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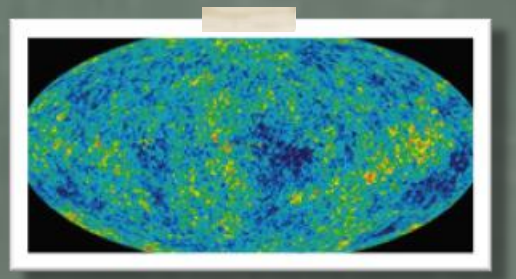
Effective Field Theory in Cosmology

Clues for cosmology from fundamental physics



Outline

- Motivation and Overview
- Effective field theories throughout physics
  - Decoupling and 'naturalness' issues
- Quantum effects in gravity: Fighting the 'split brain'
  - Quantifying theoretical error
  - What justifies the classical approximation?
- Relevance to cosmology
  - A brief cold shower
  - Inflation, Dark matter and Dark energy

1. Motivation and Overview

§ What controls the classical approximation in cosmology?

§ Embedding into microscopic physics as an important clue

1.1 What Controls the Classical Approx?

§ The importance of quantifying theoretical error

Quantum field theory is a precision science: e.g. QED

$$a_\mu = 159652188.4(4.3)\ 10^{-12} \text{ (exp)}$$

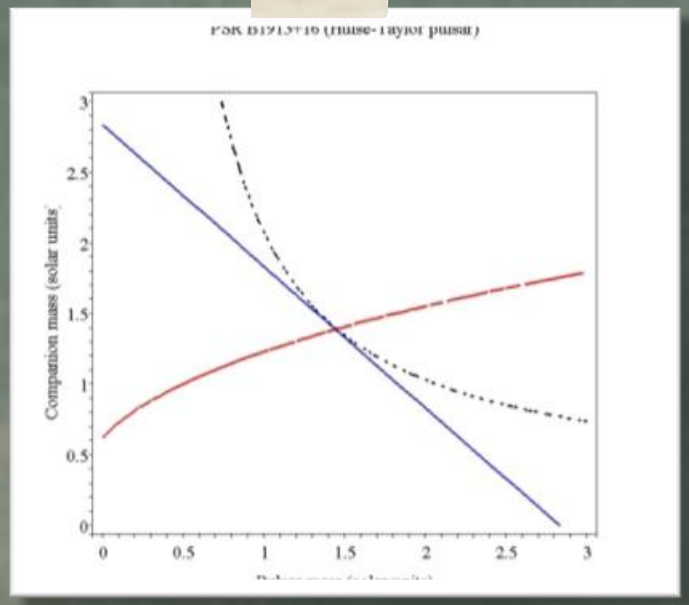
$$a_\mu = 159652140(27.1)\ 10^{-12} \text{ (th)}$$

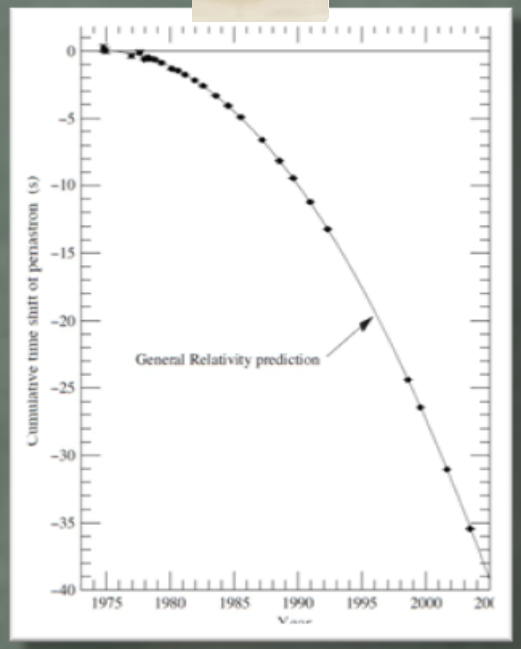
Renormalizability of QED is important part of calculability

1.1 What Controls the Classical Approx?

§ The importance of quantifying theoretical error

General Relativity is also a precision science: e.g. solar system tests, binary pulsar, etc..





