A robust approach to f(R) gravity

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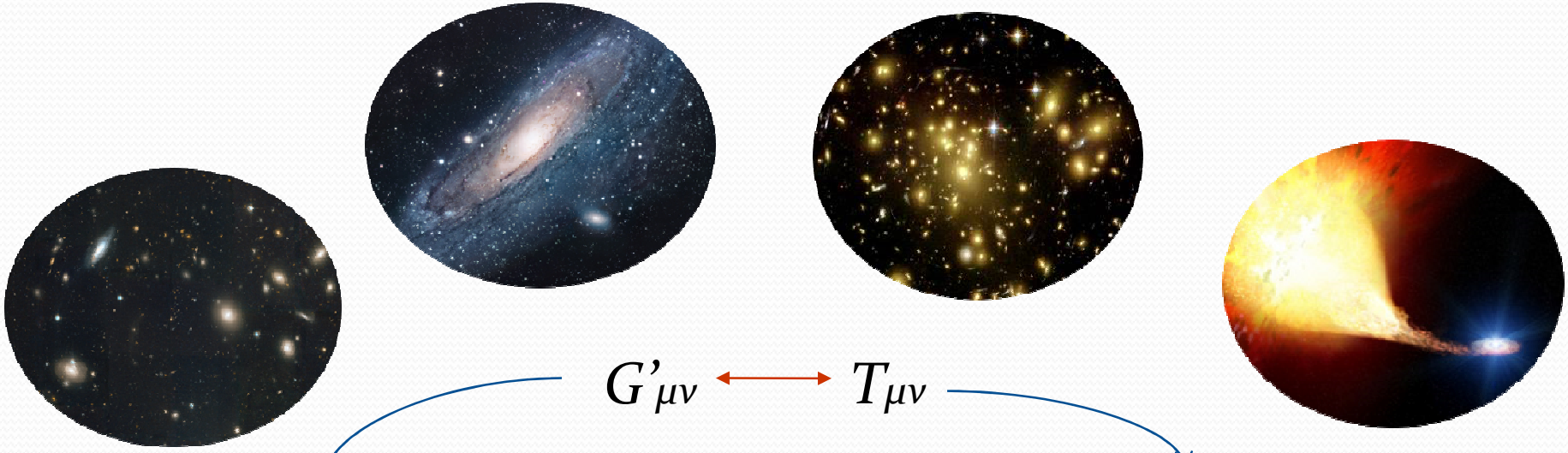
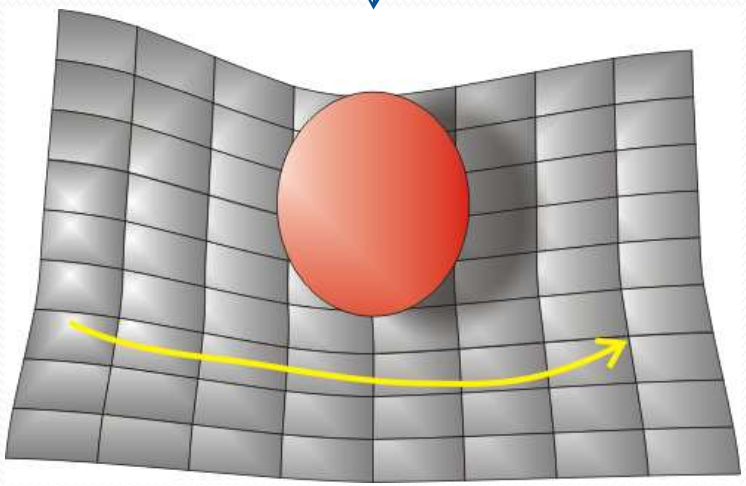
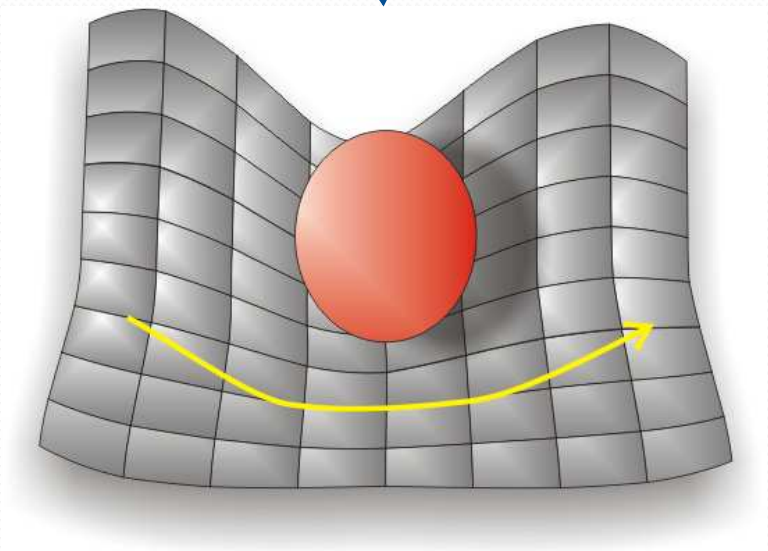
Geometry and energy content



