Cosmology with Strong Gravitational Lensing

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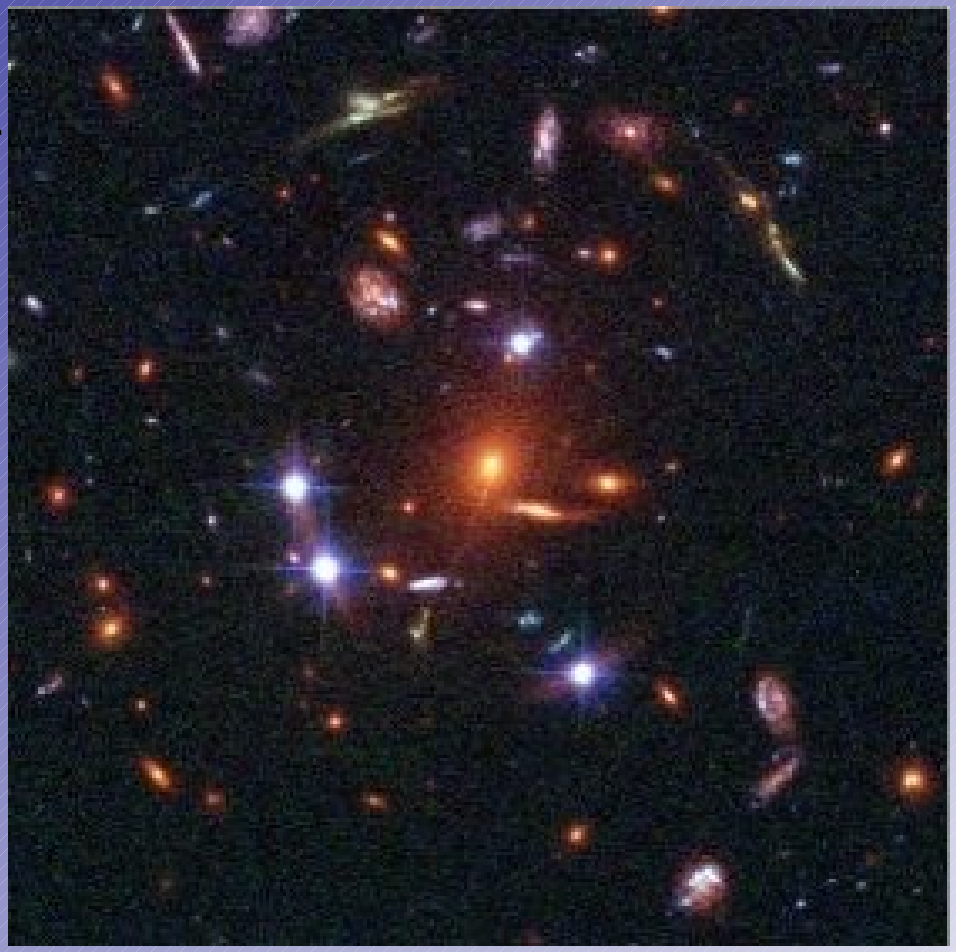
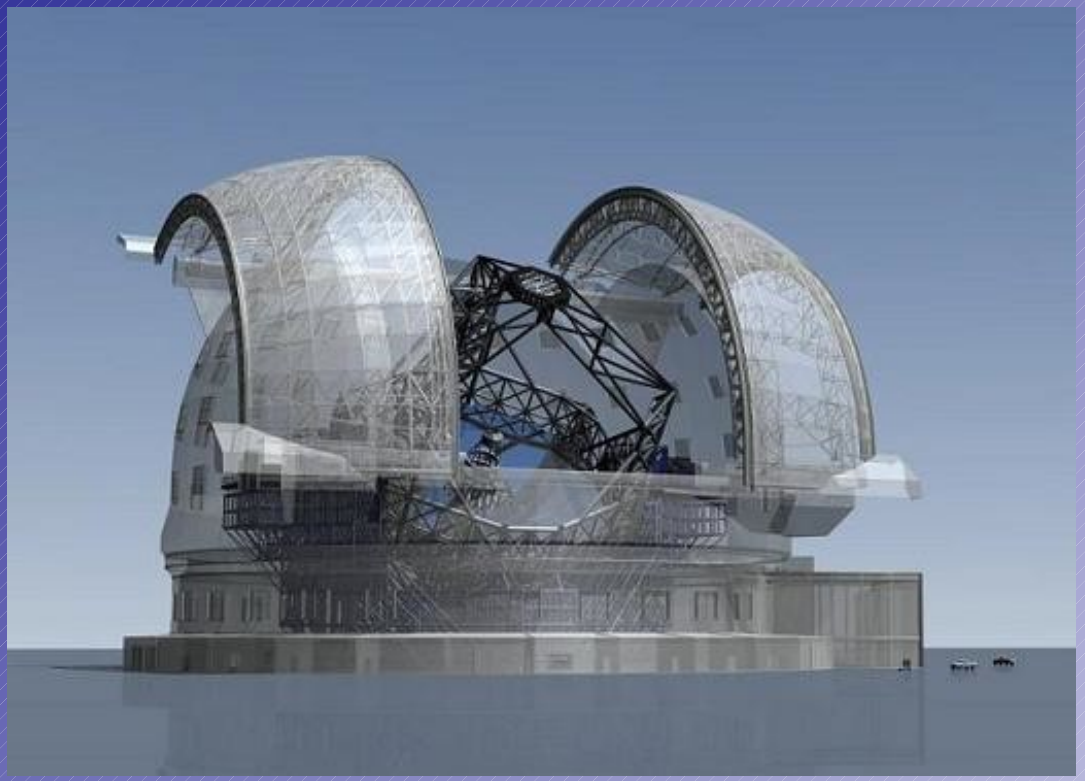
The idea

