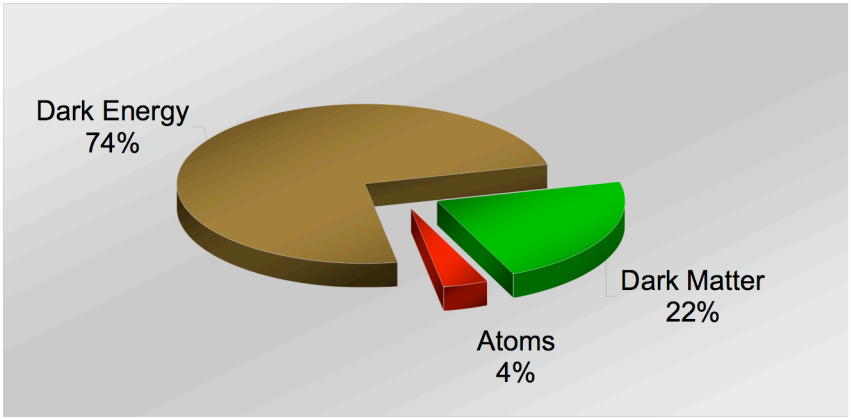
M87 © Anglo-Australian Observatory
Photo by David Malin


Correlation Between Black Hole Mass and Bulge Mass
Black hole mass
One billion solar masses
One million solar masses
No black hole
Increasing
Mass of central bulge


45.0
40.0
35.0
30.0
25.0
12:23:20.0
15.0
10.0
05.0



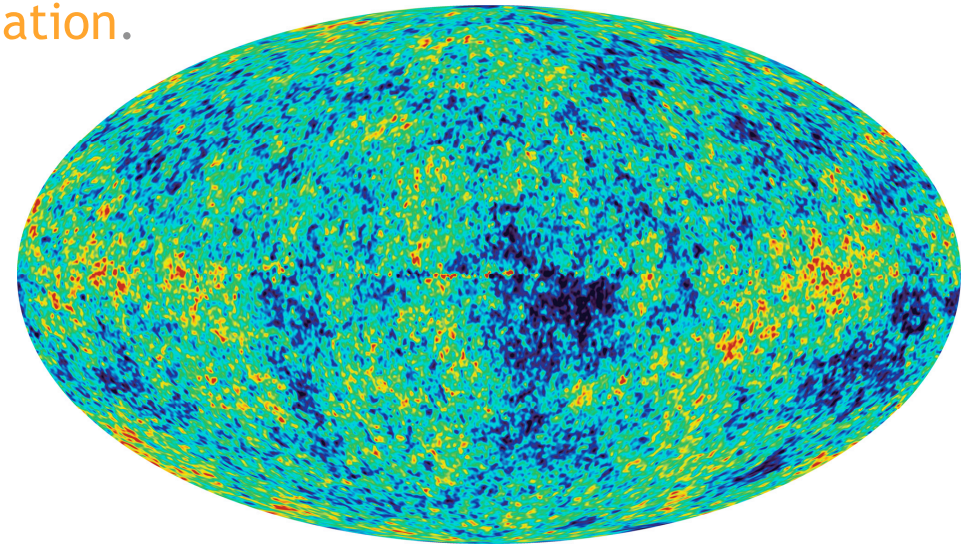
The initial conditions of cosmological structure formation are known:



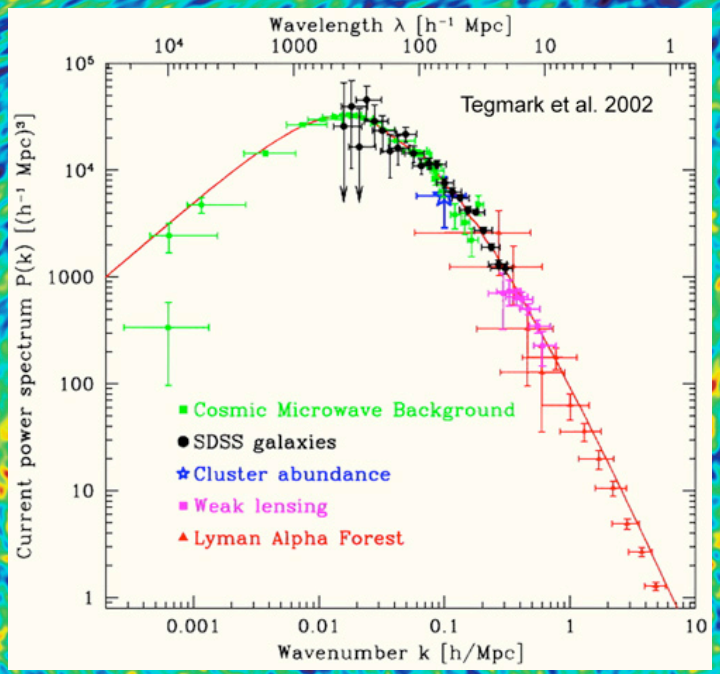
THEORY: inflation

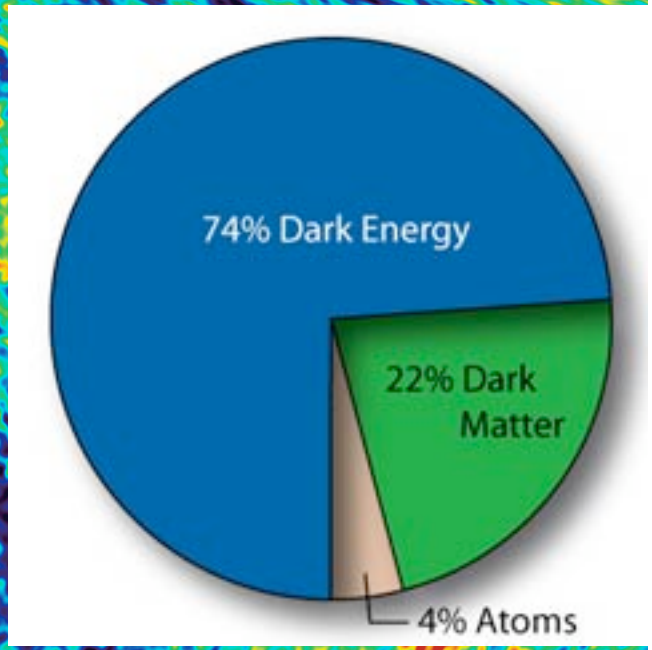
- quantum fluctuations result in small variations in the density from place to place.

- During inflation length scales are blown up by an enormous factor and these fluctuations become the "seeds" for structure formation.

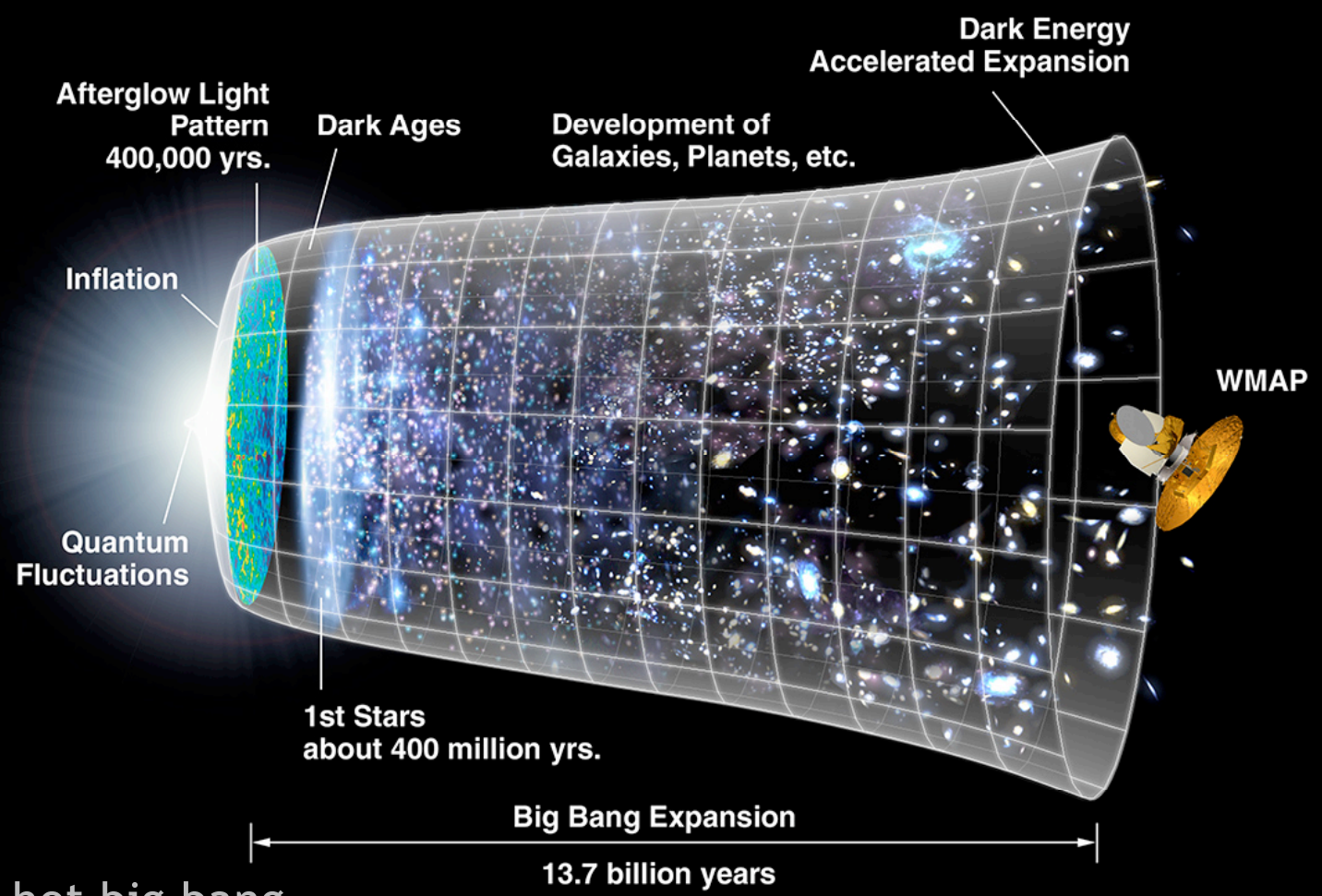


The rate at which structure forms depends on the initial power spectrum of matter fluctuation + expansion rate





Our standard paradigm for Structure formation in the Universe



- There was a hot big bang
- Structure formation was seeded by inflation
- Most matter is dark, non-baryonic and weakly interacting
- The universe is spatially flat and vacuum-dominated at present

NASA/WMAP Scier

What happens to these fluctuations as the Universe evolves?