$$G'_{\mu\nu} \longleftrightarrow T_{\mu\nu}$$



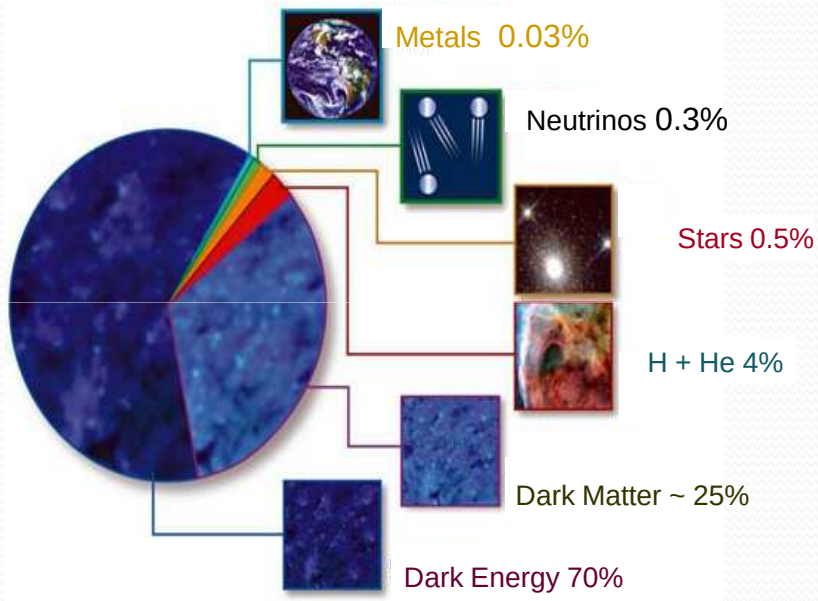


Standard General Relativity

- Hilbert-Einstein action

$$S_{\mathrm{EH}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g}\, R$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} + g_{\mu\nu} \Lambda$$