1.1 What Controls the Classical Approx?

§ The importance of quantifying theoretical error

General Relativity is also a precision science: e.g. solar system tests, binary pulsar, etc..

$$dP/dt = -2.408(10)\ 10^{-12} \quad \text{(exp)}$$

$$dP/dt = -2.40243(5)\ 10^{-12} \quad \text{(th)}$$

Comparison meaningless if size of quantum effects is unknown

1.1 What Controls the Classical Approx?

§ The quantum-gravity split-brain mentality:

- Because gravity is not renormalizable (unlike QED), quantum mechanics and gravity cannot be treated within the same framework

- The size of quantum effects cannot be computed for gravitational situations, even for macroscopic systems

1.1 What Controls the Classical Approx?

§ The quantum-gravity split-brain mentality:

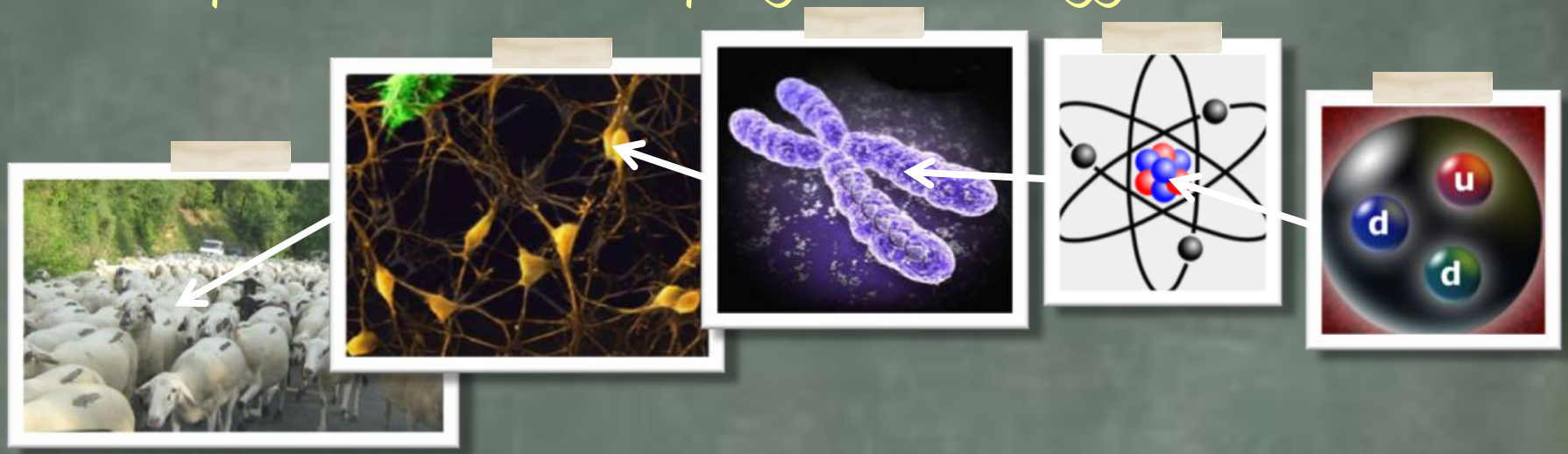
$$\hbar = 1$$
$$G_N = 0$$

$$\hbar = 0$$
$$G_N = 1$$

If interpreted as an EFT quantum corrections are calculable, even for gravity, provided they involve only long distances...

1.2 Microphysics as a Crucial Clue

§ The puzzle of 'non-decoupling' in cosmology



- Decoupling: usually most details of small-distance physics are not required to understand long-distance physics
- Why should particle physics (or string theory) have anything useful to say about cosmology?

1.2 Microphysics as a Crucial Clue

§ The ~~puzzle~~ opportunity of 'non-decoupling' in cosmology

– Decoupling doesn't preclude long-distance physics depending on a few of the details of short-distance physics

- For example, although atoms can be understood without knowing much about nuclear physics, you do need to know some things (like the nuclear mass, charge, possibly dipole moment, etc...)

