Non-linear structure in the Universe
Cosmology on the Beach

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“Strong” non-linearity

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Limited options

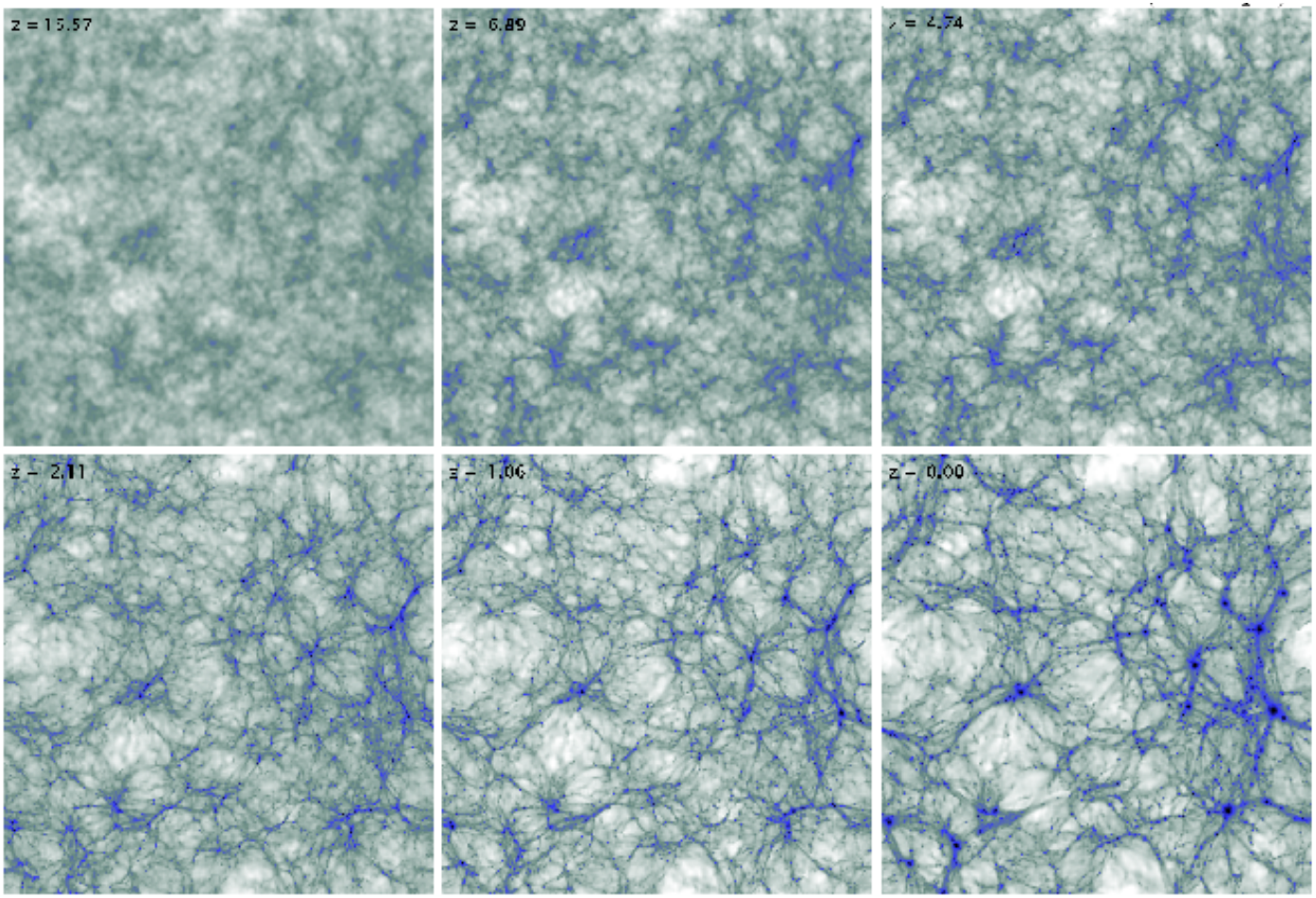
- Beyond a certain scale, linear perturbation theory breaks down
  - Definition of “non-linear scale”?
- At this point we have few options:
  - Analytical models of non-linear growth.
    - Zel’dovich approximation.
    - Spherical top-hat collapse.
  - Perturbation theory.
    - Realm of validity? Convergence criterion?
    - Good for small corrections to almost linear problems.
  - Direct simulation.
    - Numerical convergence.
    - What models to run?
    - Missing physics.

Zel'dovich approximation

- Assume particles move in a straight line with their linear perturbation theory velocity.
- Defines a mapping from initial (Lagrangian) position, q, to final (Eulerian) position, x:
  - $x=q+\Psi$ with $\Psi(q,t)=D(t)\Psi(q)$ and $\Psi_i=d\Phi/dq_i$
  - $\Psi_k = -ik/k^2\ \delta_k$
- If the initial field is uniform, the final density is the Jacobian of this mapping.
  - $\rho\sim[(1-D\alpha)(1-D\beta)(1-D\gamma)]^{-1}$
  - α,β,γ e-values of $-d^2\Phi/dq_i dq_j$
- Collapse takes place first along largest e-value (pancake/sheet), then middle (filament) then final (halo).