- To measure small redshift differences in the multiple images of strong gravitational lensing systems
- This difference is equal to the value of the Hubble parameter at the redshift of the source by the time delay between the images





- Ideal candidates: high-z QSOs lensed by massive galaxy clusters
- Ideal instrument: CODEX on E-ELT

The method I

- The redshift of a source:

$$1 + z_s = \frac{a(t_o)}{a(t_e)} = \frac{1}{a(t_e)}$$

- The observed and emitted time intervals of a varying source are

$$\Delta t_o = (1 + z_s)\,\Delta t_e$$

- The time delay between two lensed images of the same source is

$$\Delta t(\mathbf{x_1}, \mathbf{x_2}, \mathbf{y}) = \tau(\mathbf{x_1}, \mathbf{y}) - \tau(\mathbf{x_2}, \mathbf{y})$$

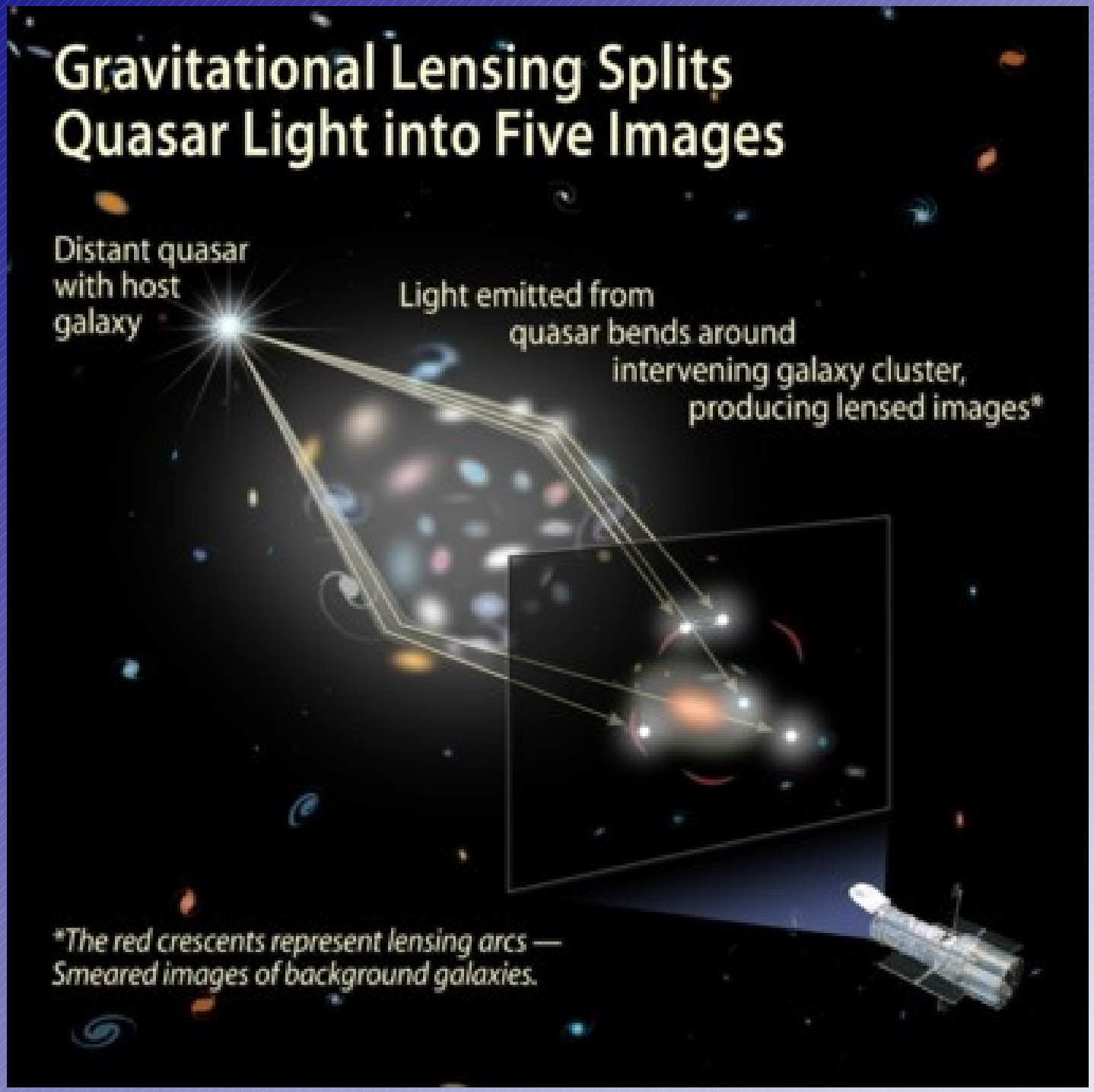
- where the Fermat potential is

$$\tau(\mathbf{x}, \mathbf{y}) = \frac{1 + z_l}{c}\,\frac{D_{ol} D_{ls}}{D_{os}}\left(\frac{|\mathbf{x} - \mathbf{y}|^2}{2} - \psi(\mathbf{x})\right)$$

- a function of the deflection potential

$$\psi(\mathbf{x}) = \frac{D_{ls}}{D_{ol} D_{os}}\,\frac{2}{c^2}\int \Phi(D_{ol}\mathbf{x}, z)\,\mathrm{d}z$$

Time delays


Gravitational Lensing Splits
Quasar Light into Five Images
Distant quasar
with host
galaxy
Light emitted from
quasar bends around
intervening galaxy cluster,
producing lensed images*
*The red crescents represent lensing arcs —
Smeared images of background galaxies.

Time delays for a SIS

- A simple model for the total mass distribution of a lens is a Singular Isothermal Sphere (SIS)