1.2 Microphysics as a Crucial Clue

§ The ~~puzzle~~ opportunity of 'non-decoupling' in cosmology

- Decoupling doesn't preclude long-distance physics depending on *a few* of the details of short-distance physics

- Most cosmological models rely on long-distance features that do depend on the details of short-distance physics
  - the existence of scalars with small masses, $m \leq H$
  - the existence of small vacuum energies, $G_N \rho \leq H^2$
  - the existence of exotic states or dynamics at very low energies

1.2 Microphysics as a Crucial Clue

§ The opportunity of 'non-decoupling' in cosmology

- Cosmological observations in themselves are unlikely to be able to definitively distinguish amongst all the extant models

- The difficulty of embedding a model into a realistic UV completion is an important clue, which when combined with observations may make it possible to identify the model of interest

1.2 Microphysics as a Crucial Clue

§ Non-decoupling as a double-edged sword

– If short-distance physics does not decouple from cosmology, why doesn't this make it impossible to make cosmological predictions without first solving quantum gravity?

- e.g. is an inflationary interpretation of CMB fluctuations robust?

§ The dangers of 'Planck slop'

– Even if heavy physics decouples, why can't small Planck-suppressed interactions screw up cosmology?

2. Effective Field Theory throughout physics

§ Decoupling: integrating out heavy particles

– Examples from Weak, Strong and Electromagnetic interactions

§ General features of low-energy effective theories

2.1 Integrating Out Heavy Particles

§ Consider a system with a hierarchy of scales: light particles with mass m and heavy particles with mass M

– Compute an observable (like light-particle scattering or bound-state energy levels) involving energy E much smaller than M:

$$E^2\,\sigma = f\left(\alpha, \frac{E}{m}, \frac{E}{M}, \frac{\mu}{E}\right)$$

$$\approx\; f_0\left(\alpha, \frac{E}{m}, \frac{\mu}{E}\right) + \left(\frac{E}{M}\right)^2 f_1\left(\alpha, \frac{E}{m}, \frac{\mu}{E}\right) + \cdots$$

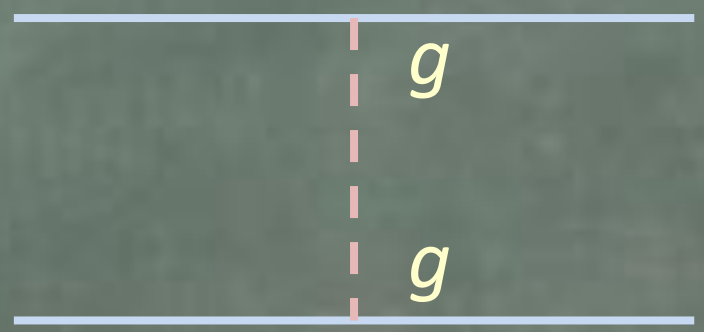
up to possible logs of E/M. (α here denotes dimensionless coupling, renormalized at renormalization scale μ)

2.1 Integrating Out Heavy Particles

§ Calculations are simpler the earlier one expands in 1/M

– With E much less than M only virtual heavy particles contribute

e.g.



if light-particle scattering takes place due to heavy-particle exchange.

– Most efficient to expand in 1/M directly in propagator, before evaluating observables

2.1 Integrating Out Heavy Particles

§ Expanding propagator in 1/M gives local result:
ultimately a consequence of the equivalence principle

$$G(x,y) = \int \frac{dk}{(2\pi)^4} \frac{e^{-ik(x-y)}}{k^2 + M^2} \approx \frac{1}{M^2}\delta(x-y) + \cdots$$

2.1 Integrating Out Heavy Particles

§ Expanding propagator in $1/M$ gives local result:

When used in the amplitude the result is as if computed with a direct 4-point interaction involving only light fields, with strength $G = g^2/M^2$

$$L_{eff} = G(\psi\gamma^\mu\psi)(\psi\gamma_\mu\psi)$$

2.1 Integrating Out Heavy Particles

§ This local 'effective' interaction expresses (at order $1/M^2$) the effects of virtual heavy-particle exchange in any process