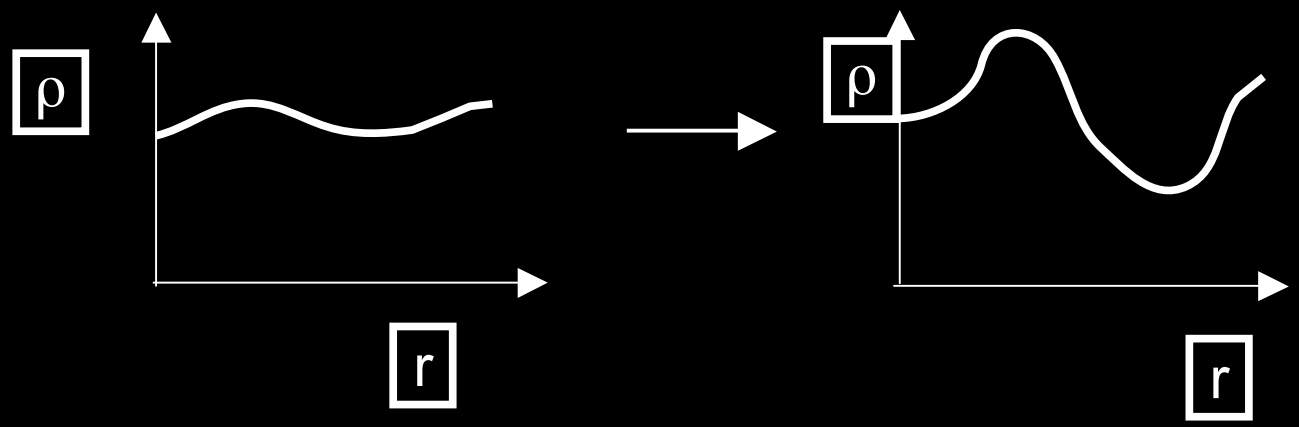
Fluctuations amplifier: Gravity

- Fluctuations grow through **gravitational instability**.

  -Regions with a higher density than the mean ($\rho > \langle\rho\rangle$) will exert a greater gravitational pull on the surrounding matter, and so attract it. They will then become even denser (ρ increases.)

  -Regions with less density than the mean will lose matter to the denser regions, and so become even less dense (ρ decreases.)

The evolution of density between two different times might look like this.



With density fluctuations, the rich get richer, and the poor get poorer...

Time machine

http://timemachine.gigapan.org/wiki/Early_Universe

Studying structure formation therefore involves calculation of how fluctuations with small density contrasts evolve under the process of gravitational instability.

-in general this is a very difficult problem

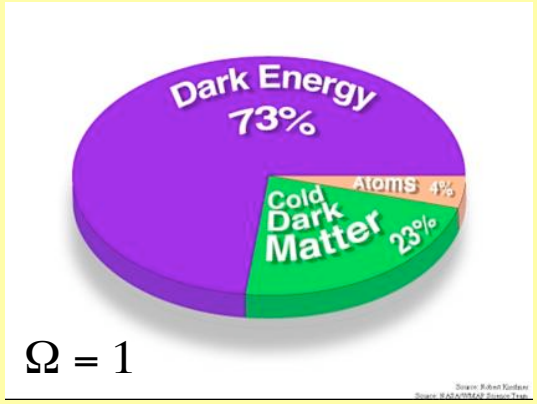
-most work is done using computers

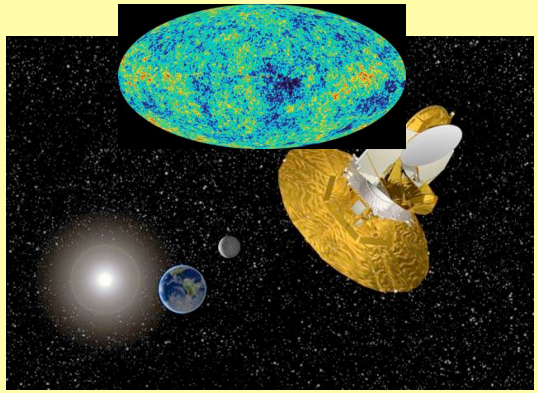
Cosmological Simulations

- We can simulate ab-initio structure formation:

  well-specified initial conditions : inflation

  + free parameters

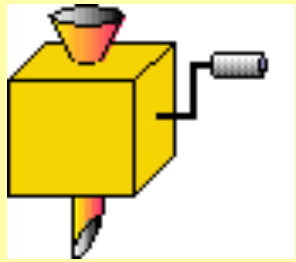


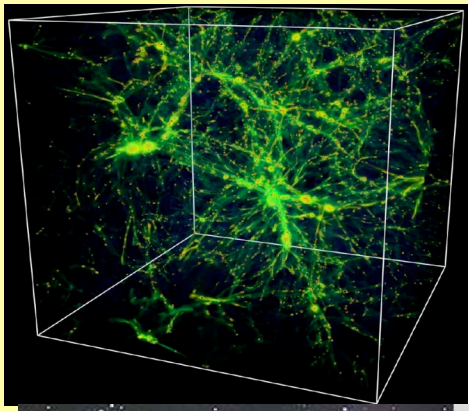


+

physics:

gravity
hydrodynamics
radiative processes
star formation
black holes



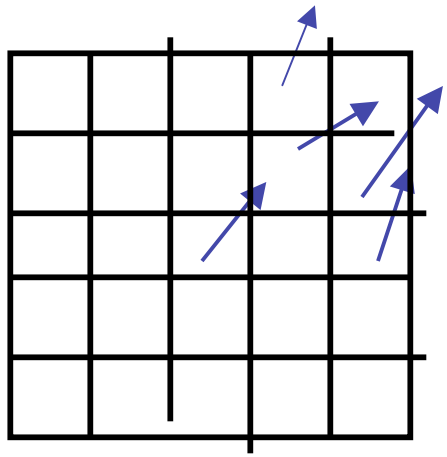


methods: how to represent a fluid

Eulerian

discretize space

representation on a mesh
(volume elements)

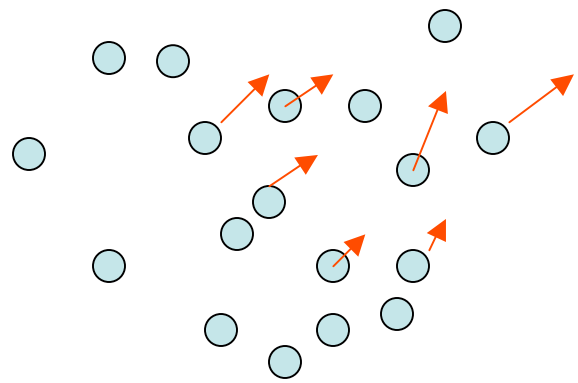


principle advantage:
high accurancy (shock capturing)
low num. viscosity

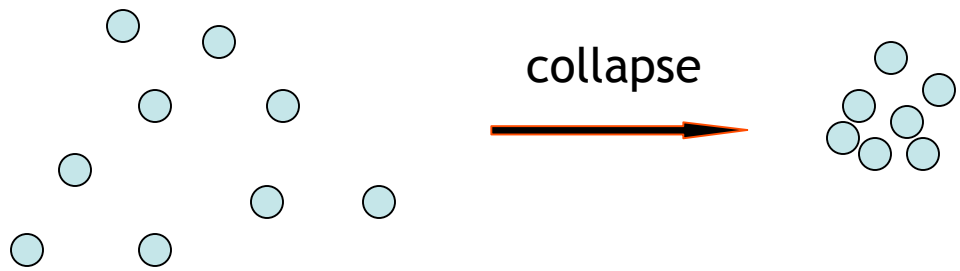
Lagrangian

discretize mass

representation by fluid elements
(particles)



principle advantage:
resolution adjusts automatically
to the flow



Cosmological Simulations

DARK MATTER: only appreciable interaction is GRAVITY

(Stars)

N-body METHOD: mass discretised in particles modeled as collisionless fluid governed by Boltzmann Eq. -
grav potential solved with Poisson Eq.

GAS: baryons in the universe can be modeled as 'ideal' gas: HYDRO EQUATIONS-
Euler eq. continuity, energy + EOS

SPH Method: fluid represented by particles smoothed by local kernel averaging
Hydro eqns solved in its Galilean-invariant Lagrangian Formulation.

- Radiative cooling of gas within halos (dissipation)

- cooling catastrophe & overproduction of stars
- the true real structure of the ISM is known to be multiphase
- collapse of gas halted by numerical resolution not by physics

Star formation and feedback:

currently impossible:
need to develop effective subgrid
models motivated by physical model of the ISM

Multiphase model of the ISM- (Springel & Hernquist 2003)

