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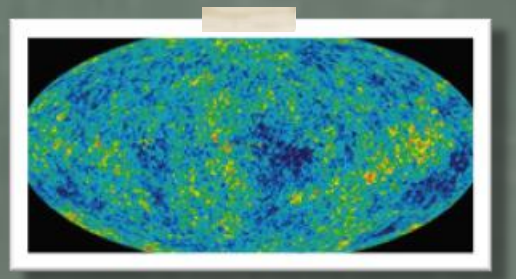
Effective Field Theory in Cosmology

Clues for cosmology from fundamental physics



Outline

- Motivation and Overview
- Effective field theories throughout physics
  - Decoupling and 'naturalness' issues
- Quantum effects in gravity: Fighting the 'split brain'
  - Living with gravity's non-renormalizability
  - Naturalness issues
- Relevance to cosmology
  - A brief cold shower
  - Inflation, Dark matter and Dark energy

3. Quantum Effects in Gravity

§ Living with gravity's non-renormalizability

§ Naturalness and fine-tuning

3.1 Living with gravity's non-renormalizability

§ What does general relativity not renormalizable mean?

– compare GR, expanded about a backgroun

$$g_{mn} = \bar{g}_{mn} + h_{mn}/M_p$$

$$L = \sqrt{-g}\ \ M_p{}^2\ R$$

$$= (\partial h)^2 + \frac{1}{M_p}\ h(\partial h)^2 + \frac{1}{M_p{}^2} h^2(\partial h)^2 + ..$$

Inverse powers of mass

Dimensionless

– with QED

$$L = (\partial A)^2 + \psi\partial\psi + \mathrm{m}\psi\psi + \mathrm{e}\ A\,\psi\psi$$

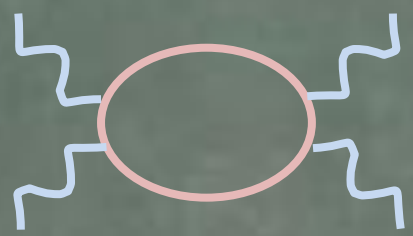
3.1 Living with gravity's non-renormalizability

§ Why do you care? Imagine calculating photon-photon scattering in QED:

using

$$L = (\partial A)^2 + \psi\partial\psi + \mathrm{m}\psi\psi + \mathrm{e}\, A\,\psi\psi$$

to evaluate



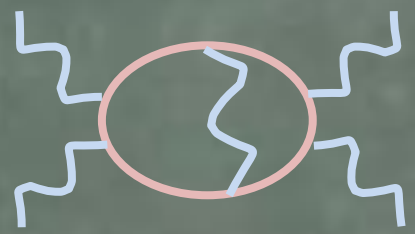
gives

$$A = e^4\, E^4 \int \frac{d^4p}{(2\pi)^4}\, \frac{1}{(p^2+m^2)^4} \cong \frac{e^4 E^4}{(4\pi)^2\, m^4}$$

3.1 Living with gravity's non-renormalizability

§ Two noteworthy features:

– The integral converges, and continues to do so once higher orders are included (more powers of e don't require a more divergent integral)



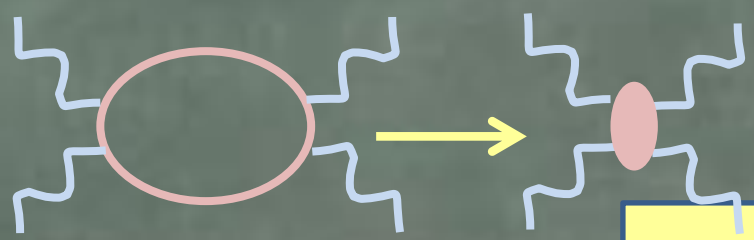
Adds 3 extra internal lines plus two vertices and one extra loop

$$A = e^6 E^4 \int \left(\frac{d^4p}{(2\pi)^4}\right)^2 \frac{1}{(p^2+m^2)^4} \frac{1}{p^2} \frac{1}{(p+m)^2}$$