– Effective hamiltonian (depending only on the light fields) is defined so that it reproduces the time evolution of the full theory at low energies, order by order in powers of E/M, for arbitrary low-energy states

$$U(t,t') = \exp[-iH(l,h)(t-t')] = \exp\left[-iH_{\text{eff}}(l)(t-t')\right]$$

full theory

light fields only

2.1 Integrating Out Heavy Particles

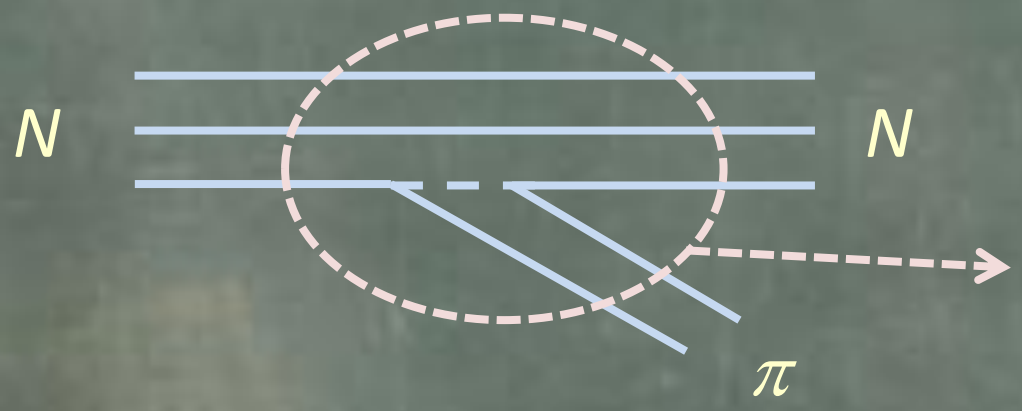
§ Since H and H_{eff} are equivalent as quantum operators, effective interactions can be generated by and used in loops

– e.g. how muons affect low-energy electron scattering in QED

2.1 Integrating Out Heavy Particles

§ Low-energy degrees of freedom can be qualitatively different from high-energy ones

– e.g. pions, or atoms, or planets can be 'elementary' fields at low energies while their constituents are 'elementary' at high energies



$$L_{eff} = \frac{1}{F}\partial_\mu \pi (N\gamma^\mu N)$$

if $E < 4\pi F \sim 1$ GeV

2.2 General lessons

- The virtual effects of heavy particles at low energy can be captured by a local effective theory involving only light fields
  - Unitary: because at energies below their mass, energy conservation ensures heavy particles are not produced if not initially present
  - Local: provided the agreement between full theory and low-energy theory is order-by-order in $1/M$
  - Nonrenormalizable: effective interactions have couplings proportional to negative powers of M

2.2 General lessons

§ In principle 'integrating out' heavy particle generates all possible effective interactions consistent with low energy particle content and symmetries

$$L_{eff} \propto \sum_{dn} \frac{c_{dn}}{M^d} O\left(\partial^d \left(\phi/v\right)^n\right)$$

- Infinite number of possible effective interactions generated
- Scale M dictated by mass of particle integrated out
- Couplings c_{dn} calculable in terms of microscopic parameters

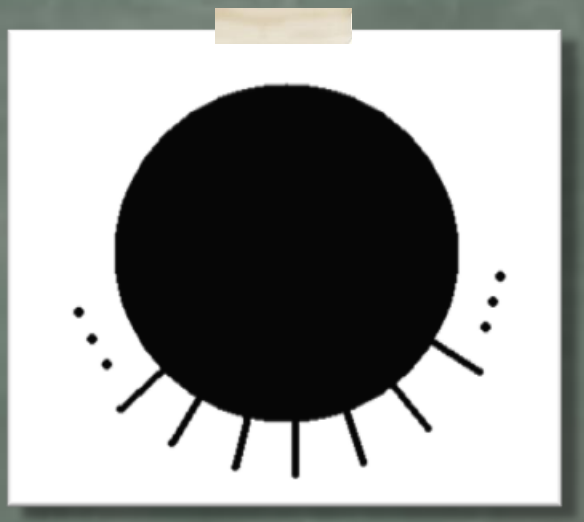
2.2 General lessons

- Better accuracy requires higher orders in 1/M which involves more kinds of effective interactions
  - Although effective theory includes in prinicple an infinite number of effective interactions

$$L_{eff} = f^2 M^2 \sum_{kn} \frac{c_{kn}}{M^k} \, O\left(\partial^k, \left(\phi/_v\right)^n\right)$$

the theory is nonetheless predictive, since only a fixed number contribute to any given order in 1/M

