Coincidence problem within decaying self-interacting Bose gas

REFERENCES

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Cancun
January, 2012

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Coincidence problem

$$\Omega_{\Lambda 0} = \rho_{\Lambda 0}/\rho_{c0} \simeq .73$$

$$\Omega_{dm0} = \rho_{dm0}/\rho_{c0} \simeq .22$$

$$\Omega_{b0} \simeq .044$$

Fine-tuning problem

Today: critical density

$$\rho_{\Lambda 0} \simeq 4\times 10^{-6}\ \mathrm{GeV/cm^3}$$

Planck time: Planck density

$$\rho_{\Lambda P} = \frac{1}{(2\pi)^3}\int^{M_P} d^3k\sqrt{k^2+m^2} \simeq 3\times 10^{114}\ \frac{\mathrm{GeV}}{\mathrm{cm}^3}$$

Thermodynamics of interacting Bose gas

Single-particle energy

$$\varepsilon = \frac{p^2}{2m} + v_0 n, (v_0 < 0)$$

(valid at low T and T~T_c)

Occupation number

$$\bar{n}_k = \frac{1}{e^{(\epsilon_k - \mu)/T} - 1}$$

single gas particle non-interacting Bose plus an ***interaction term***.

For $T < T_0 = \frac{3.31\hbar}{g^{2/3} m} n^{2/3}$ the ***chemical potential*** is

$$\mu = v_0 n$$

Total energy

The ***total energy*** can be obtained by integrating the single-particle energy into the ***Bose distribution***. When $T<T_{0,}$

$$E = \frac{gVm^{3/2}}{\sqrt{2}\pi^2}\int d\epsilon\epsilon^{3/2}\frac{1}{e^{(\epsilon_k-\mu)/T}-1} + \frac{v_0}{V}\int NdN = E^c + \frac{v_0}{2}\frac{N^2}{V}$$