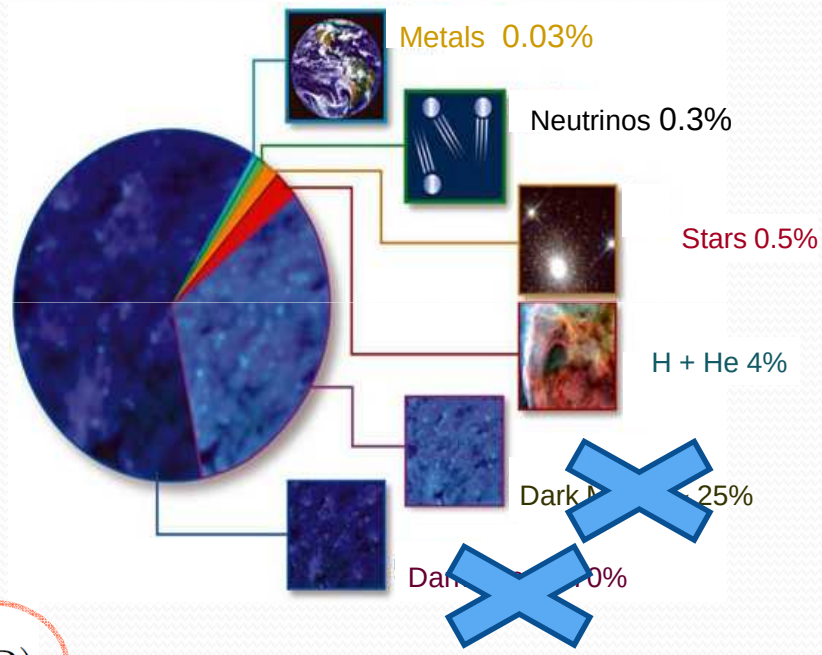


F(R) gravity theories

- Einstein-Hilbert action

$$S_{\rm EH} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,R$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu} + g_{\mu\nu}\Lambda$$



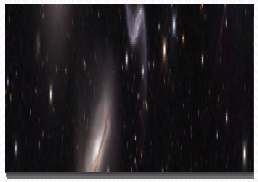
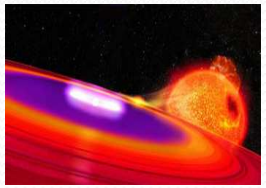
- Action for f(R) gravity

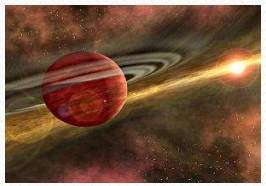
$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,f(R)$$

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - [\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box]\,f'(R) = \kappa T_{\mu\nu}$$

Test that must to pass f(R) candidates

- Solar system observations.
- Existence of stars and compact objects
- SN Ia observations
- Gravitational waves (Binary pulsar's observations)
- ETC.



Standar treatment:

Equivalence with Scalar Tensor Theories (STT)

- At the moment, most of the works in f(R), had been made by using this equivalence.
- Under this approach the potential asociated is ill defined

- Starobinsky

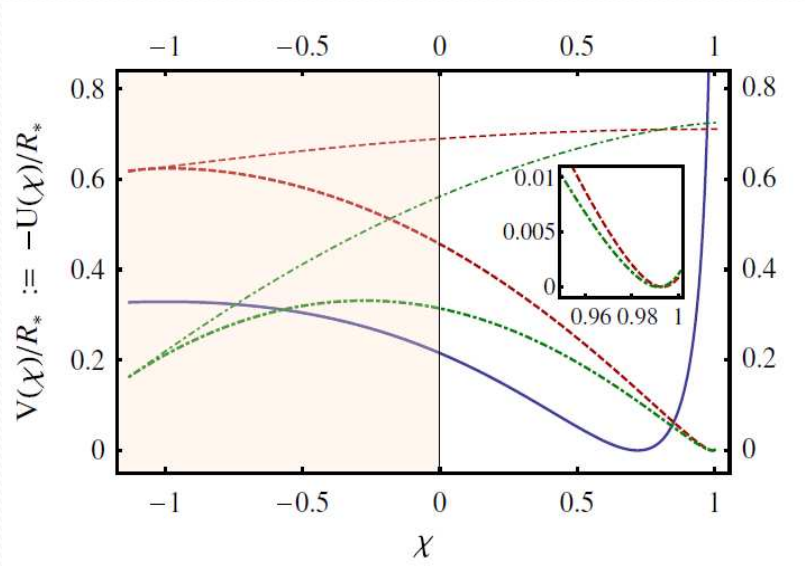
$$f(R) = R + \lambda R_0 \left[\left(1 + \frac{R^2}{R_0^{\,2}} \right)^{-n} - 1 \right]$$

- Miranda et al.

$$f(R) = R - \alpha R_* \ln\left(1 + \frac{R}{R_*} \right)$$

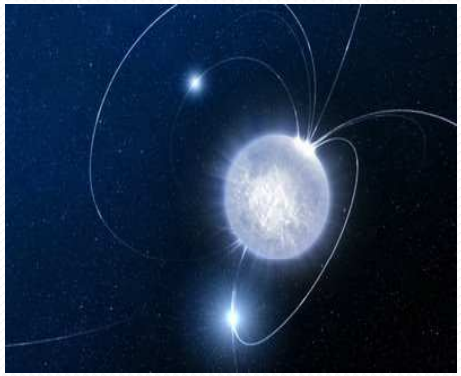
- Hu & Sawicki

$$f(R) = -m^2 \frac{c_1 \left(R/m^2 \right)^n}{c_2 \left(R/m^2 \right)^n - 1}$$



The controversy:
Existence of neutron stars in f(R) gravity

- The conclutions obteined with the standard approach are rather questionble.