The cosmic web

The Zel'dovich approximation, plus the statistics of Gaussian fields, qualitatively describes large-scale structure.



Springel, Hernquist & White (2000)

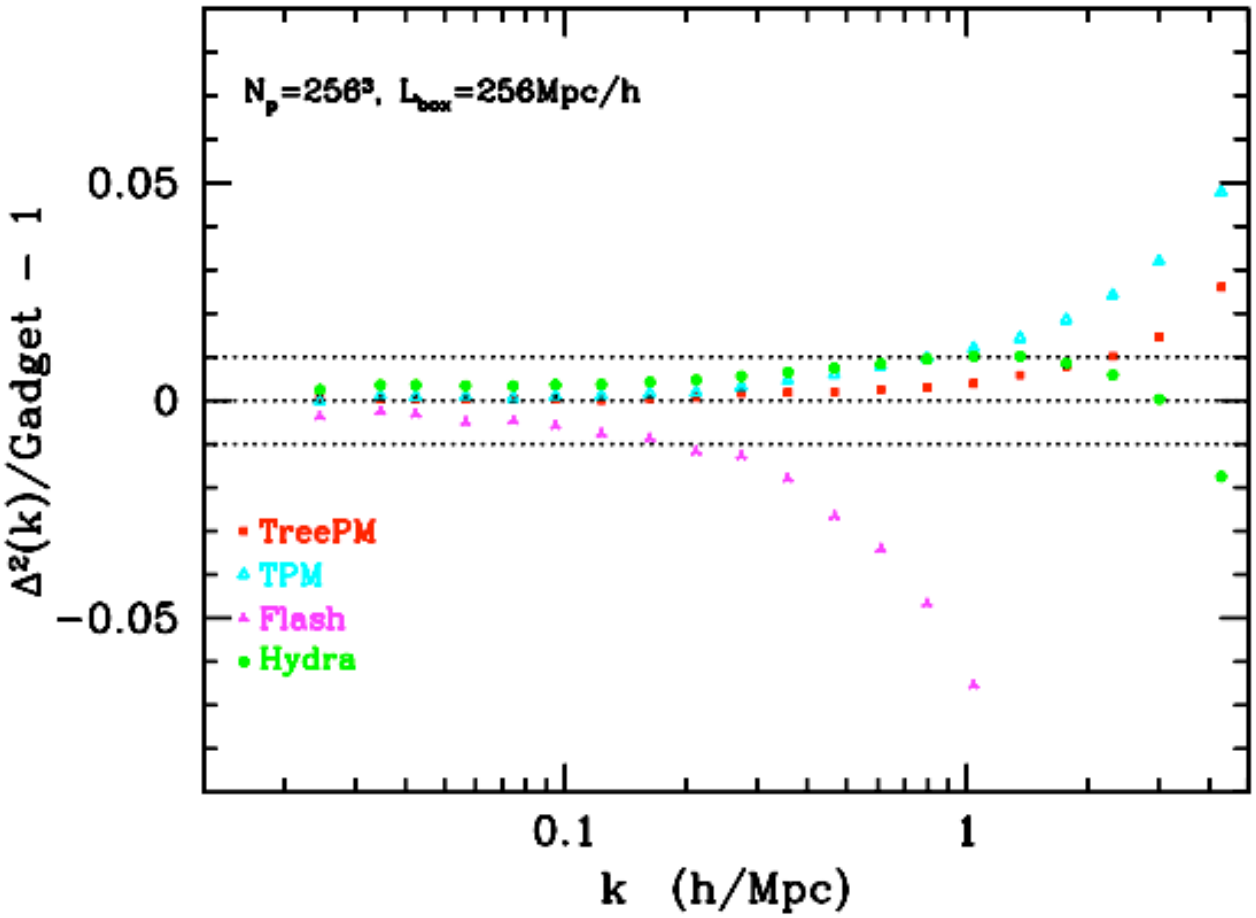
Numerical simulations

- Our ability to simulate structure formation has increased tremendously in the last decade.
- Direct simulation of the N-body problem
  - Begin at early times, but during matter domination, by displacing particles from an initial grid using 1LPT or 2LPT.
  - Monte-Carlo integration of the Vlasov equation using “super-particles” which move along the characteristics.
  - Soften the forces to avoid particle-particle scattering or integrating unphysical, tight, orbiting particles.
  - Want to approach the “fluid” limit with very large N.
  - Pure N-body codes scale “almost” perfectly.
- Our understanding of -- or at least our ability to describe -- galaxy formation has also increased dramatically.
  - Most cosmology probes observe galaxies.
  - The fundamental unit of structure theoretically is the dark matter halo.
  - Galaxies live in dark matter halos in ways we increasingly understand.

Numerical convergence

- Numerous tests of numerical convergence can be found in:
  - Heitmann et al. (2010; ApJ, 715, 104)
  - Heitmann et al. (2010; ApJ, 705, 156)
- Need to worry about
  - Starting redshift and method.
  - Force accuracy and softening.
  - Time stepping.
  - Box size.
  - Number of particles.
  - Method of computing statistic from particles.
  - How to choose *which* cosmologies to run.