3.1 Living with gravity's non-renormalizability

§ Two noteworthy features:

– Also, if photon energies are small compared with m then this is well described to order E^4/m^4 by an effective interaction



Notice: smallest mass wins

$$L = c\,(F_{mn}F^{mn})^2 \qquad c \cong \sum_i \frac{e^4}{(4\pi)^2\, m_i{}^4}$$

3.1 Living with gravity's non-renormalizability

§ The 'magic' of renormalizability in QED:

– Some amplitudes do diverge in QED, but only the following ones:

- Those with exactly two external photon lines and no external electron lines: $E_\gamma = 2$ and $E_e = 0$.
- Those with exactly two external electron lines and either zero or one external photon line: $E_\gamma = 0$ or 1 and $E_e = 2$.

$$A = e^2\, E^2 \int \frac{d^4p}{(2\pi)^4}\, \frac{1}{(p^2+m^2)^2} \cong \frac{e^2 E^2}{(4\pi)^2} \ln\frac{\Lambda}{m}$$

3.1 Living with gravity's non-renormalizability

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$$L = \delta z_1 \, (\partial A)^2 + \delta z_2 \psi \partial \psi + \delta m \, \psi\psi + \delta e A \, \psi\psi$$

Precisely the same form as interactions in L, so divergences can be absorbed into parameters of L

3.1 Living with gravity's non-renormalizability

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dimensional regularization

$$A = e^2\, E^2 \int \frac{d^4p}{(2\pi)^4}\, \frac{1}{(p^2+m^2)^2} \cong \frac{e^2 E^2}{(4\pi)^2} \ln\frac{E}{m}$$

3.1 Living with gravity's non-renormalizability

§ Repeat for graviton graviton scattering in GR:

using

$$L = (\partial h)^2 + \frac{1}{M_p} h(\partial h)^2 + \frac{1}{{M_p}^2} h^2(\partial h)^2 + ..$$

to evaluate

gives

$$A_{no\ loops} \cong \frac{E^2}{{M_p}^2} \quad and \quad A_{loop} = \frac{E^2}{{M_p}^4} \int \frac{d^4p}{(2\pi)^4} \frac{p^6}{(p^2)^4}$$

3.1 Living with gravity's non-renormalizability

§ Two noteworthy features:

– The no-loop (or 'tree') result agrees with classic classical calculations of graviton scattering (de Witt)

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– The loop integral diverges, and higher-order loops just diverge more and more (as can be seen on dimensional grounds because the coupling has dimensions of negative powers of mass)

$$A_{loop} = \frac{E^2}{{M_p}^6} \int \left(\frac{d^4 p}{(2\pi)^4} \right)^2 \frac{p^{10}}{(p^2)^7}$$

3.1 Living with gravity's non-renormalizability

§ Non-renormalizable means all divergences cannot be absorbed into the theory's couplings (ie Newton's constant)

– But divergences can be absorbed if GR is regarded as just part of a low-energy effective theory, for which w
possible interactions allowed by symmetr

Each factor of R carries two derivatives of the metric

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{c_3}{m^2} R^3 + \cdots$$

– Additional divergences can be absorbed into the new couplings, c_i

3.1 Living with gravity's non-renormalizability

§ Can compute size of contributions of each interaction to any process.

– eg L-loop contribution to graviton scattering at energy Q involving E external lines (in dimensional regularization) and V_{ik} vertices involving i fields and k derivatives:

$$A_E(Q) \propto \left(\frac{Q^2}{M_p^{E-2}}\right)\left(\frac{Q}{4\pi M_p}\right)^{2L} \prod_{i;k>2}\left(\frac{Q}{M_p}\right)^{2V_{ik}}\left(\frac{Q}{m}\right)^{(k-4)V_{ik}}$$

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– No negative powers of Q, so loops controlled by Q/M_p, and Q/m (with the latter only important for higher-curvature terms)