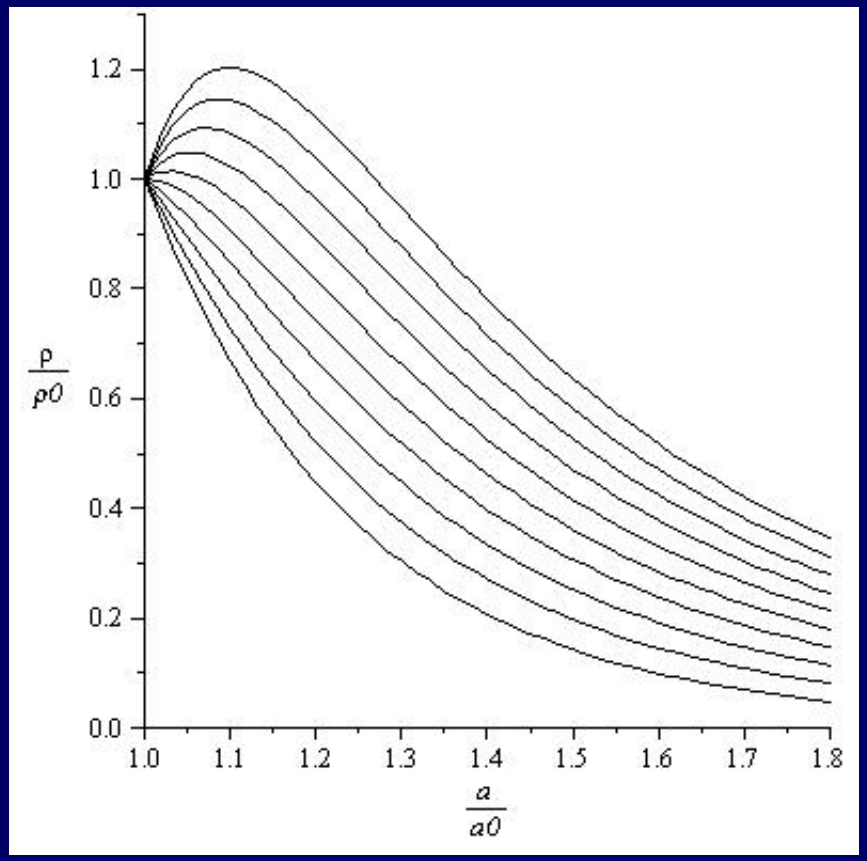
Pressure

The ***pressure*** can be obtained from the above expression,

$$p = -\left(\partial E/\partial V\right)_{N,S} \Longrightarrow p = \frac{2}{3}\rho + \frac{1}{6}v_0\frac{N^2}{V^2}$$

Interacting Bose gas as inflation?

Interactive Bose gas and radiation, no interaction between components



$$\rho = \rho_{r0}\left(\frac{a_0}{a}\right)^4 + \rho_{c0}\left(\frac{a_0}{a}\right)^5 + \frac{1}{2}v_0 n_0^2\left(\frac{a_0}{a}\right)^6$$

$$p = \frac{1}{3}\rho_r + \frac{2}{3}\rho_c + \frac{1}{6}v_0 n_0^2\left(\frac{a_0}{a}\right)^6$$

Energy density growing with a

$\Downarrow$

Super-exponential acceleration

Acceleration in terms of the total density and pressure

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{6}(\rho + 3p)$$

The universe expansion is accelerated when $\rho + 3p < 0$

The universe is accelerated super-exponentially when $\rho + p < 0$

(e.g. phantom dark energy)