Accuracy - currently demonstrated



Updated from Heitmann et al. (2007)

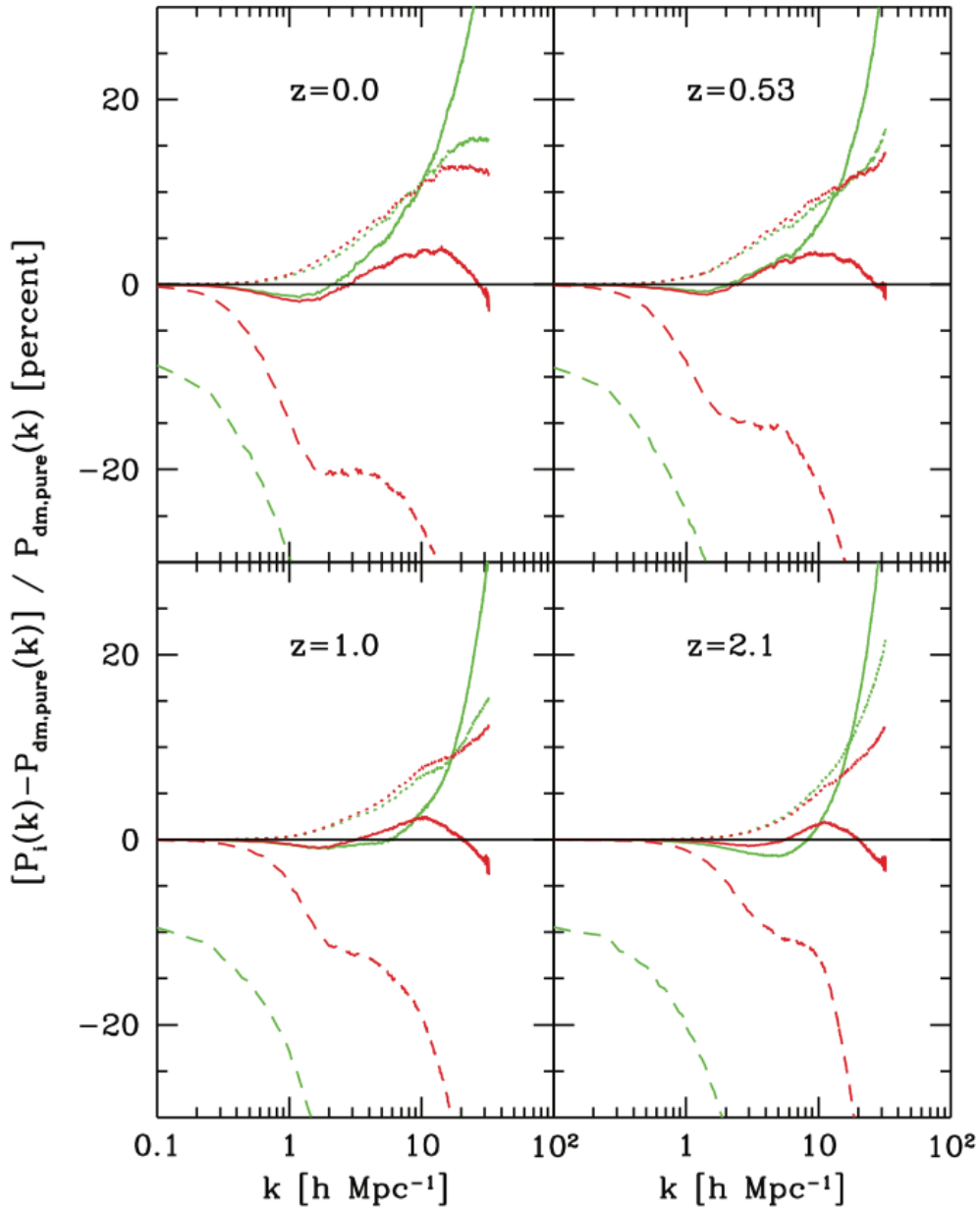
Only a sub-sample of the codes are shown here.

All codes started from the same ICs and analyzed with the same *P(k)* codes.

Extra physics

- As we go to smaller scales, we must go beyond the “pure” N-body problem and include additional physics.
  - Hydrodynamics solvers well developed.
  - Gas cools dramatically in deep potential wells, reaching high densities in a clumpy, multiphase, turbulent, magnetized ISM where it can form stars, which give off winds and radiation and go supernova injecting momentum and energy into the surrounds and have active galactic nuclei which can impart energy to their enviroments, …
- There is little scale separation between including “gas” physics and including star formation, feedback, etc. so results typically depend on sub-grid models.