3.1 Living with gravity's non-renormalizability

§ Power-counting for E-point graviton scattering at energy Q:

$$A_E(Q) \propto \left(\frac{Q^2}{M_p^{E-2}}\right)\left(\frac{Q}{4\pi M_p}\right)^{2L} \prod_{i;k>2}\left(\frac{Q}{M_p}\right)^{2V_{ik}} \left(\frac{Q}{m}\right)^{(k-4)V_{ik}}$$

- Leading order: Q^2 / M_p^{E-2}
  corresponding to $L = 0$, $V_{ik} = 0$ unless $k = 2$
- These are tree graphs built using only interactions involving precisely two derivatives (ie: classical GR)

3.1 Living with gravity's non-renormalizability

§ Power-counting for E-point graviton scattering at energy Q:

$$A_E(Q) \propto \left(\frac{Q^2}{M_E^2}\right)\left(\frac{Q}{4\pi M_E}\right)^{2L}$$

Notice also: only 4 parameters to this accuracy, so still predictive

Notice: divergences in these renormalize coefficients of these

arises in one of two ways

$L = 1$, $V_{ik} = 0$ unless $k = 2$; (ie 1 loop in GR)

or: $L = 0$, $V_{ik} = 1$ for $k = 4$, $V_{ik} = 0$ for $k > 4$

(ie tree graph with R^2 term used exactly once).

3.1 Living with gravity's non-renormalizability

§ An example of quantum prediction at low-energy

– eg: (Bjerrum-Bohr, Donoghue & Holstein)

$$V(r) = -\frac{GM_1M_2}{r}\left[1 - \frac{G(M_1+M_2)}{rc^2} + \boxed{\frac{\xi G\hbar}{r^2c^3}} + \cdots\right]$$

Finite and calculable coefficient $\xi = -\frac{167}{30\pi}$

3.1 Living with gravity's non-renormalizability

§ This is the low energy limit of what?

– Don't know, but it doesn't matter. If we know the underlying theory (eg string theory), then the constants c_i can be computed; if we don't know then they are to be obtained from experiments

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{c_3}{m^2} R^3 + \cdots$$

– Notice: higher-order terms can be (and usually are) suppressed by the lightest mass, $m \ll M_p$ integrated out (like in QED).

– Similarly: lowest-order terms (Einstein term and cosmological constant) should be enhanced by heaviest scale (M_p)

3.1 Living with gravity's non-renormalizability

§ Useful guideline for model-building

– Notice: never get anything but a series in curvatures in this way

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{c_3}{m^2} R^3 + \cdots$$

– For more exotic functions, like $f(R)$ theories, require a way to understand how quantum corrections can be controlled in order not to lose the successes of gravity for pulsars and in the solar system.

3.1 Local summary

- Classical GR is a great success of quantum gravity!
  - Can justify domain of classical approximation.
  - Identifies a hidden approximation: low energies: $E \ll m, M_p$
  - NO help with strong-field, quickly varying systems (including most interesting problems of quantum gravity).
- Useful for practical calculations:
  - eg: tracking the v/c expansion for radiation by in-spiralling compact sources (Goldberger & Rothstein), etc..
  - Allows a systematic identification of where quantum effects might be small yet significant (eg black holes, inflationary universe,. . . .).

3.2 Naturalness and fine-tuning

- How does matter change this happy picture?
  - Main change is to allow new kinds of interactions

$$L = V(\varphi) + g(\varphi)\,(\partial\varphi)^2 + M_p{}^2 f(\varphi) R + ..$$

$$V = m^4 + m^2\varphi^2 + m\,\varphi^3 + ..$$

- Low-dimension terms, like mass terms and the cosmological constant, are potentially dangerous if m is large
  - When used in loops they introduce powers of M in numerators, jeopardizing systematic suppression of loops by powers of Q/M
  - Worse: integrating out particle of mass M gives contribution with $m \sim M$ (and the heaviest mass that can contribute is most important).