Horizon problem, within inflation

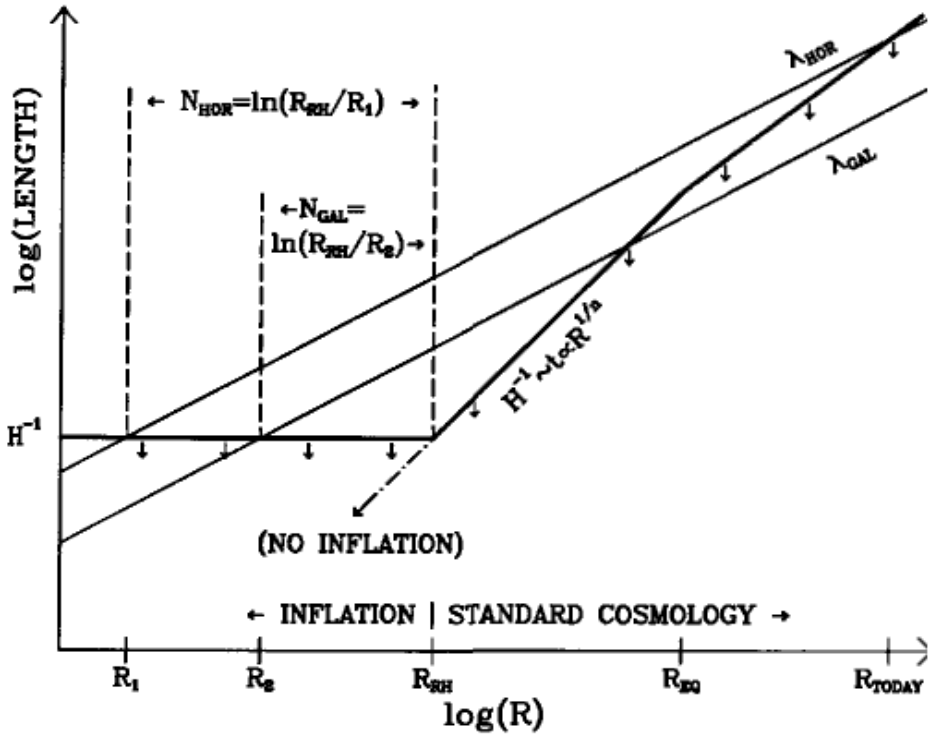
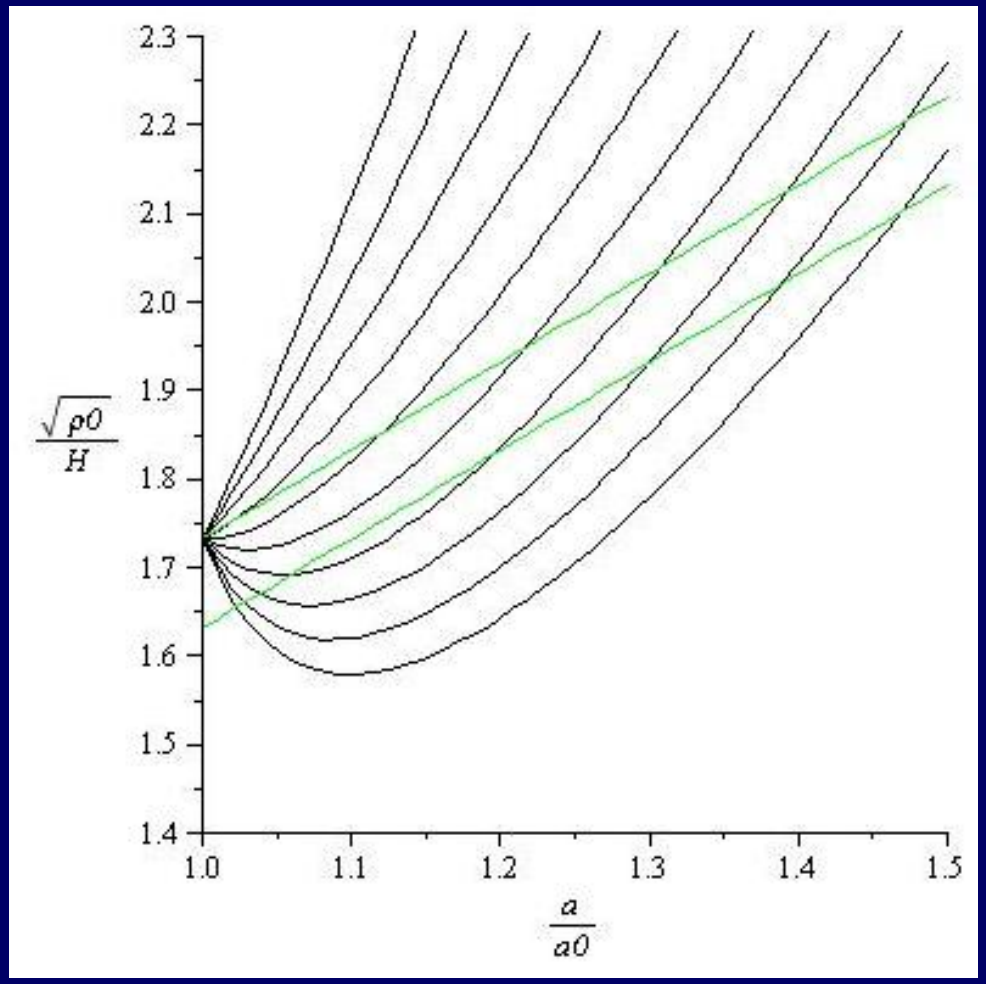


Fig. 8.4: The evolution of the physical size of the comoving scale, λ, and of the Hubble radius, H^{-1}, in the standard and the inflationary cosmologies. In the standard cosmology (i.e., no inflation) a given scale crosses the horizon but once; while in the inflationary cosmology all scales begin sub-horizon sized, cross outside the horizon ("good bye") during inflation, and re-enter again ("hello again") during the post-inflationary epoch. Note that the largest scales cross outside the horizon first and re-enter last. The growth in the scale factor $[N = \ln(R_{RH}/R)]$ between the time a scale crosses outside the horizon during inflation and the end of inflation is also indicated. For a galaxy, $N_{GAL} = \ln(R_{RH}/R_1) \sim 45$, and for the present horizon scale, $N_{HOR} = \ln(R_{RH}/R_2) \sim 53$. Causal microphysics operates only on scales less than H^{-1} (indicated by arrows). During inflation $H^{-1} \equiv H_I^{-1} = \text{const}$, and in the post-inflation era, $H^{-1} \sim t \propto R^{1/n}$ ($n = 1/2$—radiation dominated, $n = 2/3$—matter dominated).