- Kobayashi and Maeda in 2008, analized neutron stars in Starobinsky's model.
  - They found that this kind of objects can not exist for Starobinsky's model (incompressible fluid).
  - Ricci scalar diverge inside the star.
- Babichev and Langlois in 2010, work in a similar analysis for compact objects in Starobinsky's model but using state equations more realistics, they found:
  - Ricci scalar don't diverge inside the star so this kind of object could exist in f(R) gravity

- Both analysis use the equivalence with scalar tensor theories, use the same f(R) model and they found results in opposite directions.

!

- This problem is one motivation to study f(R) theories taking another point of view

A robust approach to f(R) gravity

- A simple idea:
  - We consider f(R) gravity without the mapping to their scalar-tensor counterpart, but **using the Ricci scalar as an "extra" degree of freedom**.
  - It is straightforward to write the f(R) field equation in the following way

$$f_R G_{ab} - f_{RR}\nabla_a\nabla_b R - f_{RRR}(\nabla_a R)(\nabla_b R)$$
$$+g_{ab}\left[\frac{1}{2}\left(Rf_R - f\right) + f_{RR}\Box R + f_{RRR}(\nabla R)^2\right] = \kappa T_{ab}$$

  - Taking the trace of this equation yields:

$$\Box R = \frac{1}{3f_{RR}}\left[kT - 3f_{RRR}(\nabla R)^2 + 2f - Rf_R\right]$$

  - And using this in the field equation we find

$$G_{\mu\nu} = \frac{1}{f_R}\left[f_{RR}\nabla_\mu\nabla_\nu R + f_{RRR}(\nabla_\mu R)(\nabla_\nu R) - \frac{g_{\mu\nu}}{6}\left(Rf_R + f + 2kT\right) + kT_{\mu\nu}\right]$$

  - These last two equations are the basic equations for f(R) gravity that we propose to use in every application.

Static and spherically symmetric spacetimes

- In order to test our approach, we consider a static and spherically symetric spacetime:

$$ds^2 = -n(r)dt^2 + m(r)dr^2 + r^2(d\theta^2 + sen^2\theta d\varphi^2)$$

- So, using equations proposed we have:

$$\begin{aligned} R'' &= \frac{1}{3f_{RR}}\left[m(kT + 2f - Rf_R) - 3f_{RRR}R'^2\right] + \\ &+ \left(\frac{m'}{2m} - \frac{n'}{2n} - \frac{2}{r}\right)R' \end{aligned}$$

- The componets rr, tt and θθ of the field equations give us the next differential equations:

$$\begin{aligned} m' = \frac{m}{r(2f_R + rR'f_{RR})}\Big\{ & 2f_R(1-m) - 2mr^2kT_t^t + \frac{mr^2}{3}(Rf_R + f + 2kT) + \\ & + \frac{rR'f_{RR}}{f_R}\Big[\frac{mr^2}{3}(2Rf_R - f + kT) - kmr^2(T_t^t + T_r^r) + \\ & + 2(1-m)f_R + 2rR'f_{RR}\Big]\Big\} \end{aligned}$$

$$n' = \frac{n}{r(2f_R + rR'f_{RR})}\left[r^2m(f - Rf_R + 2kT_r^r) + 2(m-1)f_R - 4rR'f_{RR}\right]$$

$$\begin{aligned} n'' = & \frac{2nm}{f_R}\left[kT_\theta^\theta - \frac{1}{6}(Rf_R + f + 2kT) + \frac{R'}{rm}f_{RR}\right] + \\ & \frac{n}{2r}\left[2\left(\frac{m'}{m} - \frac{n'}{n}\right) + r\frac{n'}{n}\left(\frac{m'}{m} + \frac{n'}{n}\right)\right] \end{aligned}$$