3.2 Naturalness and fine-tuning

- These large contributions to masses can sometimes be forbidden by symmetries
  - for spin-half and spin-one particles
  - for spin-zero cubic terms
  - supersymmetry
  - for scalar mass terms in specific case where there is a shift symmetry: $\varphi \to \varphi + c$ (corresponding to a goldstone boson)

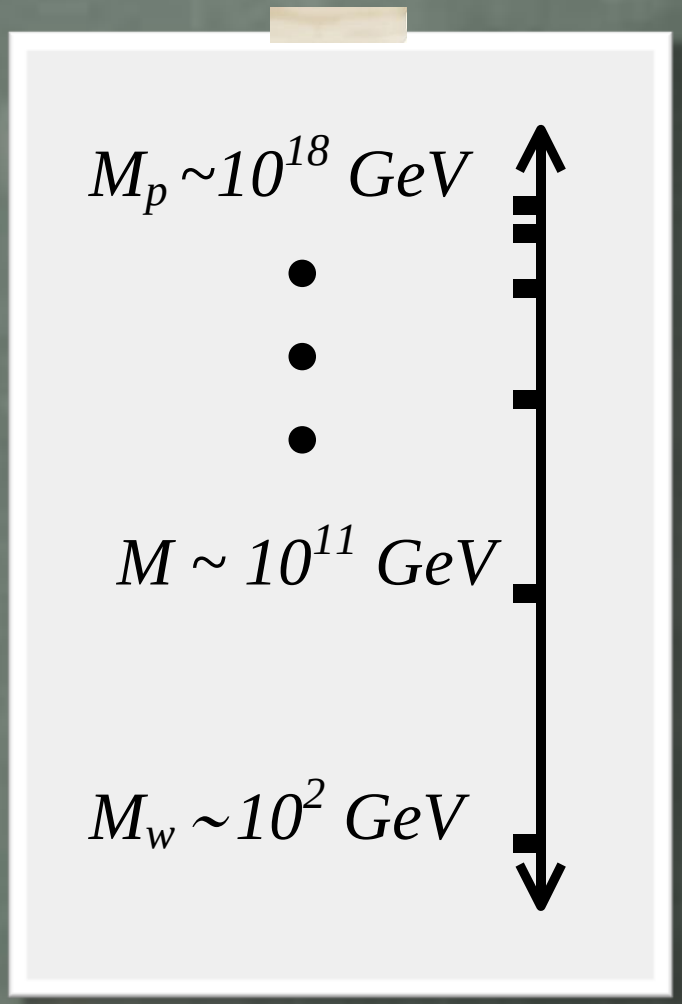
- Otherwise they are dangerous, and reflect a sensitivity to very heavy (or very short-wavelength) physics

3.2 Naturalness and fine-tuning

- Need you really care about these large contributions?
  - After all, the couplings have already been used to absorb infinite contributions from divergences