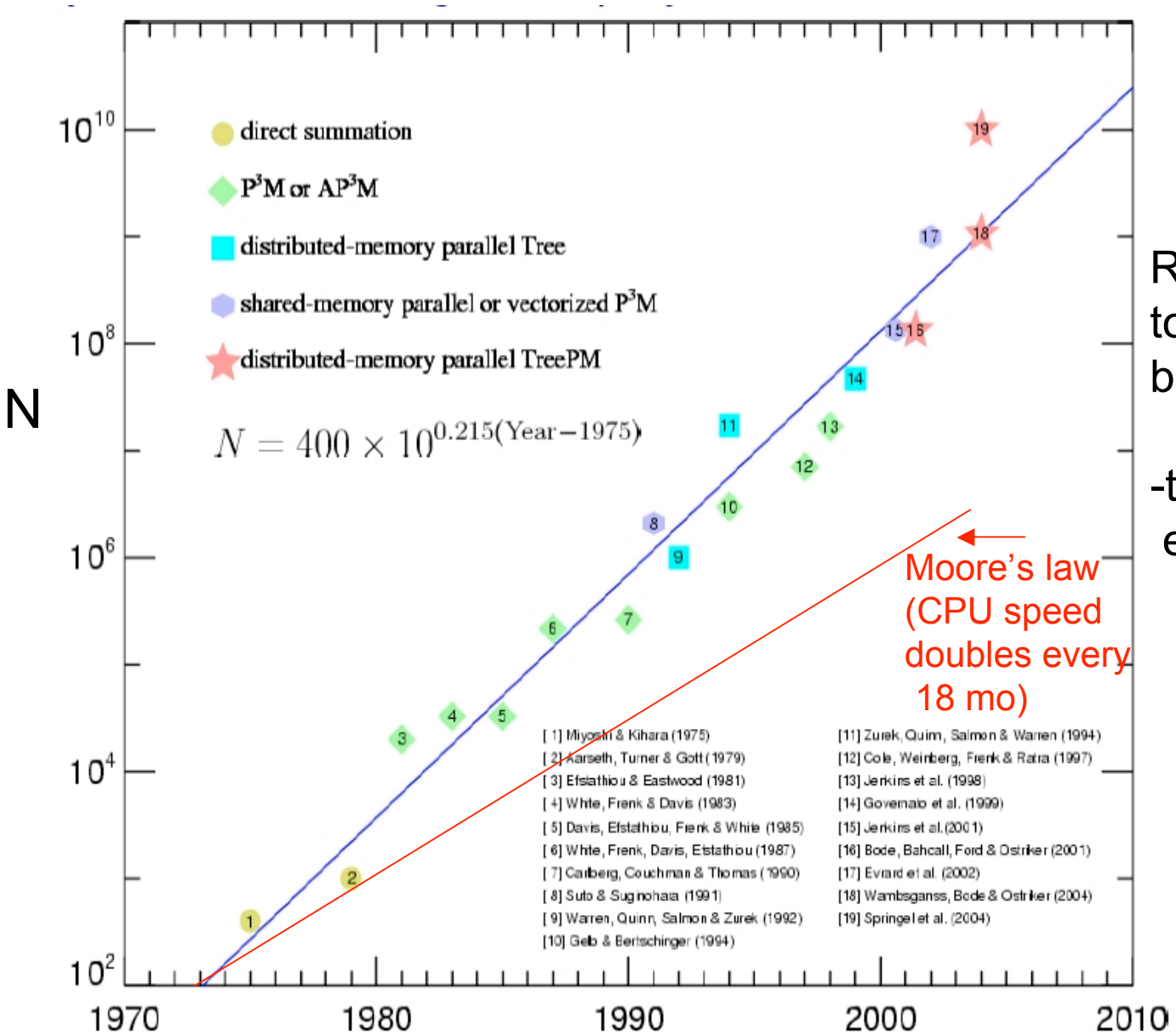
Cosmological Simulations

- conflicting requirements:

  (a) want small mass particles to resolve the relevant physics
  (b) want large volumes to obtain representative sample of the universe

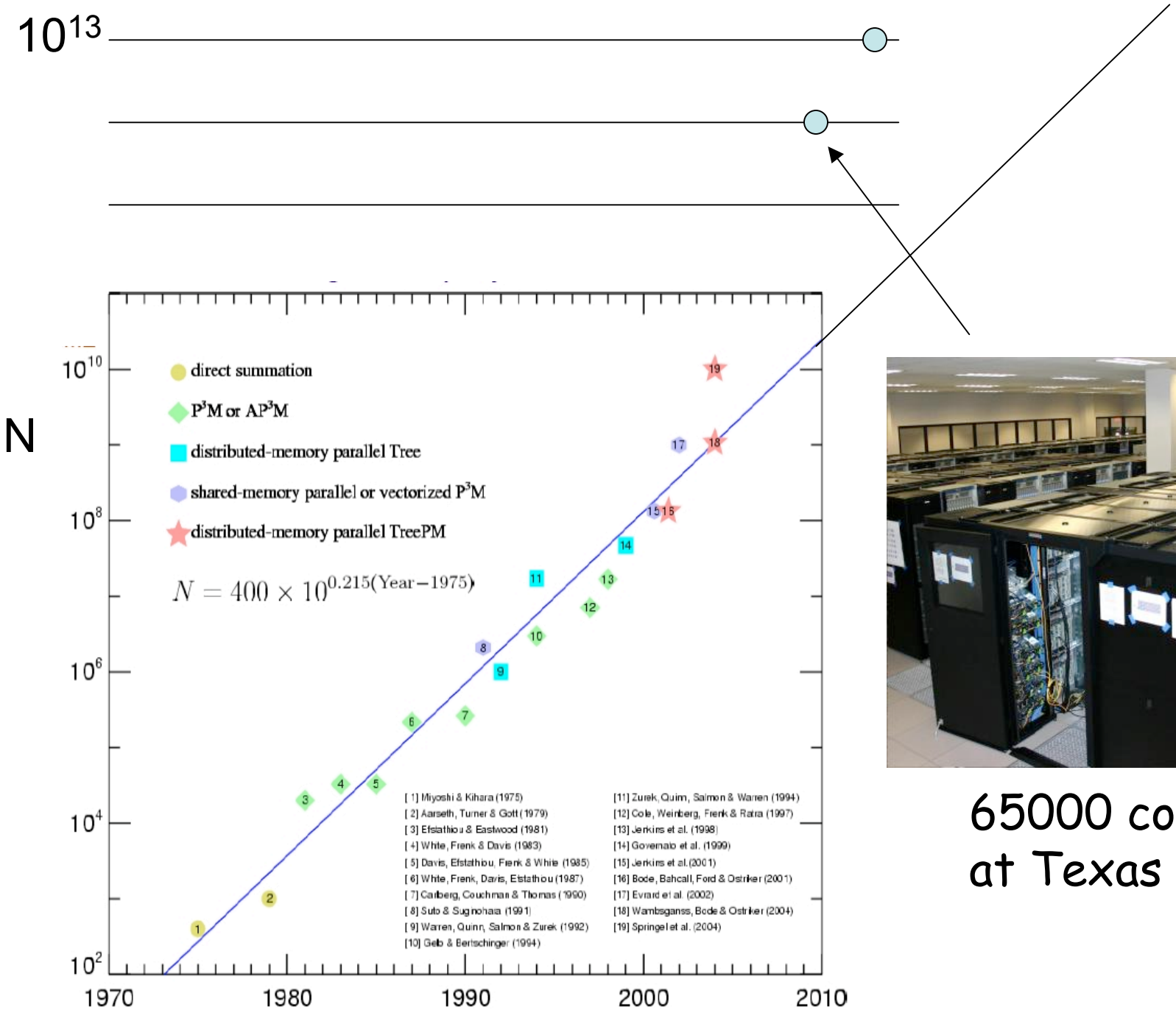
LARGE N

Number of particles vs year for gravity only simulations



Rapid rise due to parallelization, better algorithms

-these go together e.g. tree domain decomposition



65000 cores
at Texas

The particle-mesh method

Poisson's equation can be solved in real-space by a convolution of the density field with a Green's function.

$$\Phi(\mathbf{x}) = \int g(\mathbf{x}-\mathbf{x}')\,\rho(\mathbf{x})\,\mathrm{d}\mathbf{x}'$$

Example for vacuum boundaries:

$$\Phi(\mathbf{x}) = -G\int \frac{\rho(\mathbf{x})}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}\mathbf{x}' \qquad g(\mathbf{x}) = -\frac{G}{|\mathbf{x}|}$$

In Fourier-space, the convolution becomes a simple multiplication!

$$\hat{\Phi}(\mathbf{k}) = \hat{g}(\mathbf{k})\cdot\hat{\rho}(\mathbf{k})$$

→ **Solve the potential in these steps:**

(1) FFT forward of the density field
(2) Multiplication with the Green's function
(3) FFT backwards to obtain potential

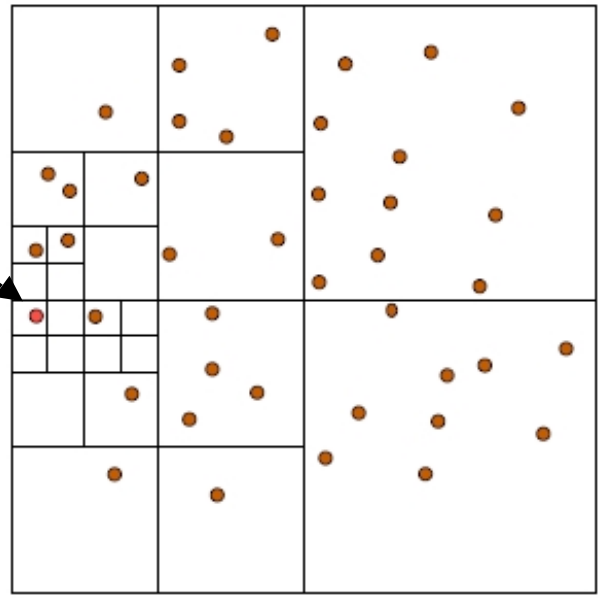
The four steps of the PM algorithm

(a) Density assignment
(b) Computation of the potential
(c) Determination of the force field
(d) Assignment of forces to particles

Tree method for gravity

To calculate force on this particle, group distant particles together using hierarchical tree structure.