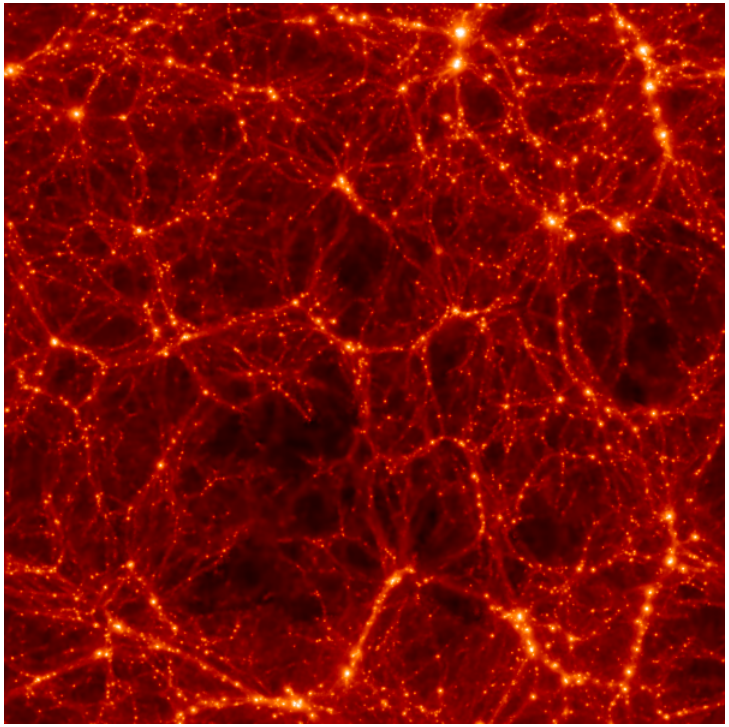
An example



One possibility, from Jing et al. (2006), for the effects of baryons (red) and baryons including star-formation and feedback (green) on the total matter (solid), dark matter (dotted) and gas (dashed).

Characteristics of LSS

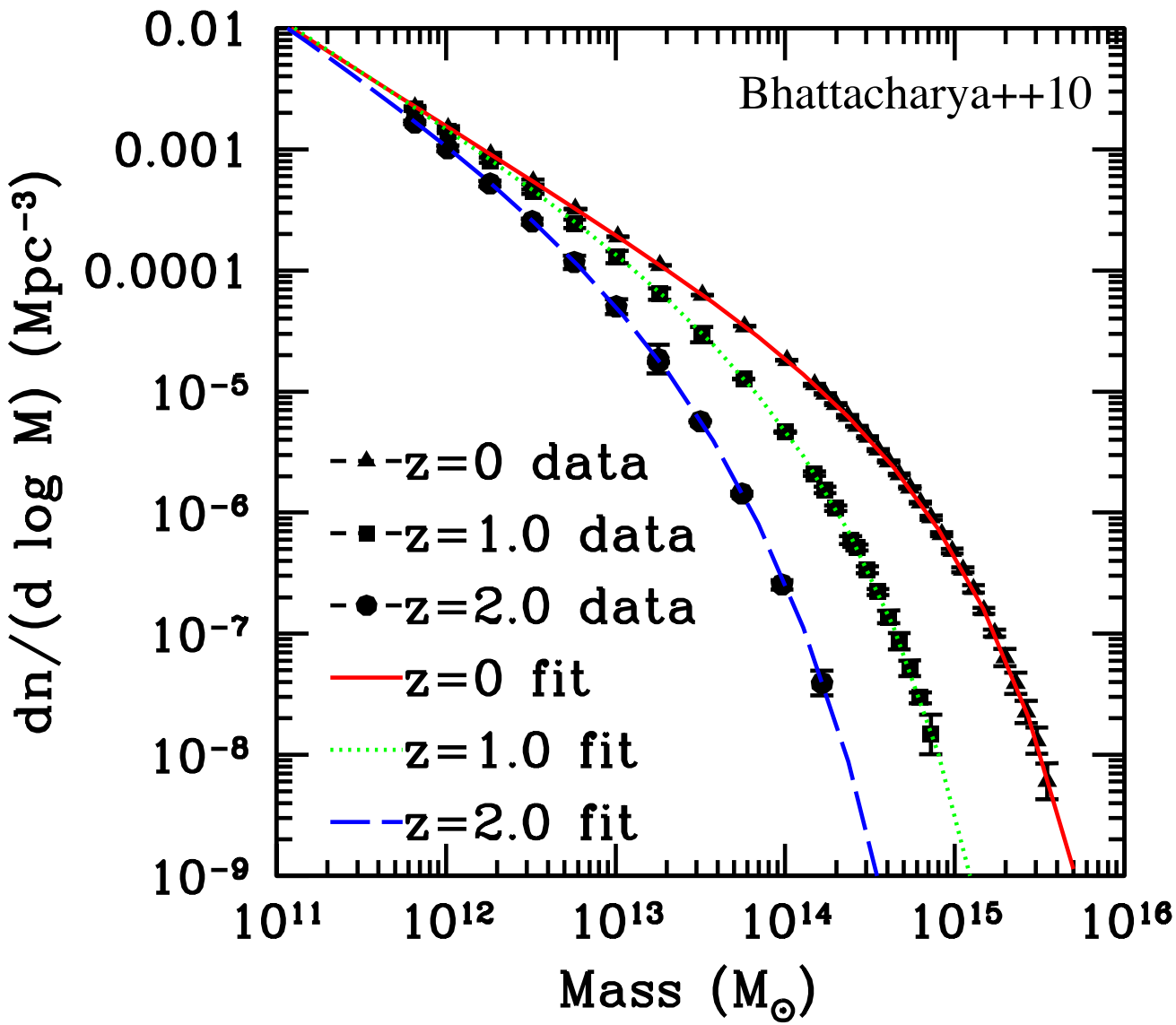
- Large-scale structure forms a beaded filamentary web of dark matter halos.
  - Number of halos vs. mass (etc.).
  - Spatial distribution of halos (vs. ?).
  - Properties of DM halos.
  - Beyond DM.