- Nobody can force you to care, but this is not how things usually work

$$L_{SM} = m_0^2 H^* H$$



3.2 Naturalness and fine-tuning

§ Effective theory can be defined at any scale

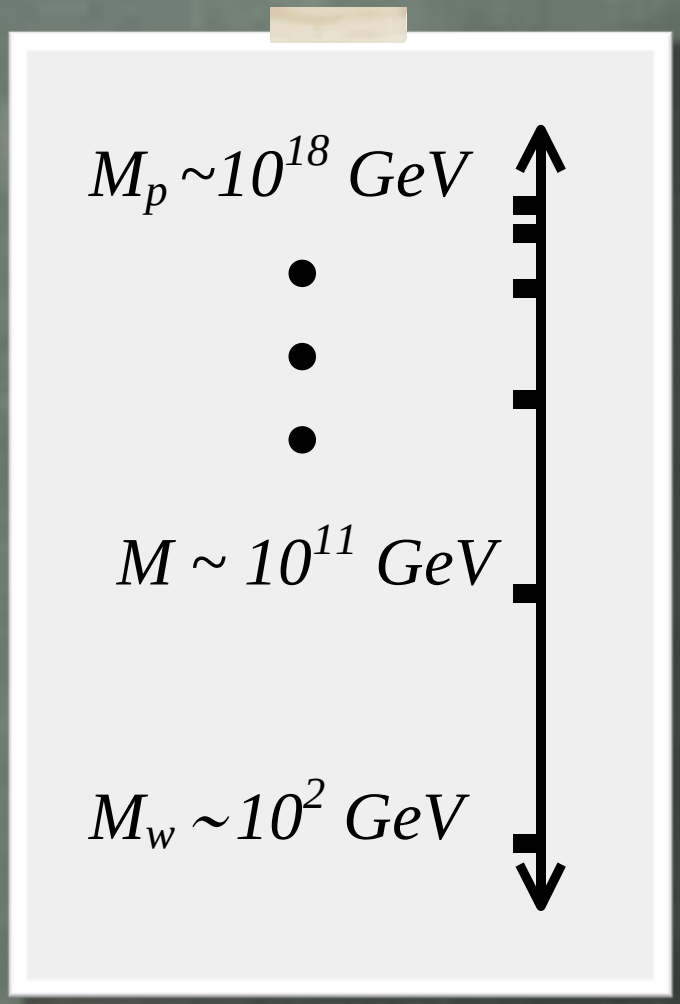
$$L = m_1{}^2 H^2 + \cdots$$

$$m^2 \approx m_1^2 + kM^2 + \cdots$$

$$L = m_0{}^2 H^2 + \ldots$$

$$m^2 \approx m_0^2 + \cdots$$

Must cancel to 20 decimal places!!



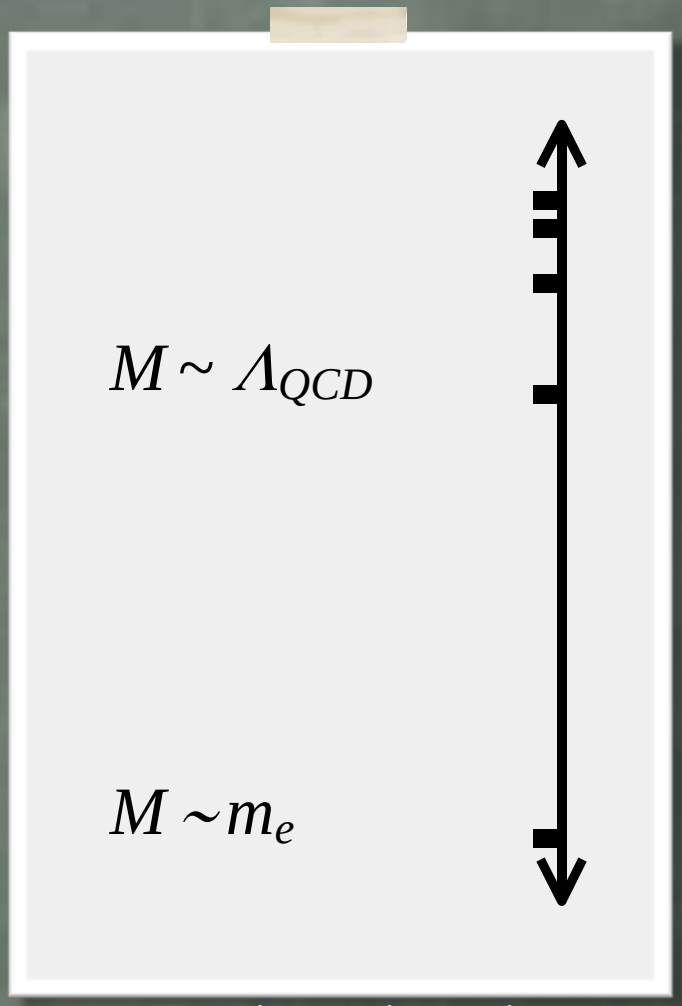
3.2 Naturalness and fine-tuning

§ Not how hierarchies usually work

– eg why are atoms large compared with nuclei?