When the parameters used predict a super-exponential inflation this range is larger.

Late Universe Acceleration

Problem in describing dark energy:

$$\rho = \rho_{r0}\left(\frac{a_0}{a}\right)^4 + \rho_{c0}\left(\frac{a_0}{a}\right)^5 + \frac{1}{2}v_0 n_0^2\left(\frac{a_0}{a}\right)^6$$

$$p = \frac{1}{3}\rho_r + \frac{2}{3}\rho_c + \frac{1}{6}v_0 n_0^2\left(\frac{a_0}{a}\right)^6$$

Interaction term decays faster than mass and kinetic terms

Generic decay term: $N=cV^x$; $N=N(t)$

*Valid in fluid exchange processes: black hole decay, cosmic ray generation, dust interaction.
*Markoffian (not dependent on previous history) .
*includes H factor in FRW equations.

Two-component model

1) Dark-energy conservation

$$d(\rho_\Lambda V) = -p_\Lambda dV + \mu_\Lambda dN_\Lambda$$

2) Total energy conservation

$$\sum_i d(\rho_i V) = \sum_i (-p_i dV + \mu_i dN_i + T_i dS_i)$$

3) 00 component of Einstein equations

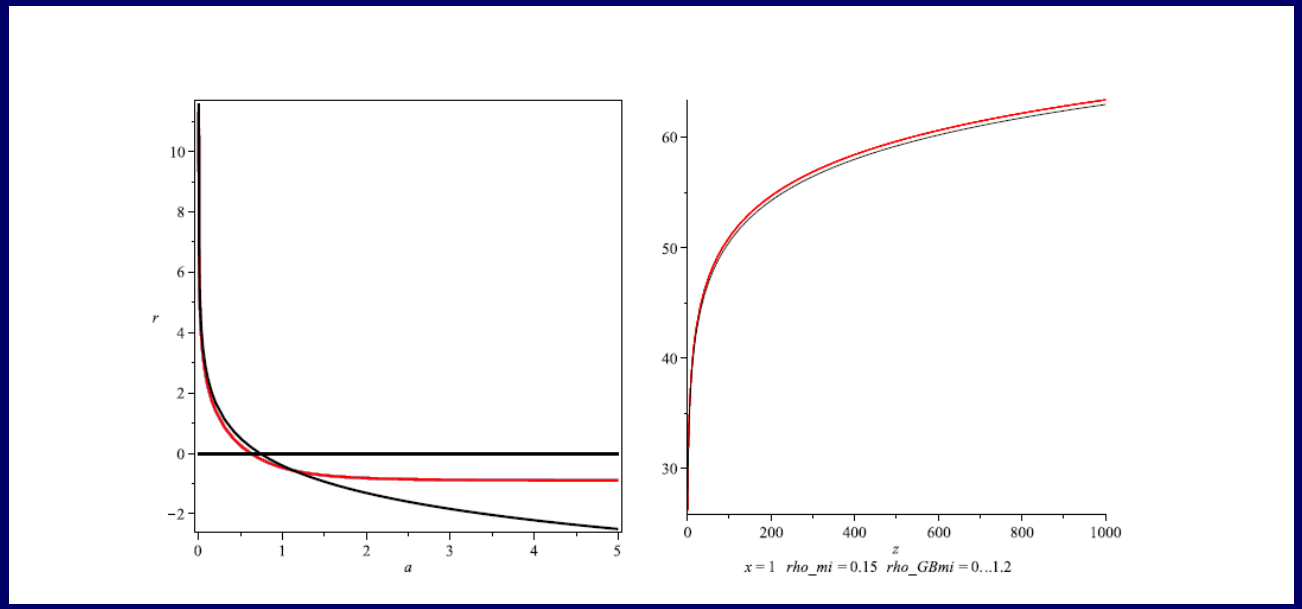
$$H^2 = \frac{8\pi}{3}\rho_c = \frac{8\pi}{3}(\rho_\Lambda + \rho_i)$$