Halo abundance

- Almost all of the mass resides in (approximately) virialized halos.
- Space density of halos depends primarily (exclusively?) on mass.
- There are a large number of low mass halos and few high mass halos.
  - Very roughly $dn \sim m^{-2} e^{-m}$
  - As time proceeds the “characteristic” mass scale increases.
- The mass function is almost cosmology independent (in scaled units).
  - This universality is not fully understood.
- Mass functions are used in many applications in cosmology.

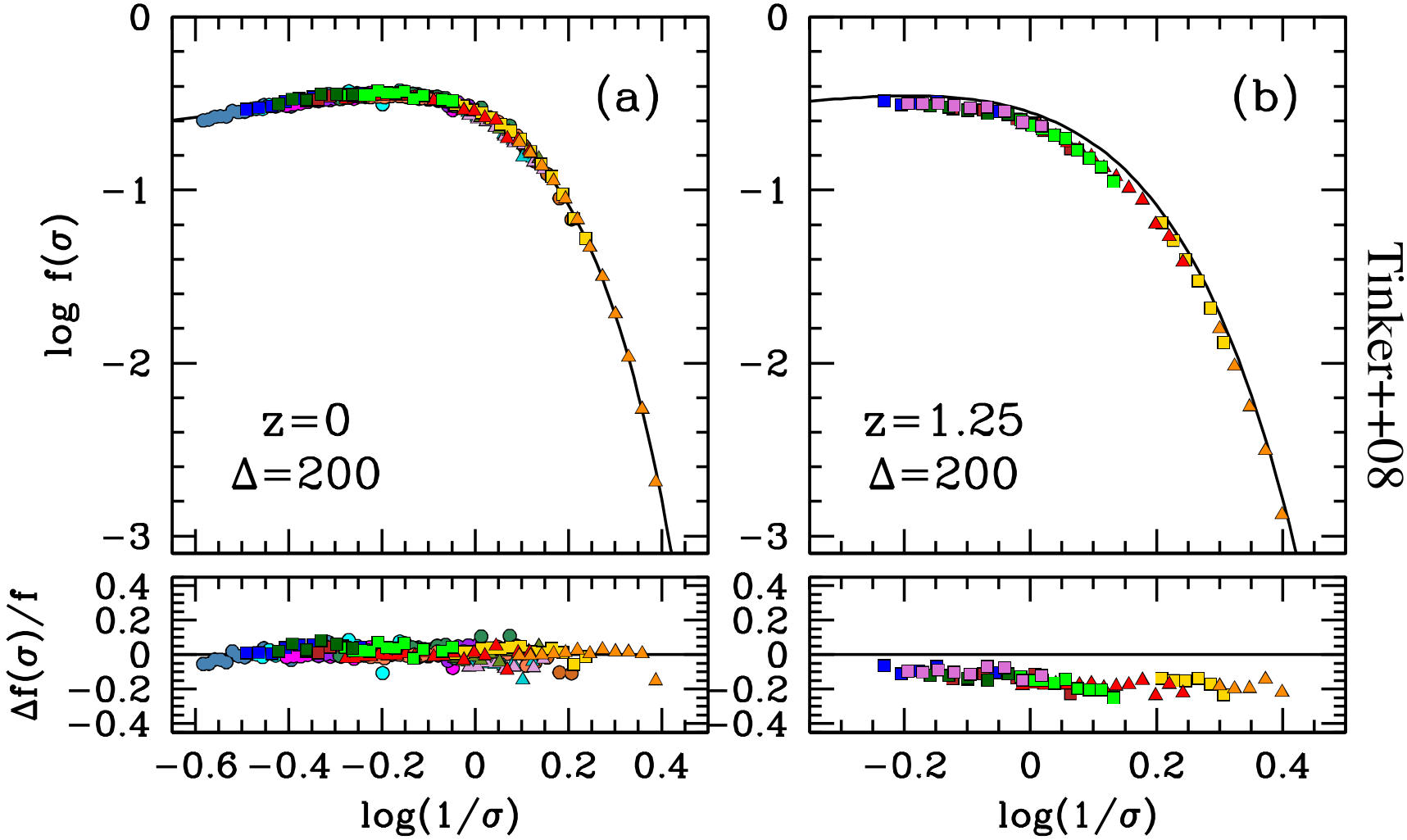
Mass function



Note the dynamic range in this figure!