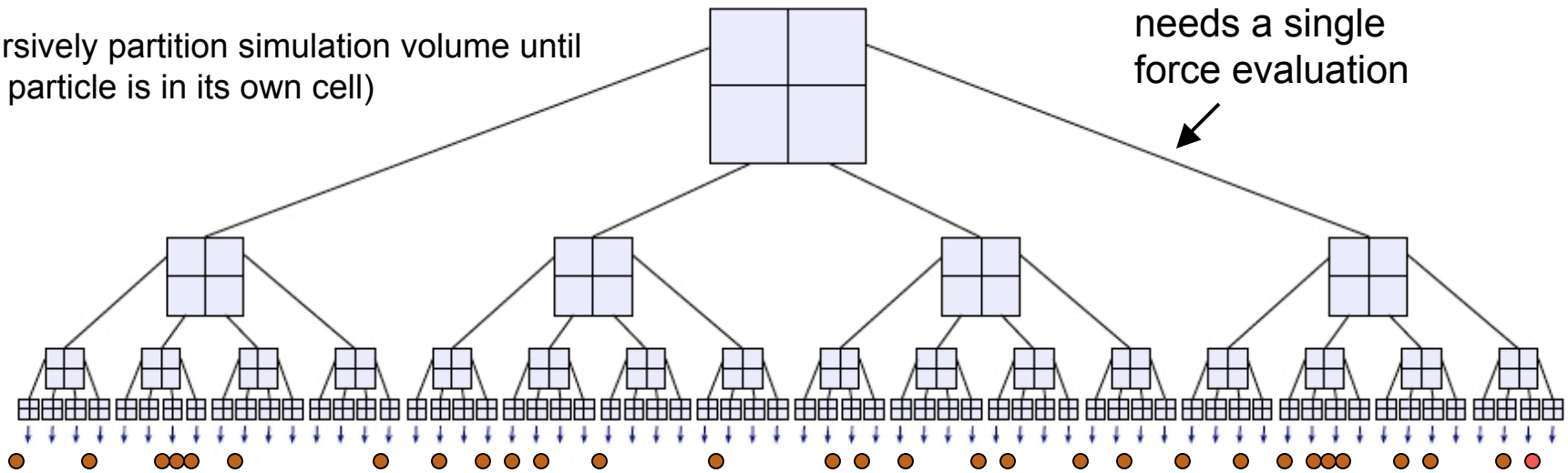
-results in ~N log N force evaluations rather than N^2



Tree structure:

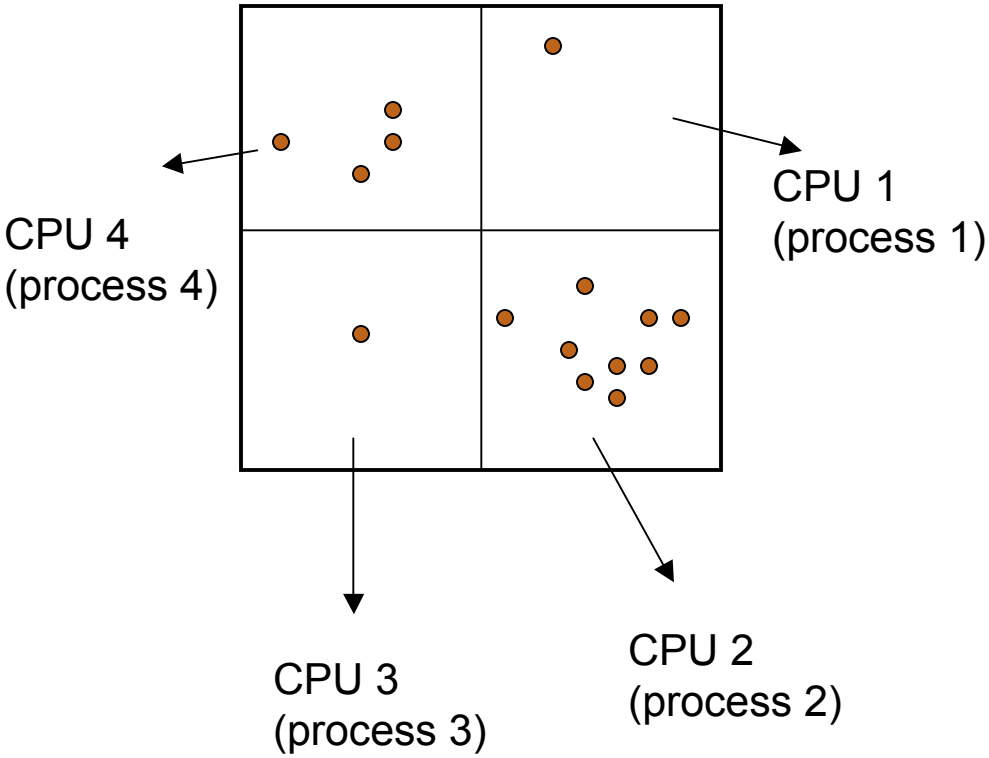
(recursively partition simulation volume until each particle is in its own cell)

Each branch needs a single force evaluation



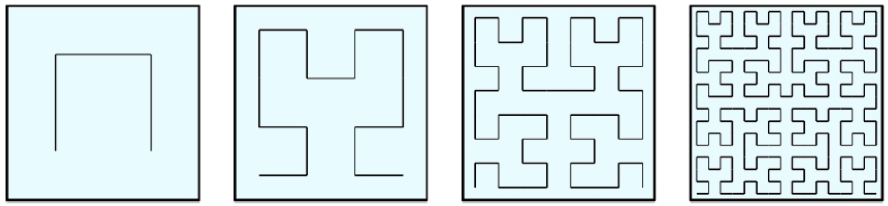
This works well on a single processor, but how do you efficiently split the force evaluations between CPUs (domain decomposition)?

Domain decomposition

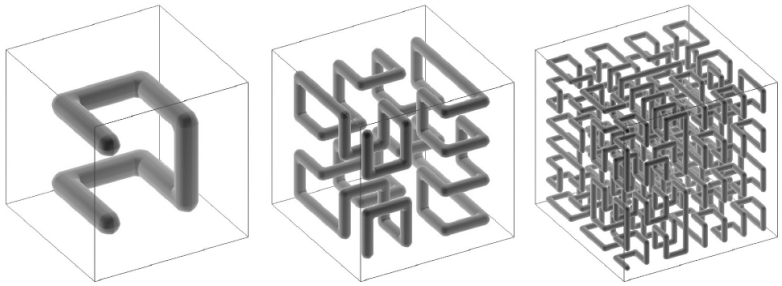


Simulations are highly clustered: With this simple partitioning of the calculation, the processes will be waiting for CPU 2 to finish.

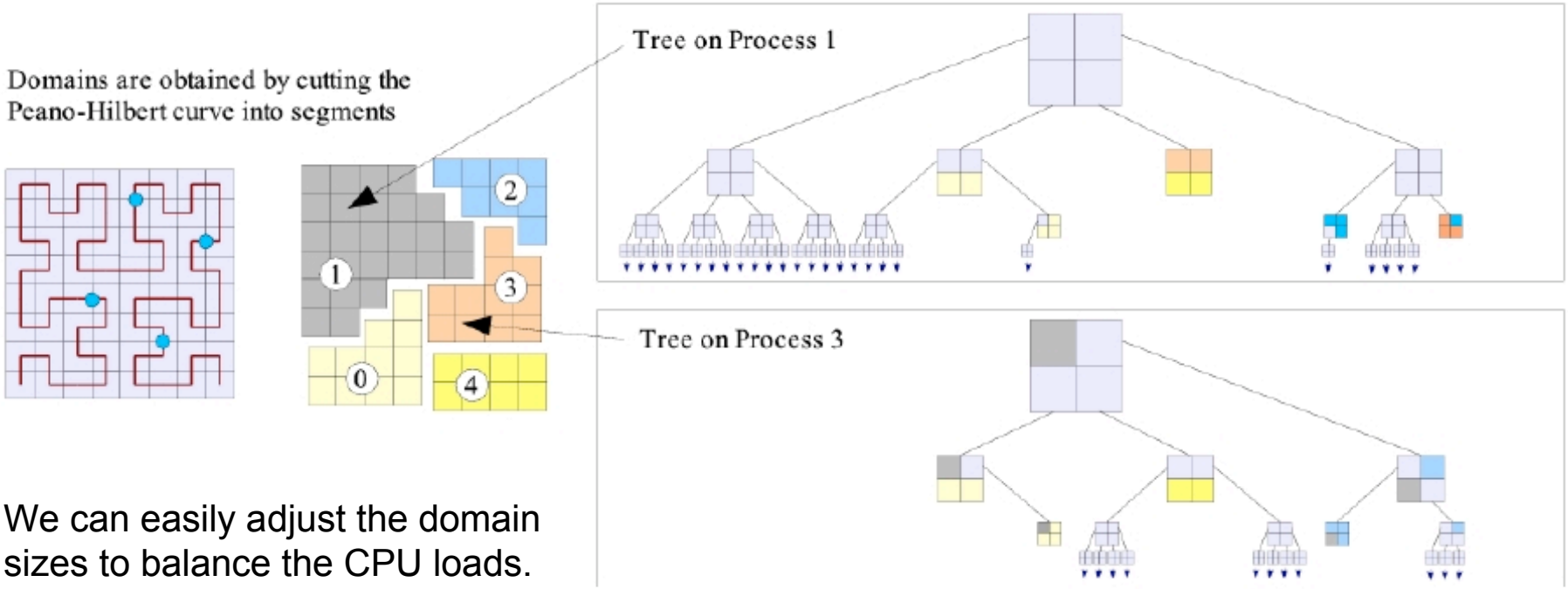
We need a better way to carry out domain decomposition for efficient parallelization.



Hilbert space filling curve (fractal)



Peano-Hilbert when generalized to 3D



We can easily adjust the domain sizes to balance the CPU loads.