Markoffian process: ***dN=N(dV/V)***

Parameter study example

$$Q = -3Hx\left(\rho_{GBm0}\, a^{3x-3} + \frac{5}{3}\rho_{c0}a^{5x-5} + 2\rho_{i0}a^{6x-6}\right)$$

Boson model vs ΛCDM



Density ratio vs a

Magnitude vs a

Main conclusions

- Attractive interaction (within a Bose gas) leads to negative pressure, and positive acceleration.
- Condensate interactive Bose model can solve the horizon problem.
- Possibility of obtaining Gaussian, scale-invariant fluctuations
- With Markoff-like dependence of N(V), scaling behavior, solving the coincidence problem.

Cosmology data

Cosmology "standard model"

parameter	symbol	WMAP-5 alone	WMAP-5 + BAO + SNe (reference)	comment
CMB temperature	T_{CMB}	2.728 ± 0.004 K	–	from (Fixsen *et al.*, 1996)
total energy density	Ω_{tot}	$1.099 \begin{smallmatrix}+0.100\\-0.085\end{smallmatrix}$	1.0052 ± 0.0064	
matter density	Ω_{m0}	0.258 ± 0.03	0.279 ± 0.015	assuming spatial flatness
baryon density	Ω_{b0}	0.0441 ± 0.0030	0.0462 ± 0.0015	here and below
cosmological constant	$\Omega_{\Lambda 0}$	0.742 ± 0.03	0.721 ± 0.015	
Hubble constant	h	$0.719 \begin{smallmatrix}+0.026\\-0.027\end{smallmatrix}$	0.701 ± 0.013	
power-spectrum normalisation	σ_8	0.796 ± 0.036	0.817 ± 0.026	
age of the Universe in Gyr	t_0	13.69 ± 0.13	13.73 ± 0.12	
decoupling redshift	z_{dec}	1087.9 ± 1.2	1088.2 ± 1.1	
reionisation optical depth	τ	0.087 ± 0.017	0.084 ± 0.016	
spectral index	n_{s}	$0.963 \begin{smallmatrix}+0.014\\-0.015\end{smallmatrix}$	$0.960 \begin{smallmatrix}+0.014\\-0.013\end{smallmatrix}$	

Standard cosmology equations

- General relativity $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$ $\quad -\nabla^2 \phi(\mathbf{r}) = G\rho(\mathbf{r})$
- FRW metric (isotropy) $ds^2 = dt^2 - R^2(t)(dx^2 + dy^2 + dz^2)$
- Flat universe (MWB) $\sum \Omega_i = 1$
- Perfect fluid at rest $T^i_{\mu\nu} = (\rho_i, p_i, p_i, p_i)$

-00 component of Einstein equations

$$H^2 = \frac{8\pi}{3}\rho_c = \frac{8\pi}{3}(\rho_\Lambda + \rho_r + \rho_m)$$

Hubble constant $H = \dot{R}/R$

-contraction of Einstein equations: energy conservation

$V \sim R^3$

within and expanding volume

$$\sum_i d(\rho_i V) = -\sum_i p_i dV$$

Parametric dependence on V,*t,R,T*, etc.