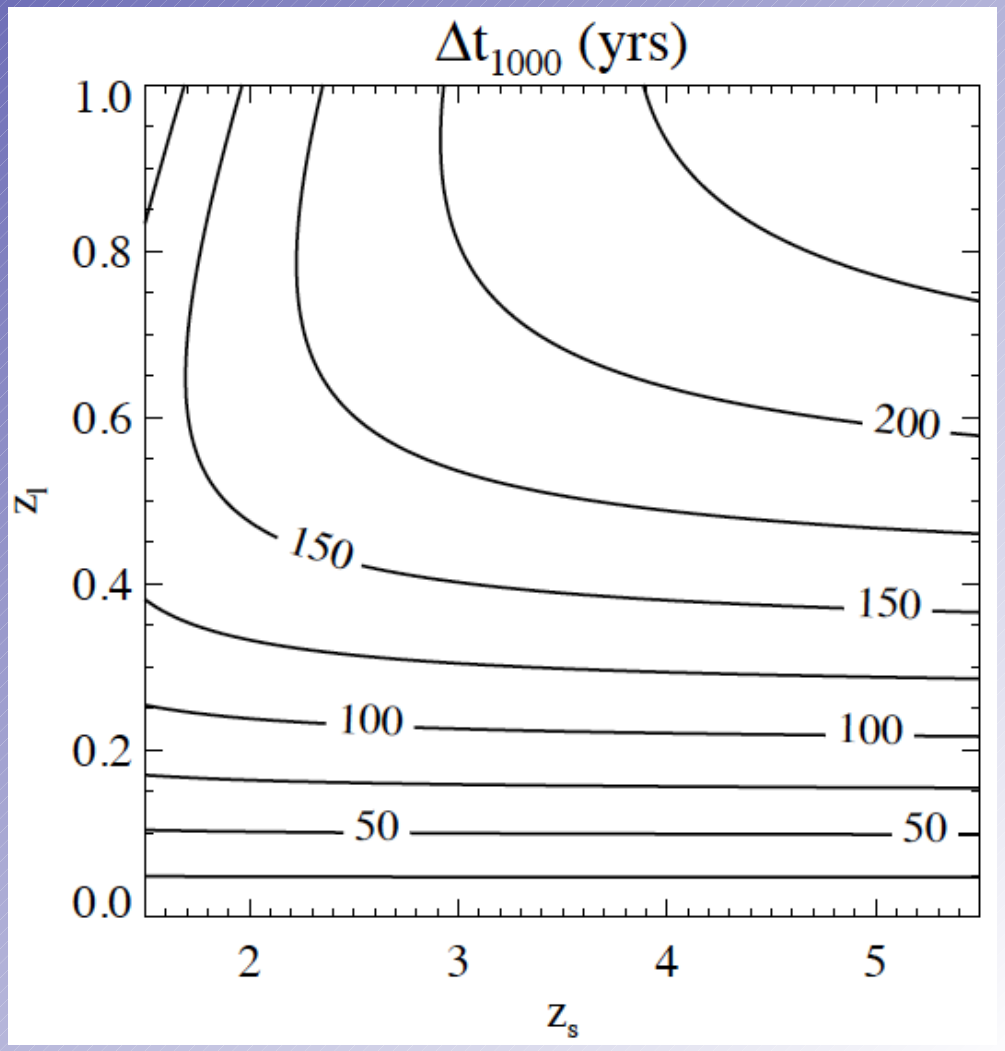
- The 3D mass density profile of a SIS decreases as $1/r^2$

- A SIS is characterized by the value of an effective velocity dispersion

- The maximum time delays between two multiple images of the same source is

$$\Delta t_{\max}(z_l, z_s, \sigma) = \frac{32\,\pi}{c^5}\,(1+z_l)\,\frac{D_{ol}D_{ls}}{D_{os}}\,\sigma^4$$

$$= \Delta t_{1000}(z_l, z_s)\left(\frac{\sigma}{1000\,\mathrm{km\,s^{-1}}}\right)^4$$



The method II

- Expanding to the first order the expression of the redshift of a source, we have

$$\Delta z_s = -\frac{1}{a^2(t_e)}\,\dot{a}(t_e)\,\Delta t_e = -H(z_s)\,(1+z_s)\,\Delta t_e = -H(z_s)\,\Delta t_o$$

- The redshift difference can be converted into a spectroscopic velocity difference:

$$\Delta v(z_s, \Delta t_o) = c\,\frac{|\Delta z_s|}{1+z_s} = c\,\frac{H(z_s)\,\Delta t_o}{1+z_s}$$