BLACK HOLES and COSMOLGY

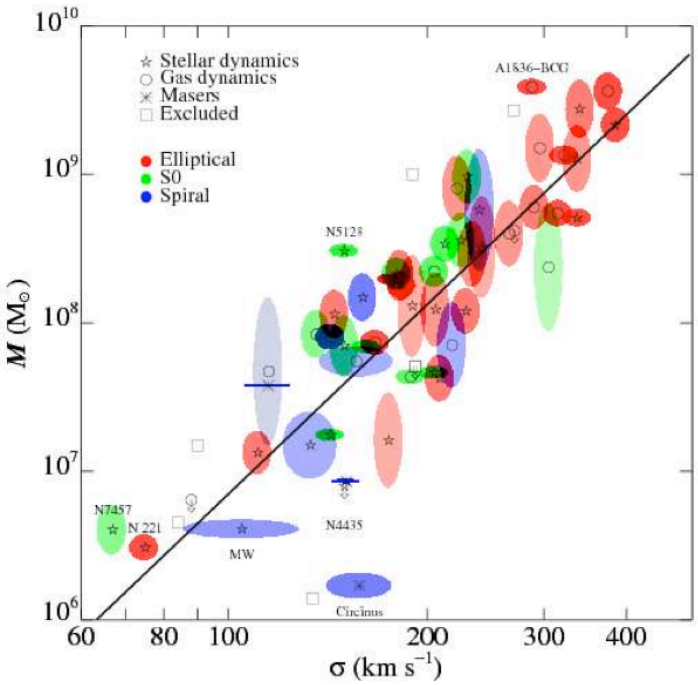
The formation, assembly history and impact of MBH - ubiquitous in the nuclei of galaxies - remain some of the main unsolved problems in cosmic structure formation studies

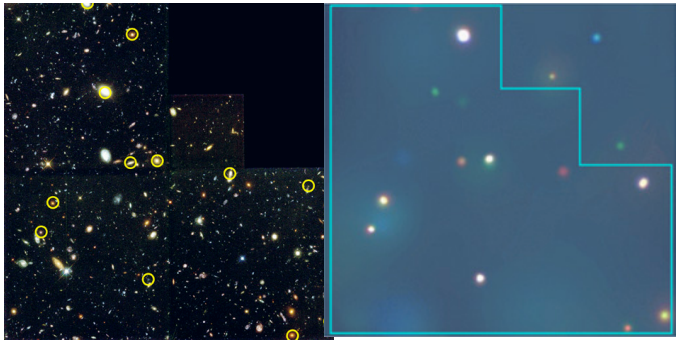
The formation, assembly history and impact of MBH - ubiquitous in the nuclei of galaxies - remain some of the main unsolved problems in cosmic structure formation studies

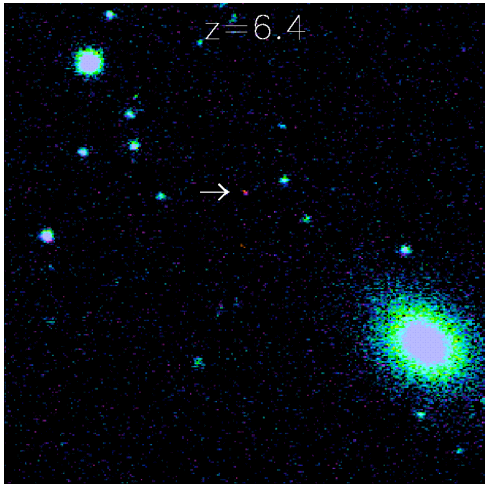
Key Questions in Galaxy evolution studies:

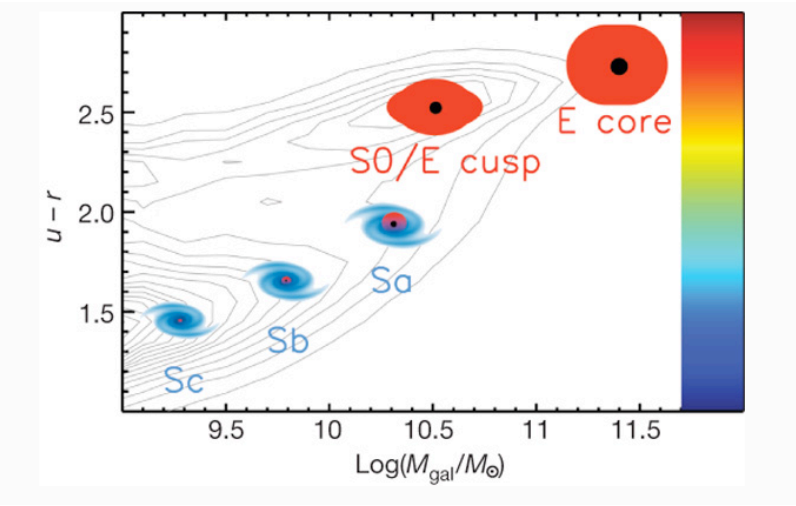
- When and how did the universe emerge from the dark ages
- How often galaxies merge and how do mergers shape the Hubble sequence /colors?
- How do “feedback” mechanisms influence galaxy formation?
- How do galaxy get their gas?

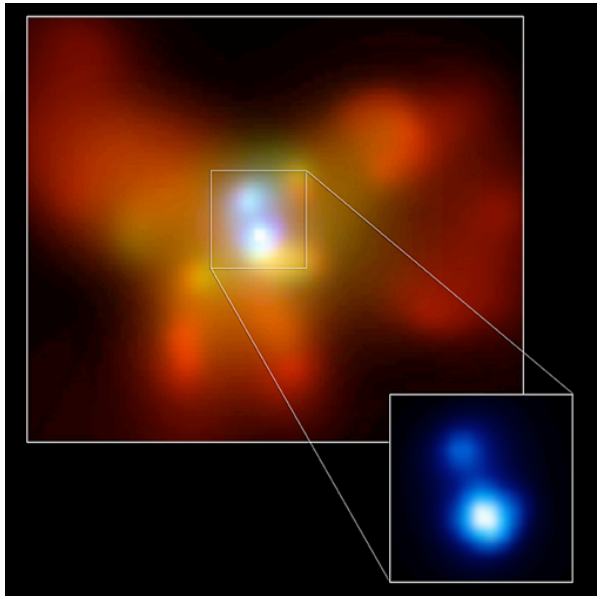
Recent observational breakthroughs….

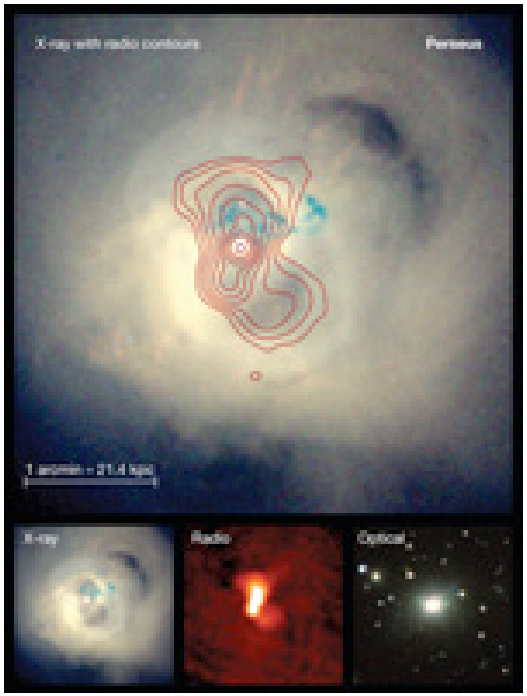






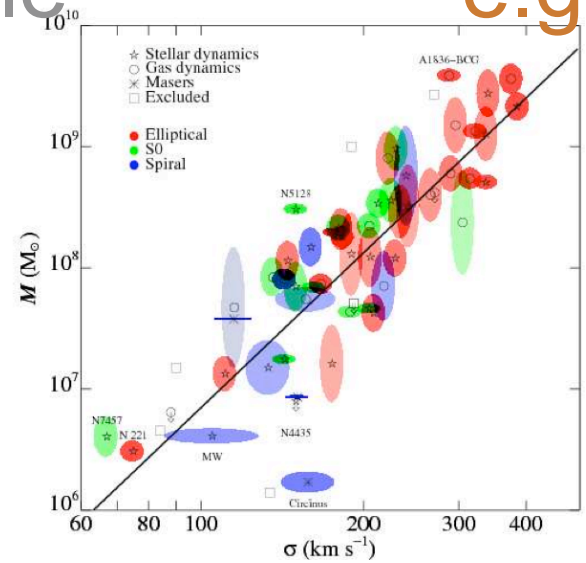






…..have not yet led to comparable theoretical progress

e.g.



SMBHs

$R_{Sch}=2GM_{BH}/c^2$

$R_{grav}=2GM_{BH}/\sigma^2$

microparsec/parsec

Stars (bulges) /galaxies

$R_{Sch}=2GM_*/\sigma^2$

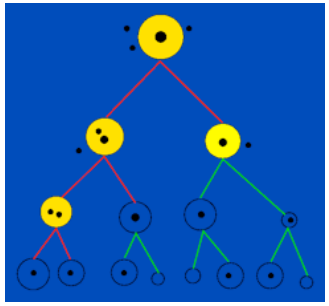
kiloparsec

DM halos?

$R_{Sch}=2GM_{Halo}/\sigma^2$

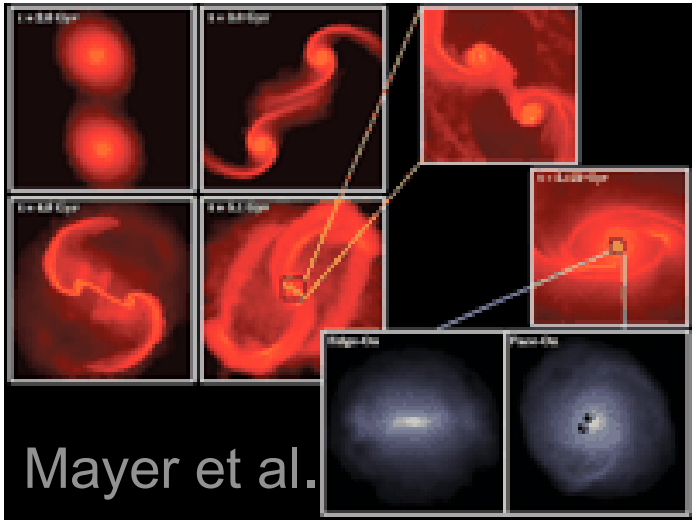
megaparsec

How/ where do MBHs seeds grow?



BH-BH mergers

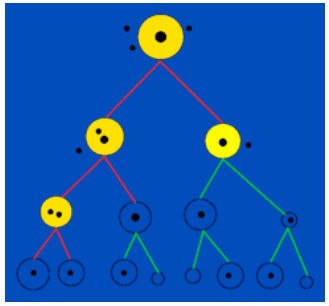
BH mass density
almost constant with time
some reshuffle in the mass function..



Gas Accretion

Total mass density of BHs
grows with time.

How/ where do MBHs seeds grow?