Stability of the model

$$N_{\rm cr} \lesssim 4\pi \frac{a_{\rm ho}}{m|v_0|}$$

we assign $a_{\rm ho}$ to the Hubble radius $1/H$

$$n_i^2|v_0| \lesssim \frac{\rho_{ri} + \rho_{ci}}{\frac{1}{2} + \frac{3m_P^2 m n_c}{2(4\pi)^2 n_i^2}}$$

Scale-factor dependence

Asymptotic limit

$$\lim_{\rho_\Lambda \to 0} \frac{\rho_\Lambda}{\rho_c} = \frac{d_i + 3\chi}{d_i - 3(w_\Lambda + 1)}$$

$$d_i = 3(w_i + 1)$$

$$-d_i/3 < \chi < 0$$

Dark energy $\rho_\Lambda \sim R^{3\chi}$

other dominant component $\rho_i \sim R^{3\chi}$

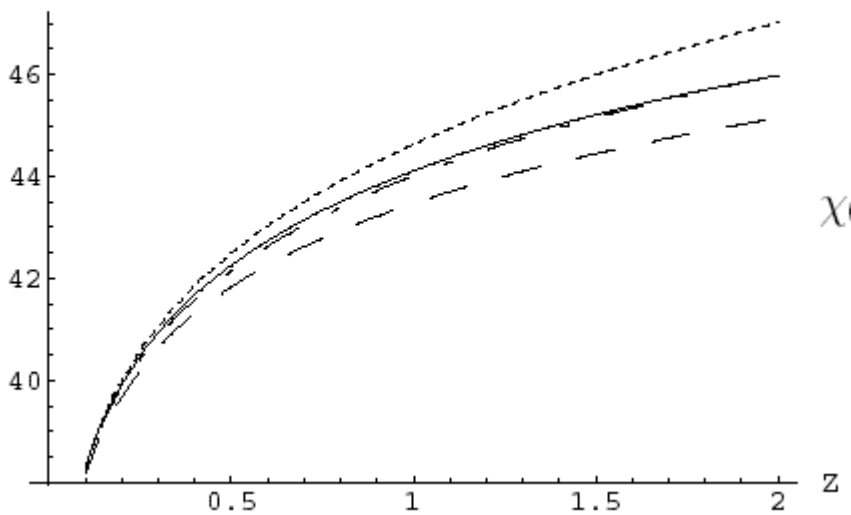
baryonic matter $\rho_b \sim 1/R^3$

Supernova data

distance-redshift relation

$$d_L = H_0^{-1}(1+z)\int_0^z dz'[\Omega_{b0}(1+z')^3 + (1-\Omega_{b0})(1+z')^{-3\chi_0}]^{-1/2}$$

$$\mu = 5\mathrm{Log}_{10}(d_L) + 25$$



$w_\Lambda = -1$

$\Omega_{m0} = 0,\ \Omega_{\Lambda 0} = 1$ (dotted)

$\Omega_{m0} = .27,\ \Omega_{\Lambda 0} = .73$ (line)

$\Omega_{m0} = 1,\ \Omega_{\Lambda 0} = 0$ (dashed)

$\chi_0 = -.48$

$\Omega_{b0} = .044$ (dot-dashed)

Entropy

The entropy of the gas can be obtained by the ***extensivity*** relation

$$E = N\mu + TS - pV$$

As $1/T = (\partial S / \partial E)_{N,V}$, the equation leads to a partial differential equation for the entropy whose solution is

$$S = CV^{2/5}\left(E - \frac{1}{2}v_0 \frac{N^2}{V}\right)^{3/5}$$

C being a numerical constant.

From the energy density evolution equation for the condensate

$$\rho_c = \rho_{c0}\left(\frac{a_0}{a}\right)^5 + \frac{1}{2}v_0 n_0^2\left(\frac{a_0}{a}\right)^6$$