$$\alpha m_e << \Lambda_{QCD}$$

$$\alpha m_e << m_p$$



3.2 Naturalness and fine-tuning

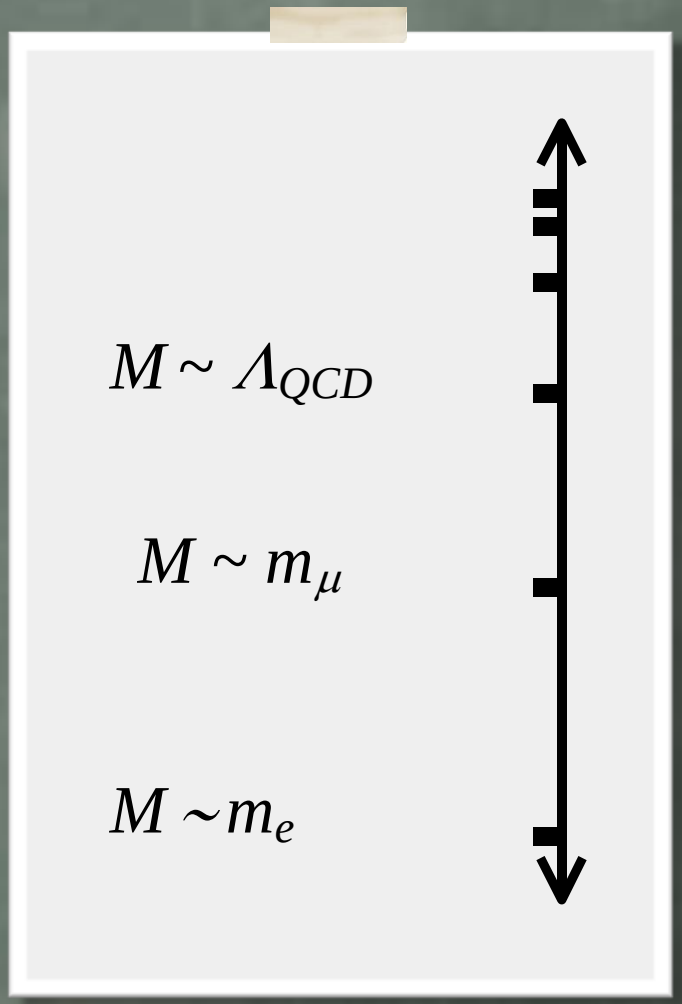
§ Not how hierarchies usually work

– eg why are atoms large compared with nuclei?