2.2 General lessons



§ Example: E-point scattering:

- L = number of loops
- V_{ik} = number of vertices with k derivatives and i fields
- Q = energy of external states

$$A_E(Q) \propto \left(\frac{f^4 Q^2}{M^2 v^E}\right)\left(\frac{MQ}{4\pi f^2}\right)^{2L} \prod_{ik} c_{ik} \left(\frac{Q}{M}\right)^{(k-2)V_{ik}}$$

2.2 General lessons

§ Utility of these effective interactions requires the low-energy expansion: $Q \ll M$ and $Q \ll f$.

- First step: specify the accuracy to which an answer is desired
- Second step: identify the order in Q/M, Q/f required to achieve this accuracy
- Third step: use power-counting to identify which effective interactions are required
- Fourth step: either compute the coefficients, c_{kn}, of these interactions from the fundamental theory; or treat these as phenomenological parameters to be found from experiment

2.2 General lessons

§ Can effective theories be used for time-dependent problems?

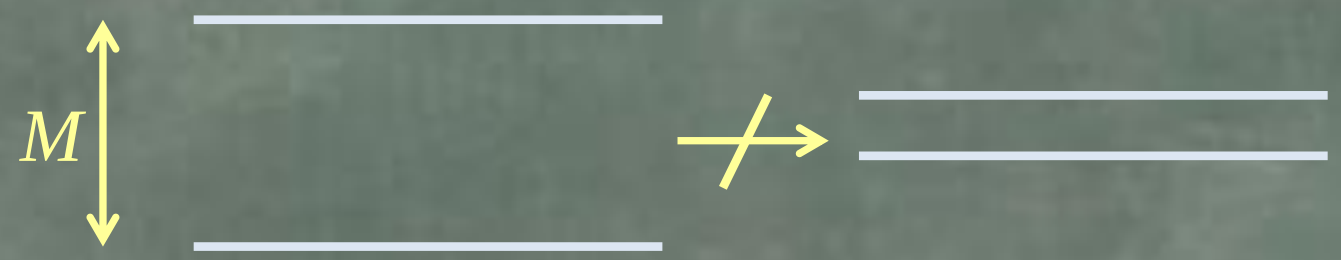
- This question has several levels:

- Q1: Are there additional conditions that must apply in order to examine time-dependent solutions in an effective theory defined about a static vacuum background?
- Q2: Since higher-derivative interactions are present, why aren't there new runaway solutions?
- Q3: Can an effective theory be set up to describe the fluctuations about a time-dependent background?

2.2 General lessons

§ For time-dependent solutions within an effective theory:

– Must check whether the low-energy criterion, $E \ll M(t)$ remains true as a function of time: must avoid level crossing:



– Additional conditions: Time-dependent solutions within the effective theory only capture time-dependence of the full theory if the evolution is adiabatic

$$\dot{\varphi} \ll M(t), \ldots$$

2.2 General lessons

§ What about runaway solutions?

– If the equations of motion were really higher order then normally acquire new, often runaway, solutions:

– e.g. if $L = \dot{q}^2 + \ddot{q}^2/M^2$ then $\ddot{q} + \ddddot{q}/M^2 = 0$

and so $q(t) = a + bt + c\,e^{Mt} + d\,e^{-Mt}$

– Key point: must perturb in the effective interactions, since they are only required to reproduce the full physics to fixed order in $1/M$

2.2 General lessons

§ Can effective theories be used for expansions about time-dependent backgrounds?

- In principle there is a problem: we distinguish light from heavy states from one another using energy as the criterion, but lose energy conservation when expanding about a time-dependent background.
- Can work if background is adiabatically evolving since in this case can define an adiabatic notion of energy at a given time, $E(t)$, as done by Cheung et al (arXiv:0709.0293)