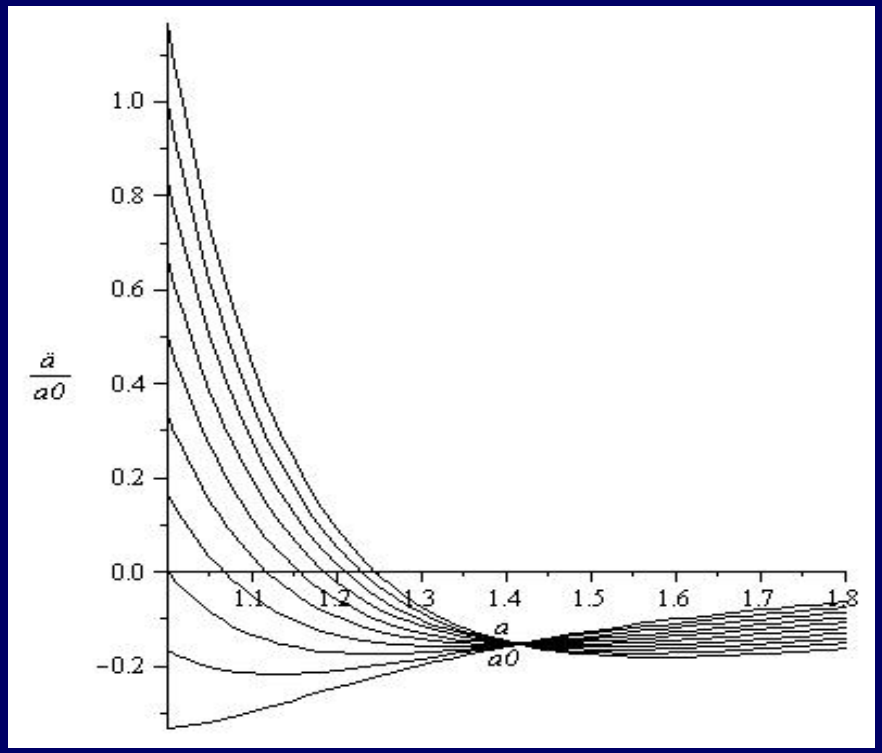
where we assume constant particle number (i.e. an adiabatic expansion).

The pressure of the interacting Bose gas is

$$p_c = \frac{2}{3}\rho_c + \frac{1}{6}v_0 n_0^2\left(\frac{a_0}{a}\right)^6$$

Acceleration with radiation term

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{6}\left(2\rho_{r0}\left(\frac{a_0}{a}\right)^4 + 3\rho_{c0}\left(\frac{a_0}{a}\right)^5 + 2v_0 n_0^2\left(\frac{a_0}{a}\right)^6\right)$$



The expansion becomes decelerated:

the interaction term decreases with the scale factor faster than the condensate and radiation terms

eventually the expansion is dominated by the radiation.

Fluctuation relations

Non-condensate component

Energy fluctuations in a volume

$$\langle (E-\bar{E})^2 \rangle = T^2 \left(\frac{\partial E}{\partial T} \right)_{N,V} = T^2 N c_v,$$

Contribution at the horizon at scale k

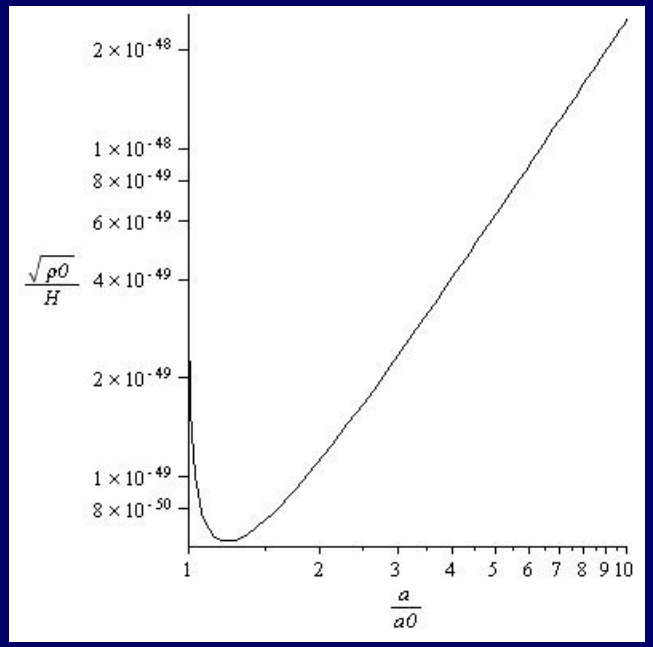
$$\delta\Omega = \frac{1}{2m^2} \rho_m V \sum_{\mathbf{k_{phys}}} k_{phys}^2 |\delta\Phi|^2,$$