Halo abundance: scaled units

$$\frac{dn}{dM} = f(\sigma)\ \frac{\bar{\rho}}{M}\ \frac{d\ln\sigma^{-1}}{dM}$$



Other fitting forms

(A detailed study of universality and numerical issues can be found in Bhattacharya++10 from which this table is taken)

MASS FUNCTION FITTING FORMULAE DERIVED IN PREVIOUS STUDIES

Reference	Fitting function f(σ)	Mass Range	Redshift range
Sheth & Tormen (2002)	$f_{ST}(\sigma) = 0.3222\sqrt{\frac{2(0.75)}{\pi}}\exp\left[-\frac{0.75\delta_c^2}{2\sigma^2}\right]\left[1+\left(\frac{\sigma^2}{0.75\delta_c^2}\right)^{0.3}\right]\frac{\delta_c}{\sigma}$	Unspecified	Unspecified
Jenkins et al. (2001)	$0.315\exp\left[-\lvert\ln\sigma^{-1}+0.61\rvert^{3.8}\right]$	$-1.2 \leq \ln\sigma^{-1} \geq 1.05$	z= 0-5
Warren et al. (2006)	$0.7234\left(\sigma^{-1.625}+0.2538\right)\exp\left[-\frac{1.1982}{\sigma^2}\right]$	$(10^{10}-10^{15})$ $h^{-1}M_\odot$	z=0
Reed et al. (2007)	$0.3222\sqrt{\frac{2(0.707)}{\pi}}\left[1+\left(\frac{\sigma^2}{0.707\delta_c^2}\right)^{0.3}+0.6G_1(\sigma)+0.4G_2(\sigma)\right]$ $\times\frac{\delta_c}{\sigma}\exp\left[-\frac{0.764\delta_c^2}{2\sigma^2}-\frac{0.03}{(n_{eff}+3)^2(\delta_c/\sigma)^{0.6}}\right]$	$-0.5 \leq \ln\sigma^{-1} \geq 1.2$	z=0-30
Manera et al. (2010)	$f_{ST}(\sigma) = 0.3222\sqrt{\frac{2a}{\pi}}\exp\left[-\frac{a\delta_c^2}{2\sigma^2}\right]\left[1+\left(\frac{\sigma^2}{a\delta_c^2}\right)^{p}\right]\frac{\delta_c}{\sigma}$	$(3.3\times10^{13}-3.3\times10^{15})$ $h^{-1}M_\odot$	z=0-0.5
Crocce et al. (2010)	$A(z)\left[\sigma^{-a(z)}+b(z)\right]\exp\left[-\frac{c(z)}{\sigma^2}\right]$	$(10^{10}-10^{15})$ $h^{-1}M_\odot$	z=0-1

$$f(\sigma) = \frac{M}{\bar{\rho}}\ \frac{dn}{d\ln\sigma^{-1}} \qquad , \qquad \int_0^\infty d\ln\sigma\ f(\sigma) = 1$$

Excursion set theory vs. peaks

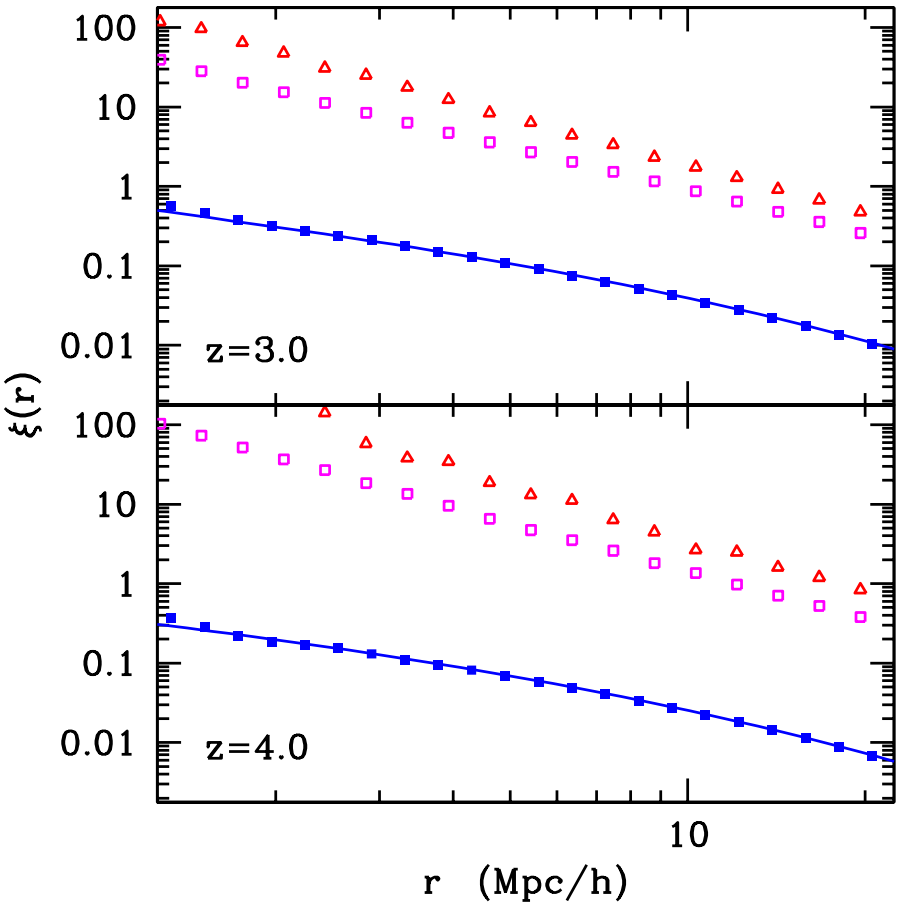
- Excursion set formalism
  - The most popular "theory".
  - The fraction of mass in halos more massive than *M* is related to the fraction of volume in which the smoothed initial density field is above some threshold, δ_c.
  - Mass function related to random walk.
    - Press-Schechter 1974; Bond, Cole, Efstathiou & Kaiser 1991.
  - Spherical collapse vs. elliptical collapse approx.
    - Mo & White, Sheth & Tormen, Zhang & Lam, …
  - How to deal with "non-locality" of halo collapse.
- Statistics of (Gaussian) peaks plus a model for halo collapse (spherical or ellipsoidal).
    - Bardeen, Bond, Kaiser & Szalay 1986
      - Based on Rice (1944; 1945) who studied 1D Gaussian fields as models of noise in communications devices.
    - Bond & Myers 1996.
    - Dalal, Lithwick & White 201X.