Gas Accretion

Total mass density of BHs grows with time.

$$L_{BH} = \text{efficiency } \dot{M}_{acc}\, c^2$$

Black hole Growth = Gas Supply = Activity

THE BLACK HOLE – GALAXY CONNECTION

The M - σ relation → AGN Feedback

(Many) Theoretical models:

BH growth self-regulated

(Silk&Rees98,Wythie&Loeb01 Fabian 99, King03,+++)

Black hole mass related to large scale properties of galaxies

$$L\, t_{dyn} = \frac{3}{2} M_{gas} \sigma^2$$

dynamical time of gal. disk

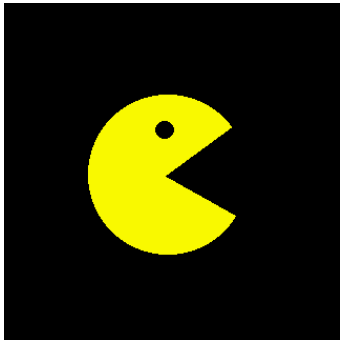
M^* / σ^3

$f_{gas}\, M_*$

$$M_{BH} \approx 2 * 10^8 M_\odot \left(\frac{\sigma}{200\ \mathrm{km/s}} \right)^5$$

BHs grow until they release sufficient energy to unbind the gas that feeds them from their host galaxy.

BHs in SPH Simulations of Galaxy formation



- **BH:** collisionless "sink" particle in the centre of galaxies

- **ACCRETION**: relate (unresolved) accretion on BH to large scale (resolved) gas distribution

$$\dot{M}_B = 4\pi \frac{(GM_{BH})^2}{(c_s^{\,2} + V_{rel}^2)^{3/2}} \rho$$

$$\dot{M}_{BH} = \min(\dot{M}_{Edd}, \dot{M}_B)$$

+ BH mergers

- **FEEDBACK:** energy extracted from the black hole (accretion) injected in the surrounding gas

$$\dot{E}_{feed} = f(\eta \dot{M} c^2) \qquad f \approx 5\%$$

BHs in Numerical Simulations

Additions in the parallel GADGET-2 code:

Implementation in SPH simulation code

- ✓ BH 'sink' particle - only feel gravity- we compute,SPH Properties of local environment (T, rho, vel)

accretion

- ✓ BH particles swallow gas stochastically from their local neighbourhoods, in accordance with the estimated BH accretion rate.

$$p_j = w_j \dot{M}_{BH} \Delta t / \rho$$

Feedback:

- ✓ BH has additional internal degree of freedom:Variable described BH mass in smooth fashion
- ✓ Feedback energy is injected kernel-weighted into the thermal reservoir of gas in BH environment

'Seed BH'

- ✓ BHs are merged if they reach small separations (smoothing lengths) and low enough relative speeds
- ✓ On-the-fly FOF halo finder detects emerging galaxies and provides them with a seed black hole

$M_{BH(seed)}=10^5 M_\odot$

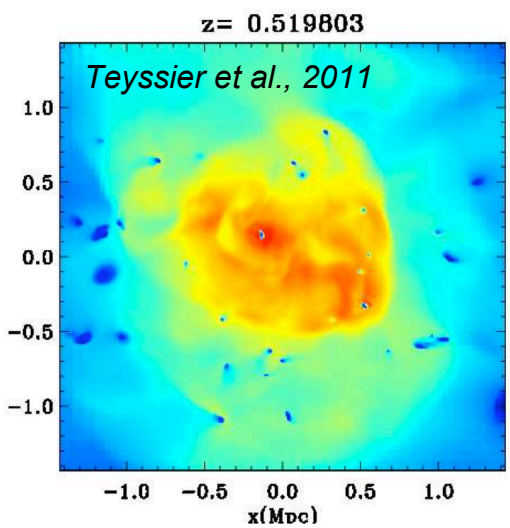
AGN in AMR (Eulearian) simulations

simulations of self-consistent jet-AGN feedback in a cosmological context

- Mimic the formation of black holes (where and when)
- Mimic the gas accretion onto black holes
- Mimic the mergers between black holes (Friend-of-friend algorithm)
- Mimic the feedback from black holes (AGN)

Teyssier, Dubois, Slyz, Devrient

With thermal input (Teyssier et al., 2011)
(see Sijacki, Di Matteo et al papers, and Booth & Schaye papers)



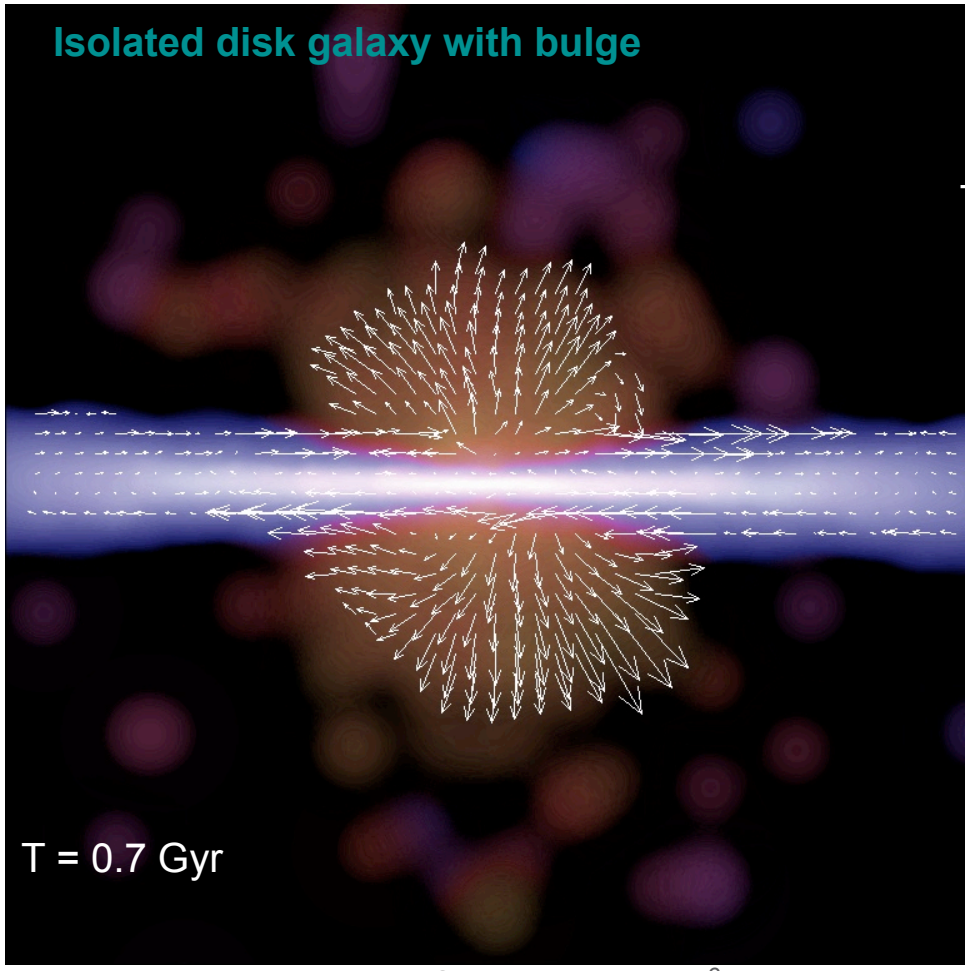
Modification of the internal energy

-> increase the gas temperature

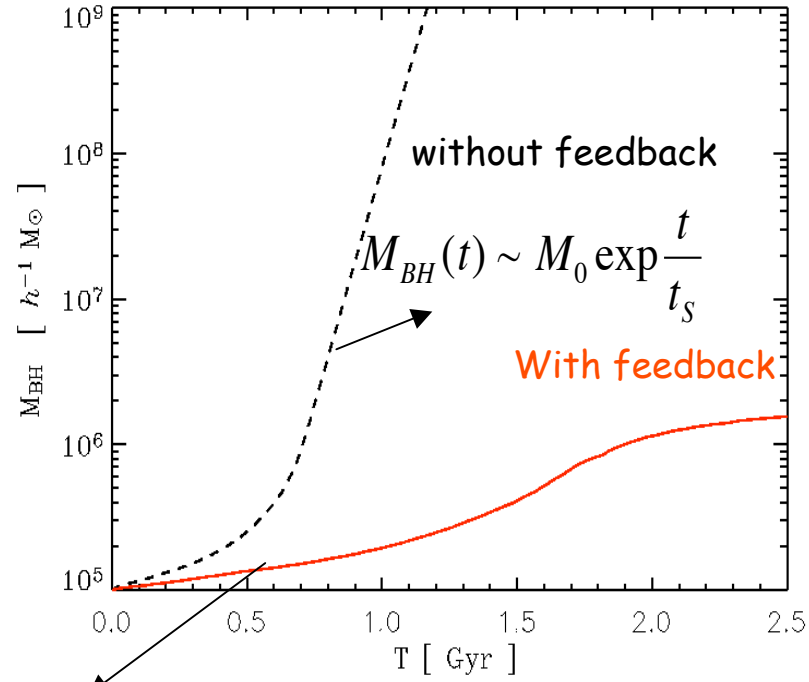
GROWTH RATE OF BLACK HOLES in ISOLATED GALAXIES