$$\alpha m_e << \Lambda_{QCD}$$

$$\alpha m_e << m_p$$

$$\delta m_e = \frac{e^4 m_e}{(4\pi)^4} \ln \frac{m_\mu}{m_e}$$

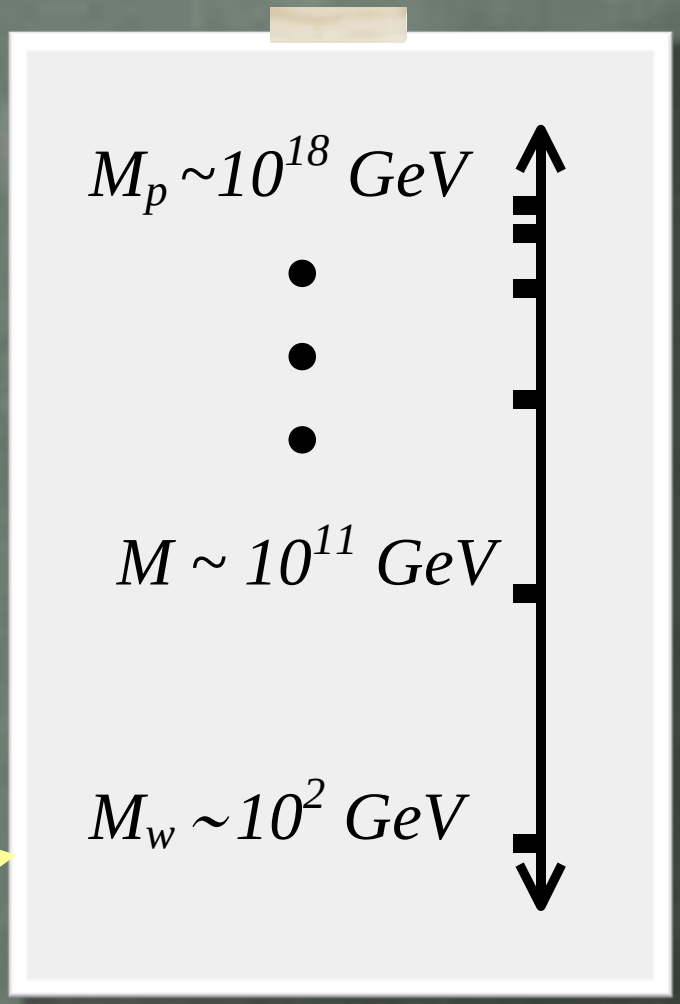


3.2 Naturalness and fine-tuning

§ For all known hierarchies of scale, our understanding has two parts:

- Why is the hierarchy small in the high-energy theory?
- Why does the hierarchy stay small as one integrates out successive particles to reach the low energies where the hierarchy is measured?

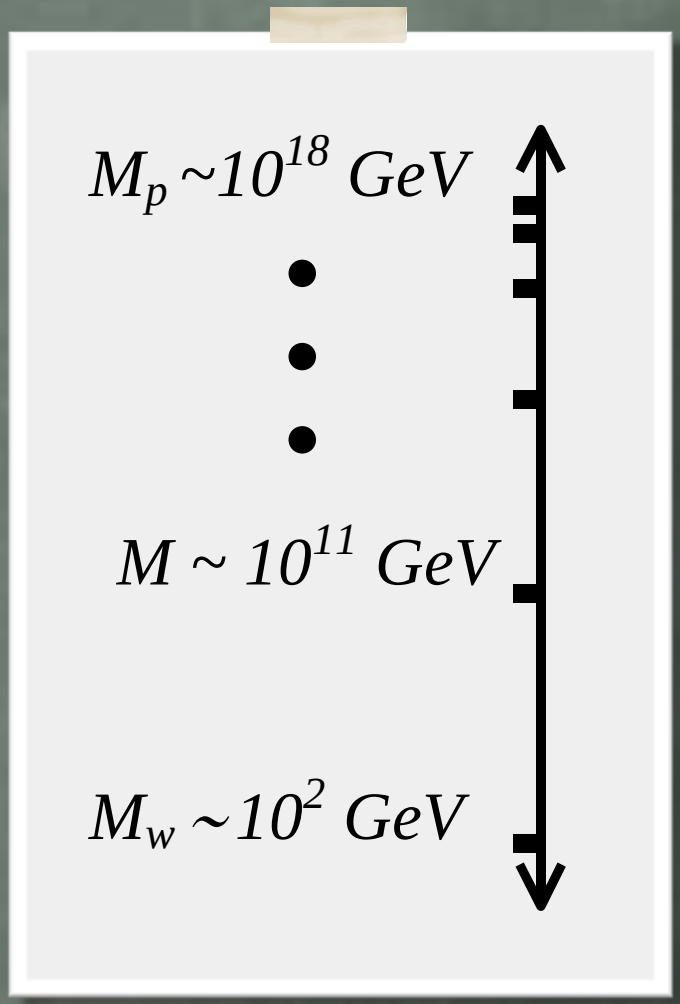
Second part must be solved at low energies!!