Excursion set theory vs. peaks

- Allow computation of mass function from statistics of initial field.
  - Choose a filter shape, and compute integrals of linear theory power spectrum and plug in formulae.
    - Not all methods self-consistent.
  - Reasonable success for mass function often improved by adjusting formulae to “fit” N-body simulations.
  - Less success for conditional mass function, merger rates etc.
  - Beware when extrapolating!

Halo bias

- The clustering of the rare, massive dark matter halos is enhanced relative to the general mass distribution
  - Kaiser 1984; Efstathiou++88; Cole & Kaiser 1989; Bond++91; Mo & White 1996; Sheth & Tormen 1999; …; Tinker++10; ...

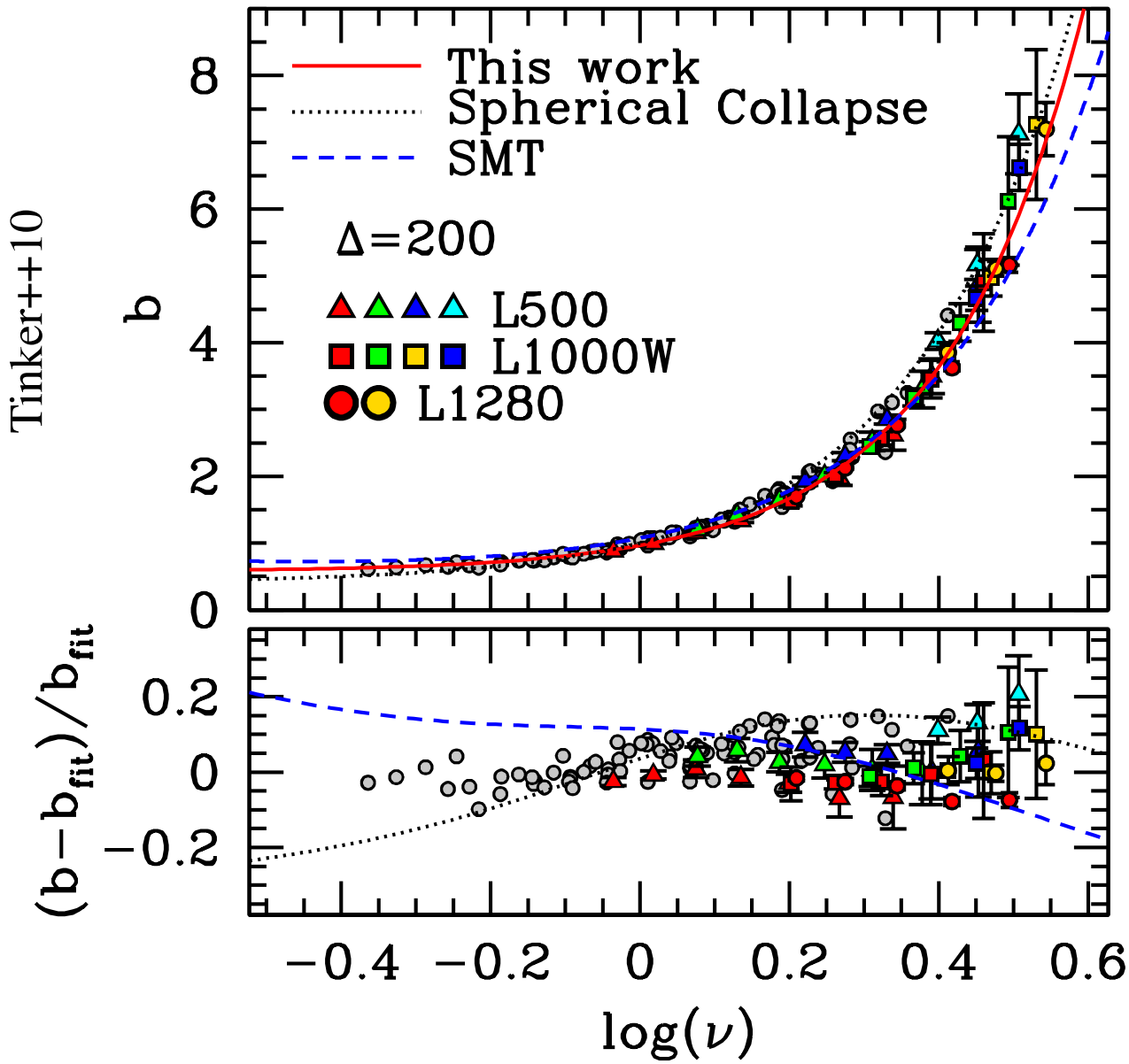


The clustering of rare halos thought to host quasars (here 10^{12} and $10^{12.5}$ M_{sun}/h) at z=3-4 is two orders of magnitude stronger than the clustering of the DM!