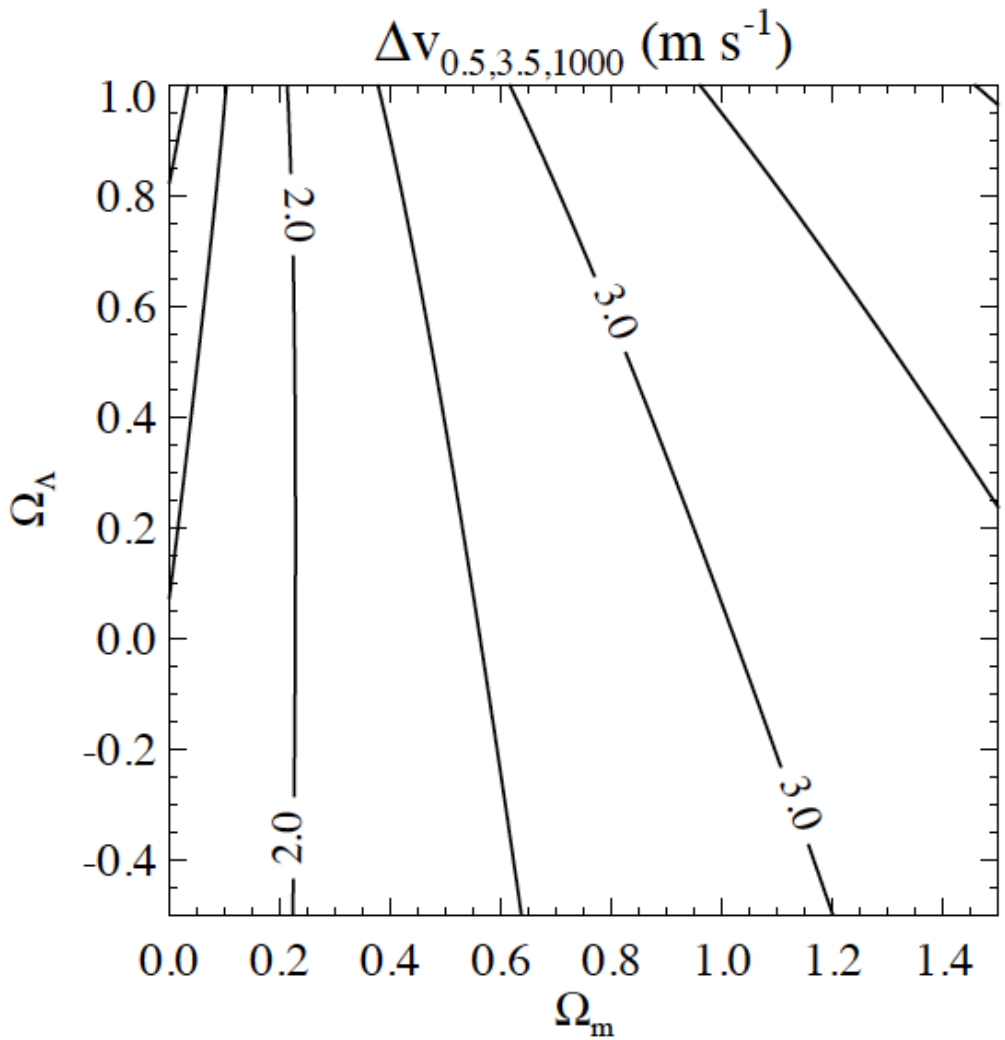
- In GR, the evolution of the Hubble parameter is linked to the mass-energy content of the Universe:

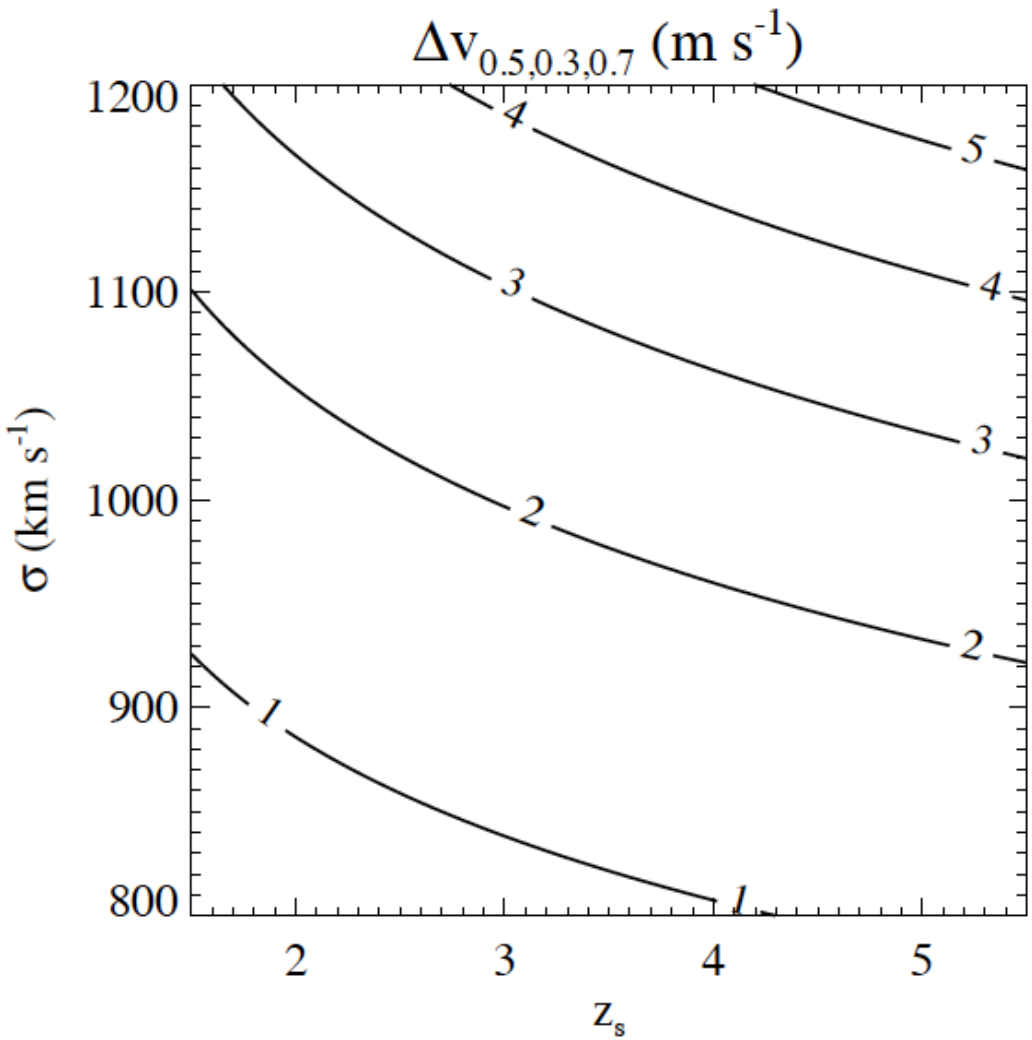
$$H(z) = H_0\left[(1+z)^4\Omega_r + (1+z)^3\Omega_m + (1+z)^2(1-\Omega_0) + \Omega_\Lambda\right]^{\frac{1}{2}}$$

Typical values

$$\Delta v_{0.5,3.5,1000}(\Omega_m, \Omega_\Lambda) = c\,\frac{H(3.5)\,\Delta t_{\max}(0.5, 3.5, 1000)/2}{1+3.5}$$