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Solving the equations

- We can note that the diferential equations for n, m and R have the form:

$$\frac{dy}{dx} = F(y_i, r) \qquad y_i = m, n, n', R, R'$$

- In order to solve this equations a Runge-Kutta algorithm is implemented in a FORTRAN code.
- Another equation wich is necesary is Tolman-Oppenheimer-Volkoff equation:

$$p' = \frac{-(\rho + p)n'}{2n}$$

- Some boundary conditions must be supplied, in this case we have
  - Regularity conditions, if r = 0 then m' = n' = 0
  - Asymptotic conditions $R \rightarrow R_1$
- In order to obtein realistic neutron stars we faces a technical difficulty:

$$\rho_{nuc} \sim 10^{14} g/cm^3 \longrightarrow \Lambda/G \sim 10^{-29} g/cm^3$$

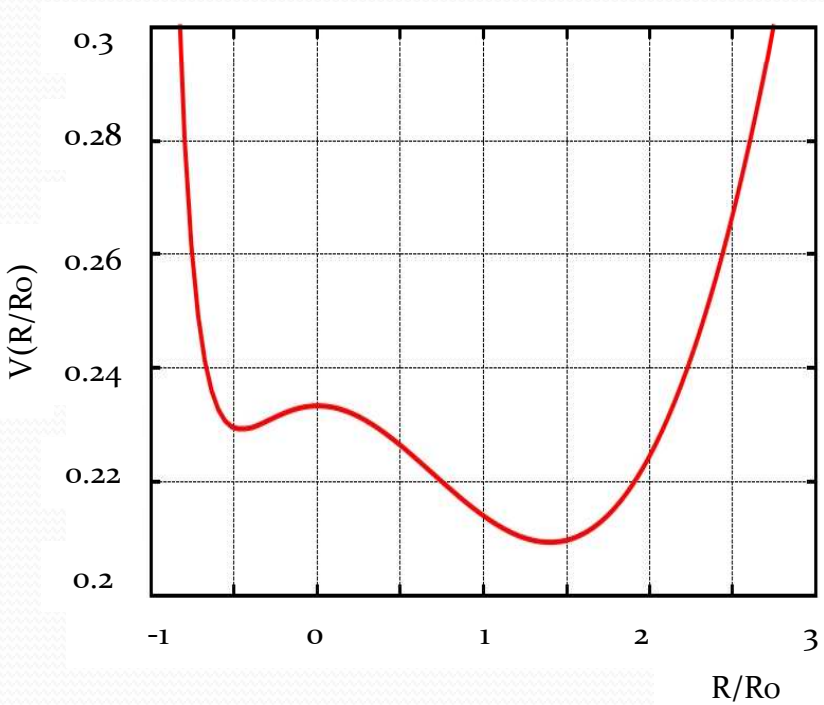
Numerical results

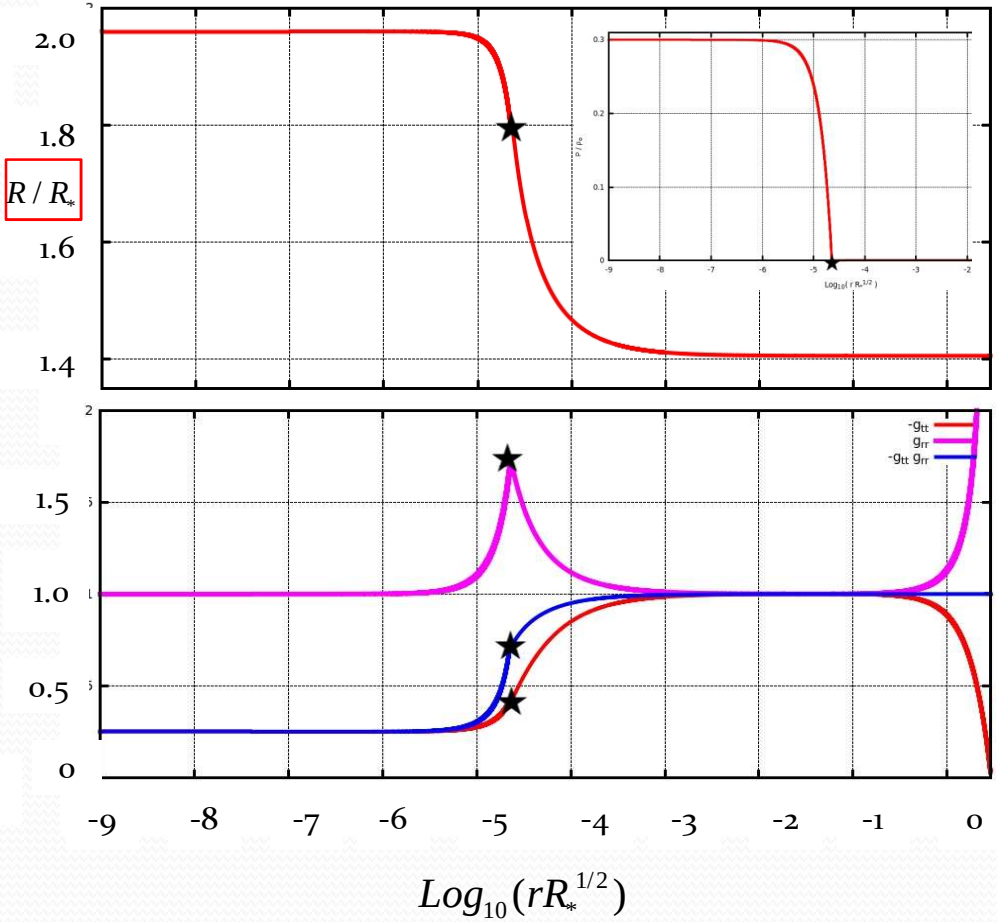
Case I: Miranda et al f(R) model

$$f(R) = R - \alpha R_* ln(1 + R/R_*)$$

- Pink g_{rr}
- Red $-g_{tt}$
- Blue $-g_{rr}g_{tt}$

$$V(\tilde{R}) = \frac{R_*^2}{6}\left\{(1+\tilde{R})\left[\tilde{R} + (6\alpha - 1)\right] - 2\alpha(3 + 2\tilde{R})ln(1+\tilde{R})\right\}$$