Wave-function fluctuation average

$$\langle |\delta\Phi|^2 \rangle = \frac{T m^2}{V \rho_m k_{phys}^2}$$

Phase-space factor

$$V d^3 k_{phys} = d^3 k$$



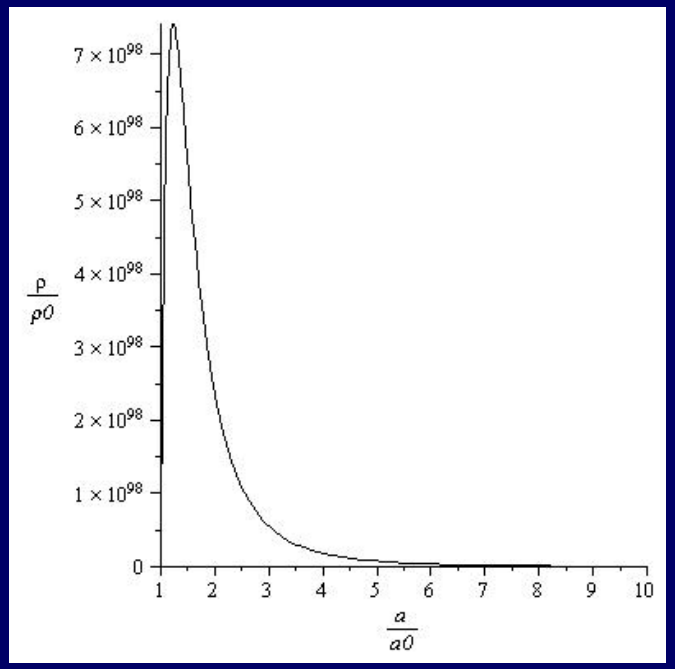
The range of wavelengths of density perturbations needed can be obtained if the Bose gas has an interaction parameter sufficiently high.

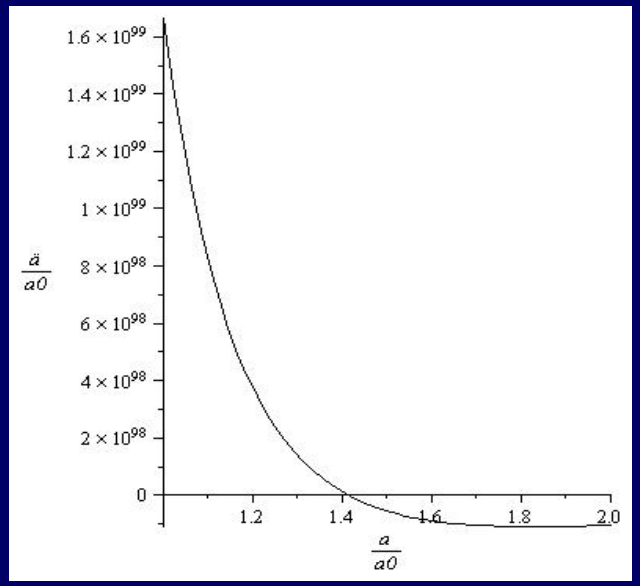
But then, which is the initial amplitude of the perturbations required?

Allowing the interaction parameter to be much larger than the initial density of condensate, predicts ρ_c <<0.

$$n_0^2 v_0 = 10^{100} \rho_{0c}$$

$$\rho_{0r} = 5 \times 10^{99} \rho_{0c}$$





The interacting Bose gas gives a natural way of obtaining an accelerated expansion.

Even with these parameters the acceleration ends soon.

Present acceleration needs decaying particle model?
Possibility of inflation-acceleration unified model