the central black hole activity blows a wind into the halo

GAS FLOWS INTO THE HALO

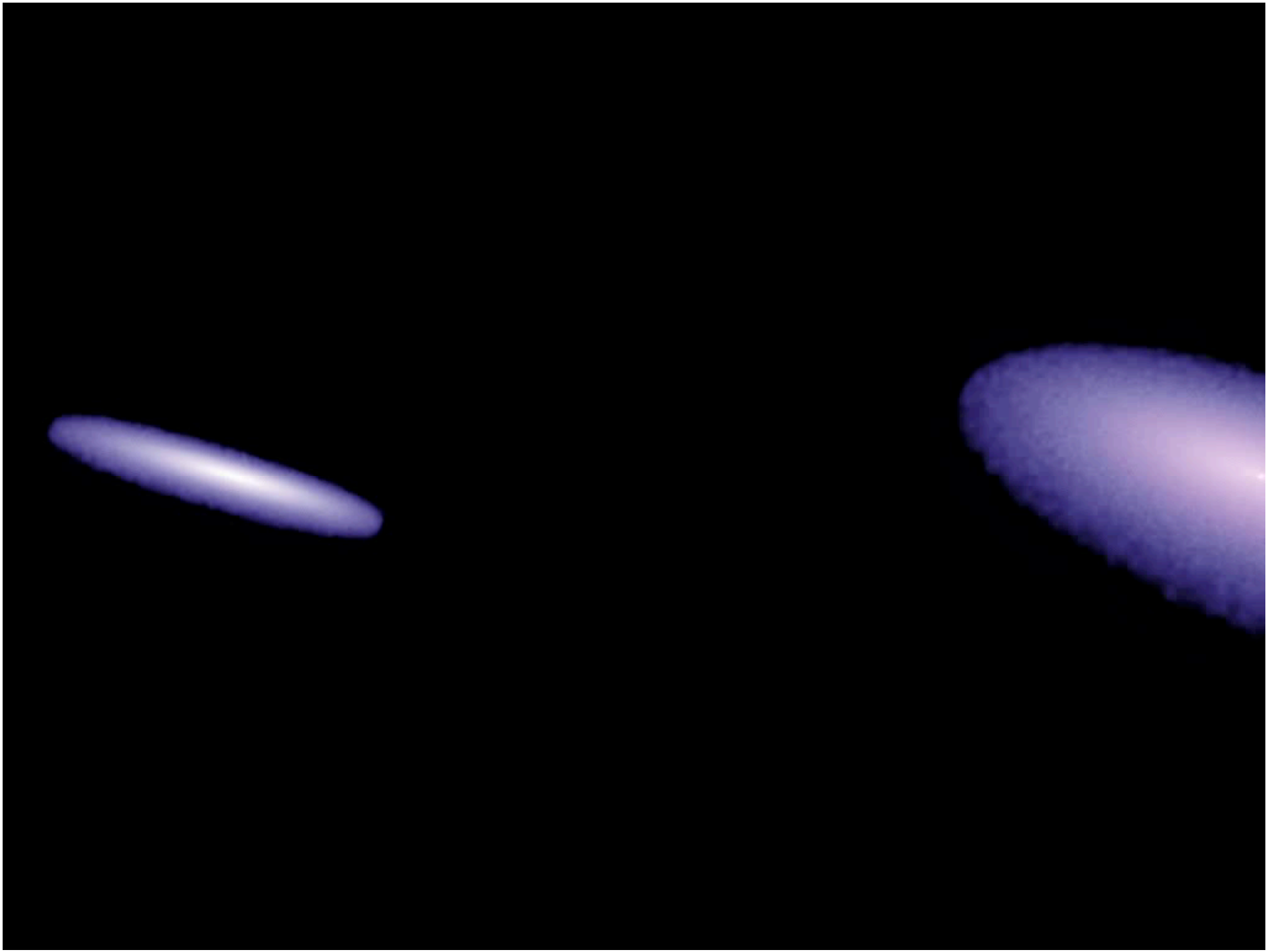


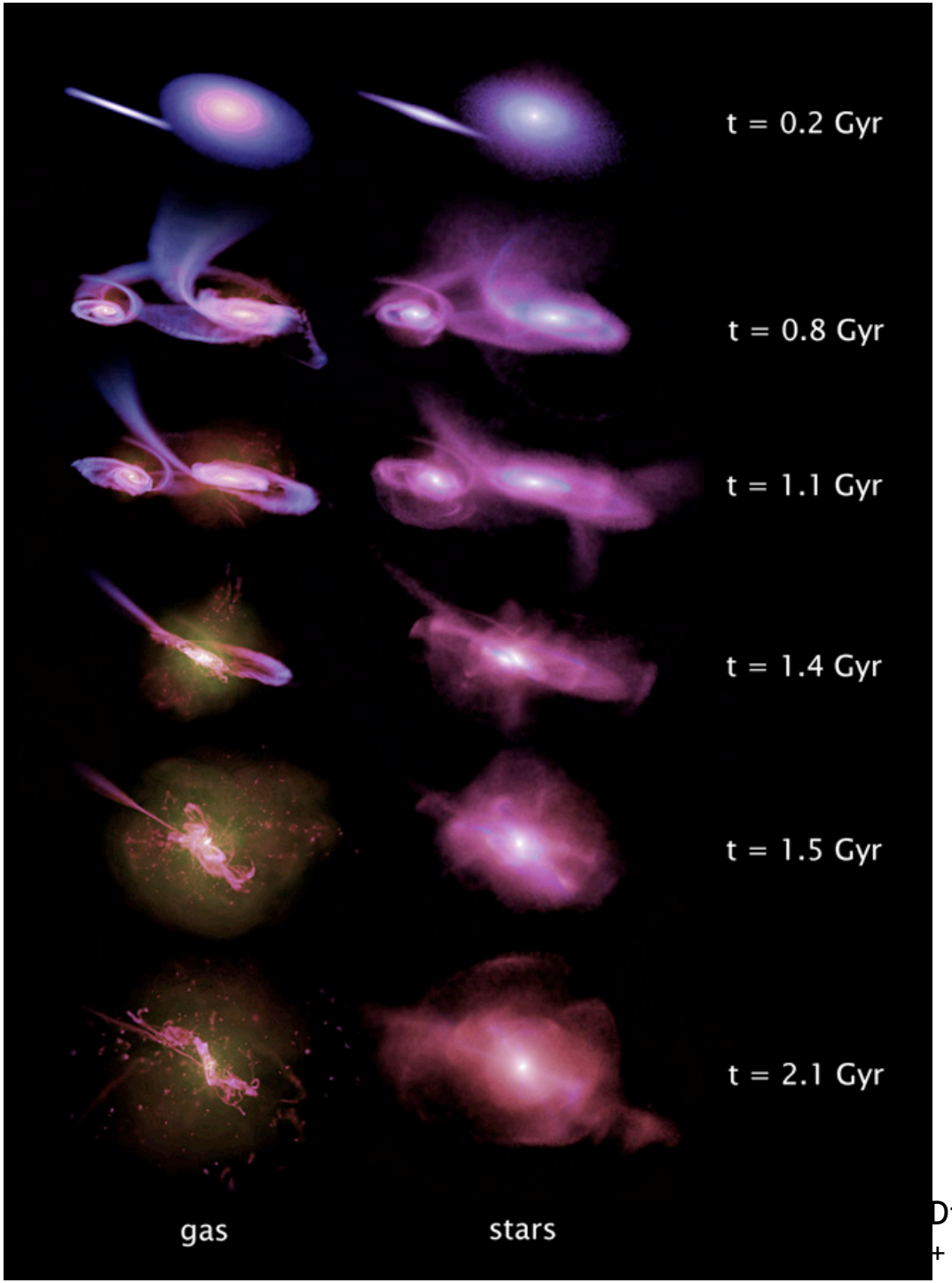
(dynamic range in gas surface density ~10^6)



Bondi growth

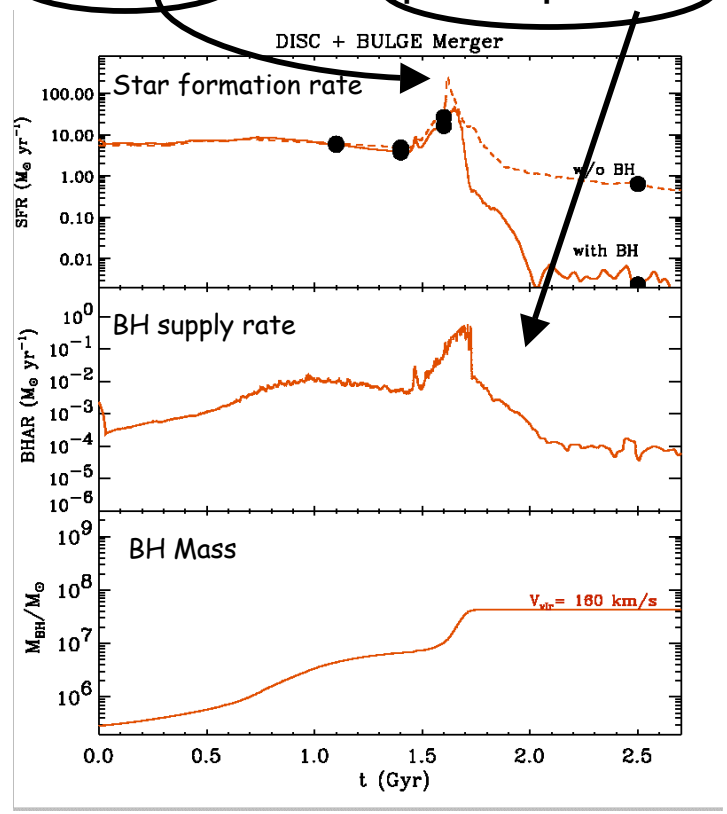
$$M_{BH}(t) \sim \frac{M_0}{1 - \chi M_0 t}$$





Time evolution of a major galaxy merger

in major-mergers between two disk galaxies, tidal torques extract angular momentum from cold gas, providing fuel for a nuclear starburst and a quasar phase