* Very small signal

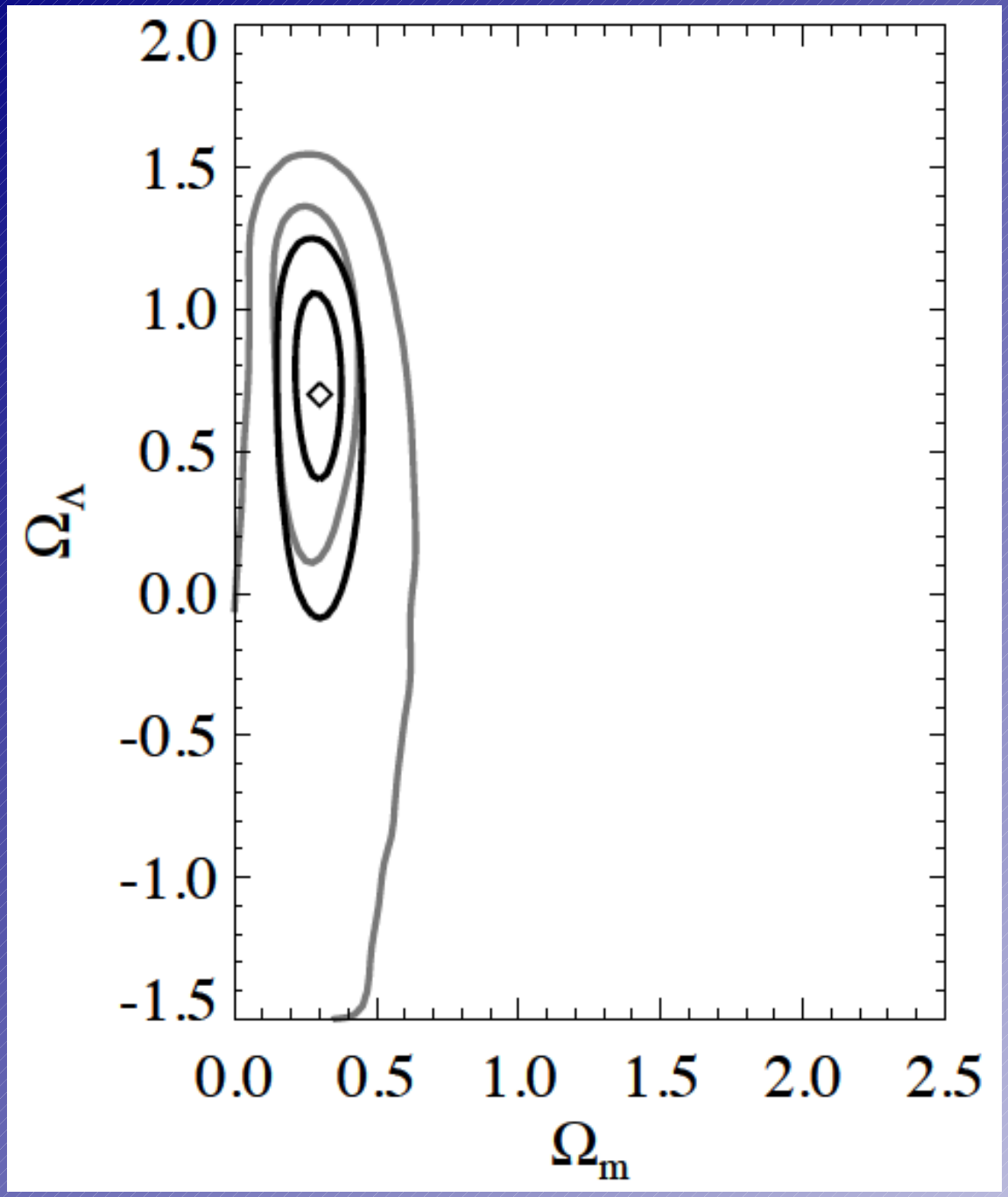




* Detectable with the E-ELT/CODEX

$$\Delta v_{0.5,0.3,0.7}(z_s, \sigma) = c\,\frac{H_{0.3,0.7}(z_s)\,\Delta t_{\max}(0.5, z_s, \sigma)/2}{1+z_s}$$

Some simulations



- 10^5 samples of 10 lensed QSOs each, with 10% or 20% errors on the observables
- The value of the matter density parameter can be determined precisely
- This method is complementary to and independent from any other cosmological probe

– The end –