3.2 Naturalness and fine-tuning

§ For the Higgs mass hierarchy this thinking leads to the three main categories of proposals for what might be seen at the LHC

- Compositeness
- Supersymmetry
- Large extra dimensions



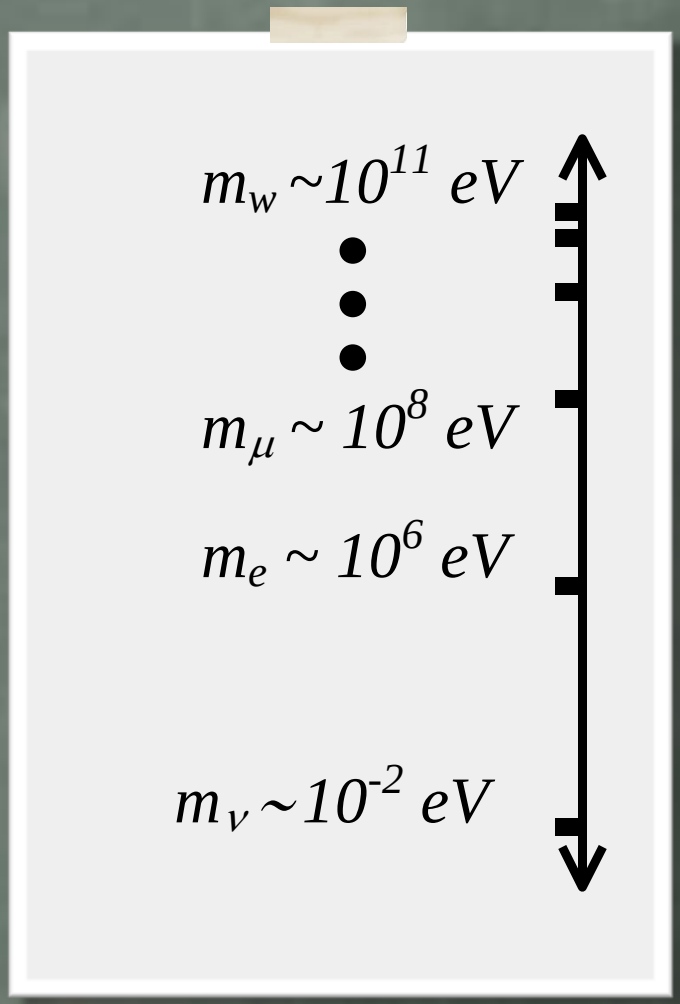
3.2 Naturalness and fine-tuning

§ What makes the cosmological constant problem hard is that the particles involved are very low energies, which we think we understand well

$$\mu^4 \approx \mu_1^4 + k_e m_e^4 + k_\nu m_\nu^4$$

$$\mu^4 \approx \mu_1^4 + k_e m_e^4 + k_\nu m_\nu^4$$

Must cancel to 32 decimal places!!



3.2 Naturalness summary

- Notice that the entire discussion has been phrased in terms of finite, renormalized masses, rather than cutoffs.
  - It is often said that naturalness problems are to do with the presence of quadratic or quartic divergences.
  - Cutoffs are used as proxies for the real problems (heavy masses), but in general we know that cutoffs do not appear in observables

$$\exp i\Gamma = \int^{\Lambda} D\varphi \ \exp[iS_{\Lambda}(\varphi)] + \cdots$$

$$\exp[iS_{\Lambda}(\varphi)] = \int_{\Lambda} D\varphi D\psi \exp[iS(\varphi,\psi)]$$

3.2 Naturalness summary

- Naturalness issues reflect a strong sensitivity of long-distance physics to short distance physics
  - It must be fixed my modifying high energy physics, starting at relatively low energies (needn't await a full quantum theory of gravity)
- Useful for guessing what kinds of new physics might be there
  - All known hierarchies are understood in a natural way: two main dangling hierarchies are the electroweak hierarchy (why the Higgs mass is small), and the cosmological constant problem (why the vacuum energy gravitates so little)