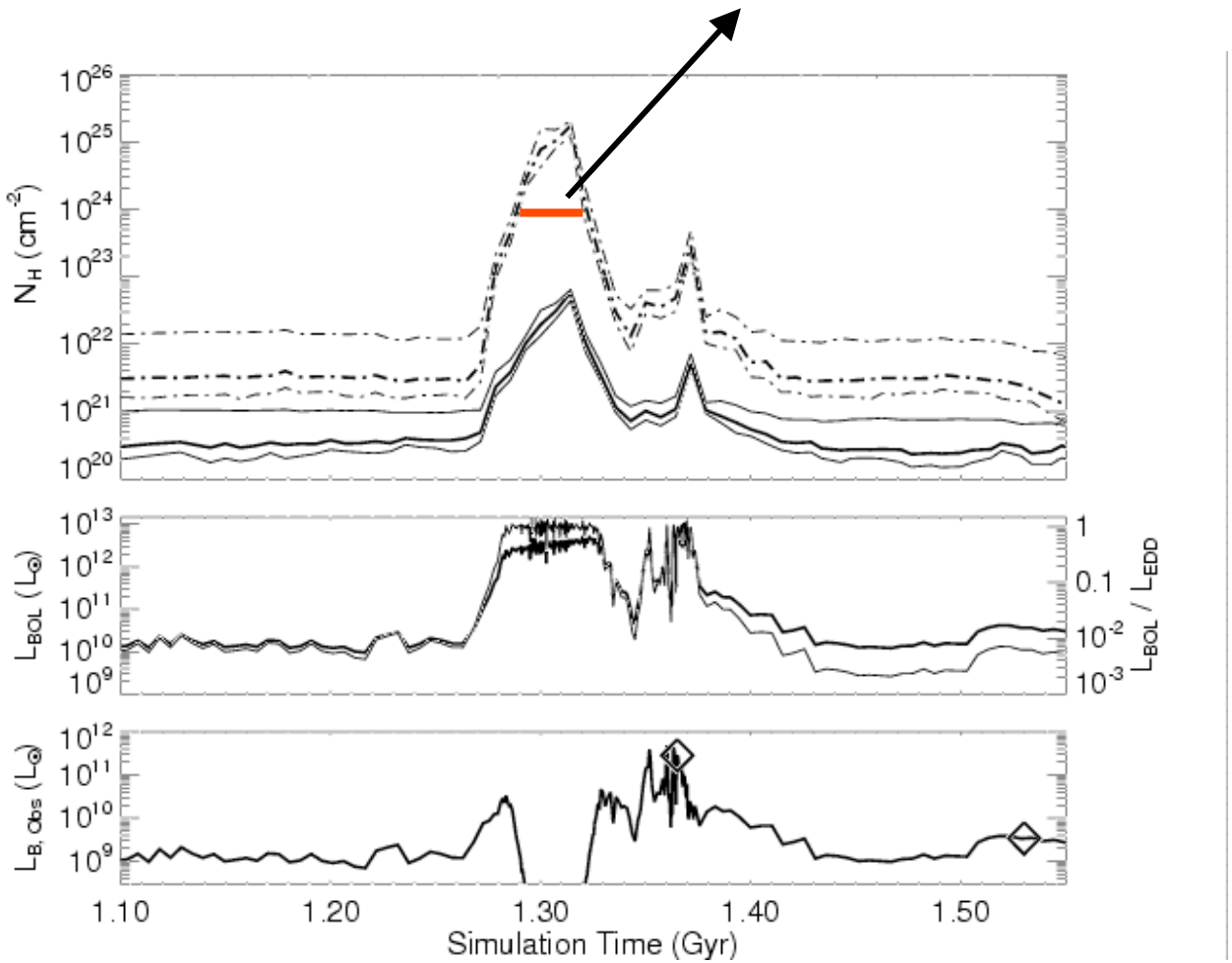


DiMatteo et al., 2005, Springel,DM,Hernquist 2005 + Hopkins et al. + Robertson et al +

PREDICTIONs FOR THE QUASAR PHASE:

large fraction of the QUASAR phase is obscured

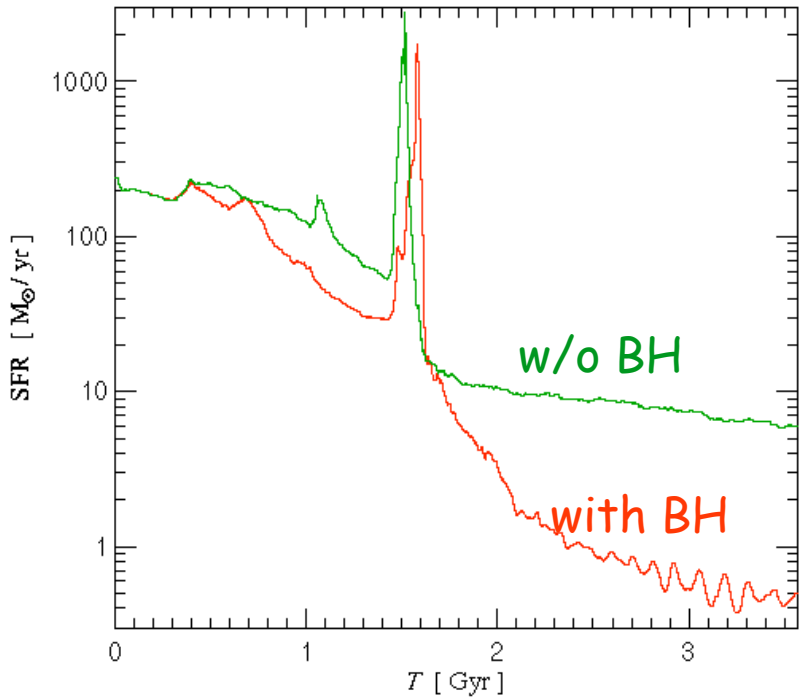
QSO obscured : time of the starburst
" " of BH growth



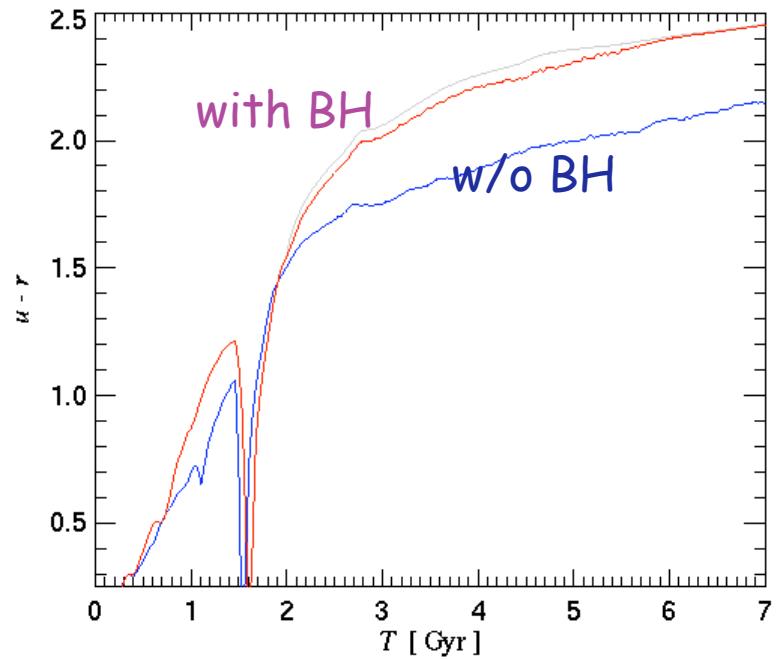
Hopkins et al. 2005

AGN feedback & REMNANT PROPERTIES

SFR evol.



Color evolution



AGN Feedback reduces SFR after the starburst

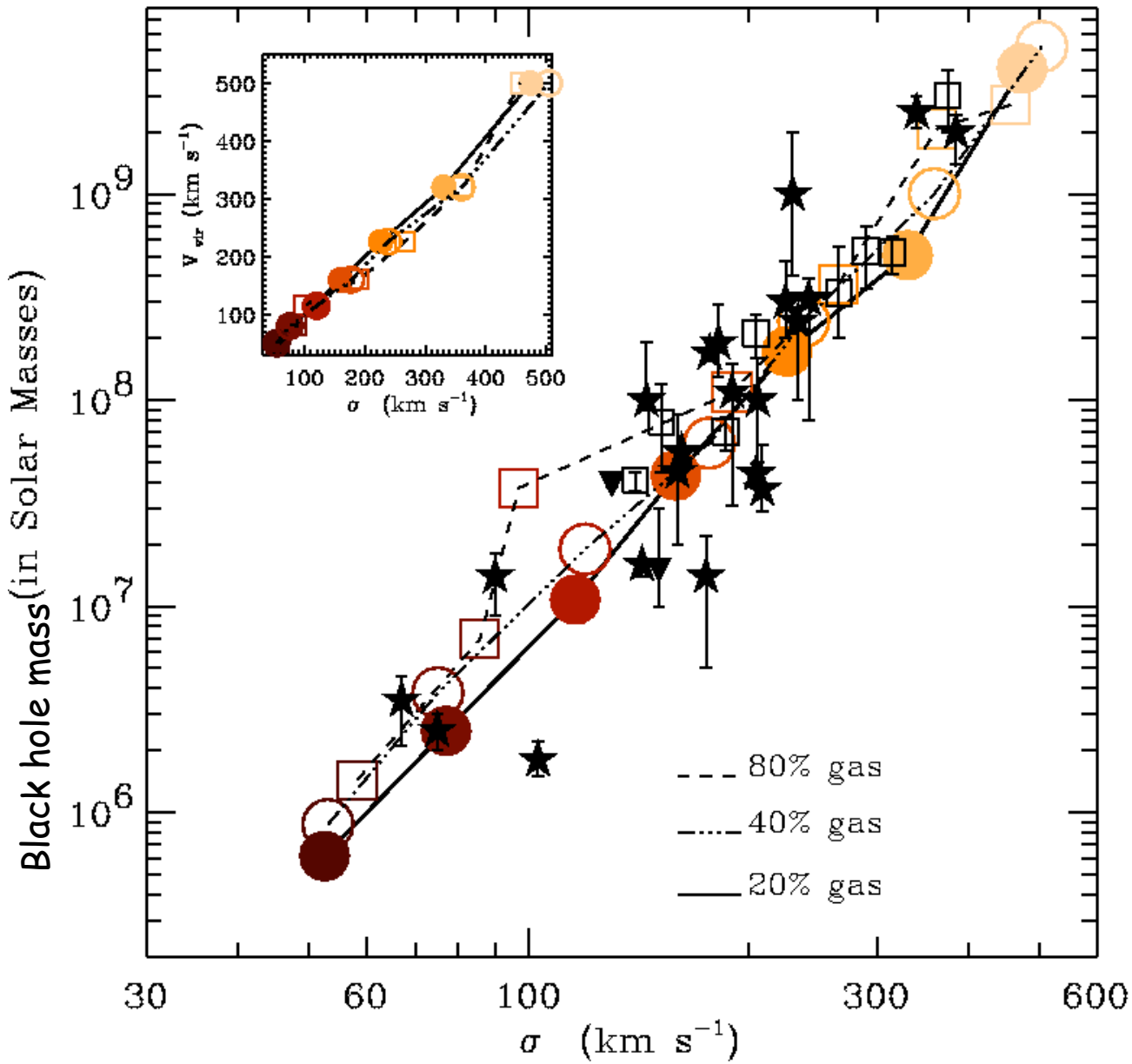
w/o BH - E formed can sustain SF over extended period
- remains relatively BLUE

With BH - Remnats redden more quickly due to efficient truncation of SFR
- red, "dead" E forms

Springel, DM & Hernquist 2005

FINAL BH MASS and SPHEROID PROPERTIES

DM, Springel & Hernquist 05



Journey into cosmological simulations with black holes

Step II: cosmological simulations

The MassiveBlack

http://www.gigapan.org/gigapans/fullscreen/69334/

visualization by Yu Feng