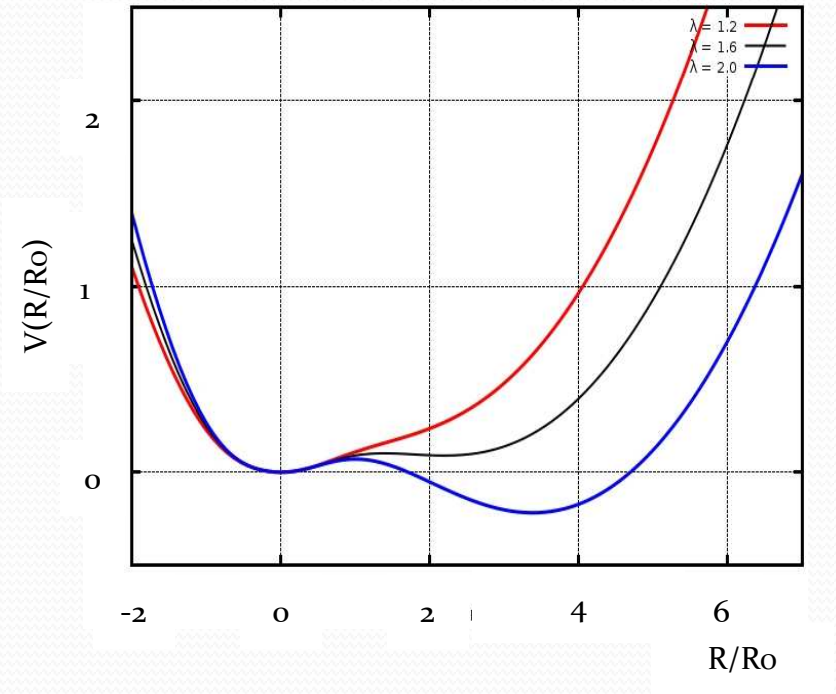
Case II: Starobinsky f(R) model

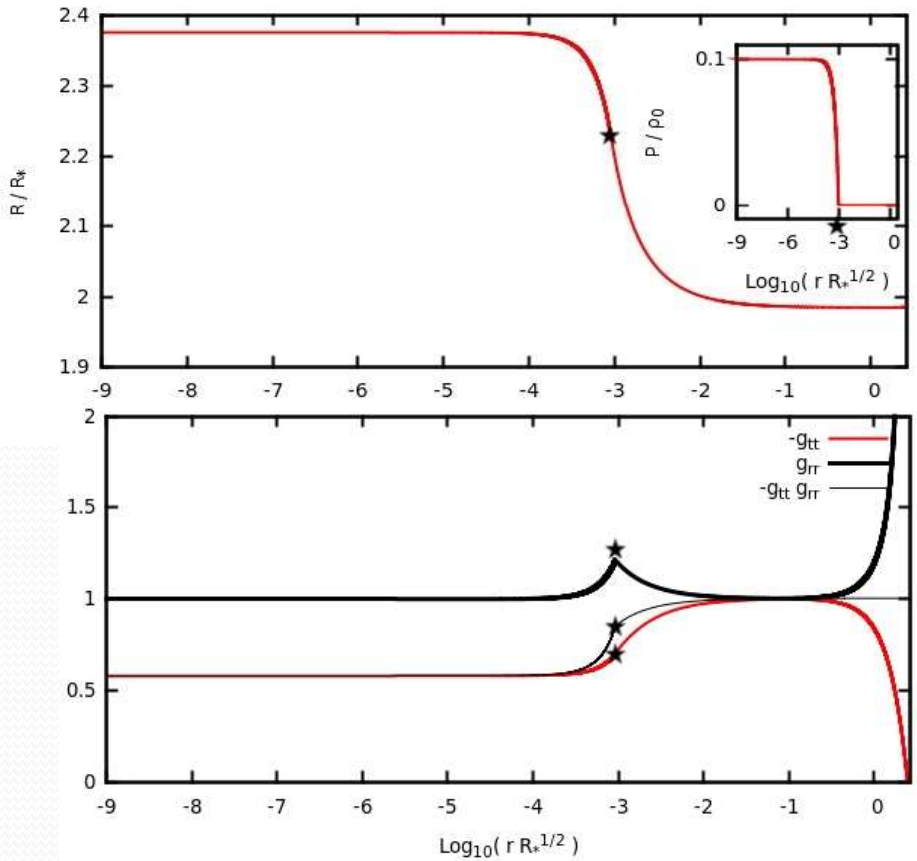
No singularity in the Ricci scalar.

$$f(R) = R - \lambda R_* \left\{ 1 - \left[1 + \left(\frac{R}{R_*} \right)^2 \right]^{-\beta} \right\}$$

- Black g_{rr}
- Red $-g_{tt}$
- Grey $-g_{rr} g_{tt}$

$$V(\tilde{R}) = \frac{R_*^2}{3} \left\{ \frac{\tilde{R}}{2} \left[\tilde{R} - 4\lambda - 2\lambda \left(1 + \tilde{R}^2 \right)^{-1} \right] + 3\lambda \arctan(\tilde{R}) \right\}$$





Conclusion

- We have devised a straigthforward and robust approach to f(R) gravity without resorting to the usual mapping to STT.
- With this method is posible to analyze in a rather transparent and well defined way several aspects of these alternative theories.
- In particular we focused on the existence of relativistic extended objects. We found that, for some f(R) models, such objects can be constructed without ambiguity.
- Building realistic neutron stars with a realistic de Sitter background in f(R) gravity still remains a technical callenge.
- In a near future we plan to study in more detail this issue with other aspects of f(R) gravity using our approach.