Halo bias

- This enhanced clustering is known as “bias”.
- Bias depends on scale [*b(r)*], but at very large scales it becomes scale-independent [*b*].
  - Bias, *b*, depends primarily on halo mass or “rarity”.
    - In simplest models $b=1+(\nu^2-1)/\delta_c$, where $\nu=\delta_c/\sigma(M)$.
    - For more accuracy, use N-body-calibrated fitting function.
    - Behavior at “extremes” can depart from fitting functions!
  - Numerical simulations now large enough to test for the dependence on halo formation history and other properties.
    - Dependencies on formation redshift, internal structure, and spin.
    - Gao++05; Wechsler++06; Harker++06; Bett++07; Wetzel++07; Jing++07; Gao&White07; Angulo++08

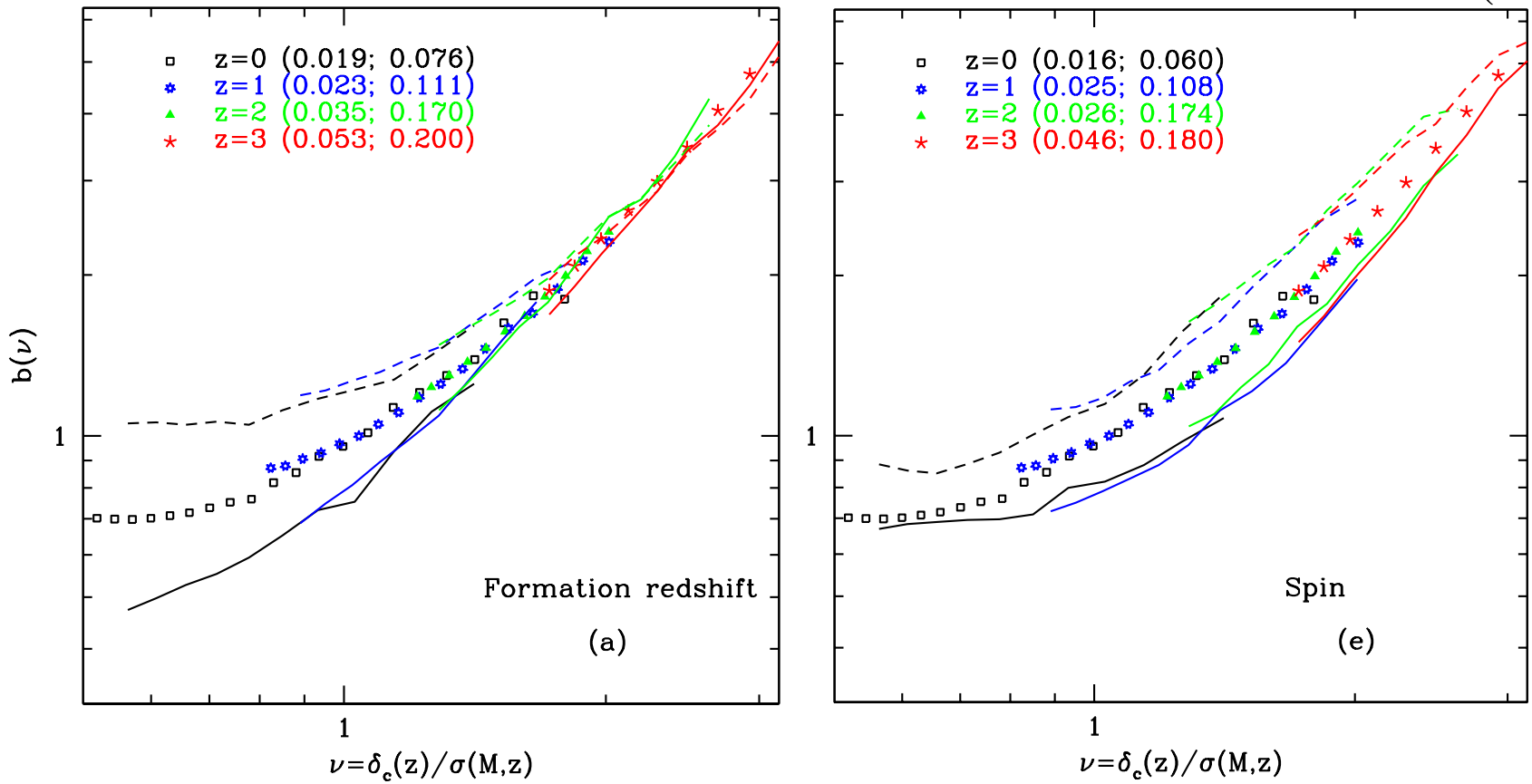
Halo bias in simulations



Halo bias increases with increasing halo mass at fixed redshift, or with increasing redshift at fixed mass.

Assembly bias

Gao & White (2007)



Solid (dashed) lines show halos in lower (upper) 20% of halos split on property labeled.

Assembly bias

- Assembly bias is quite difficult to explain in the "standard" excursion set formulation.
  - Mass function is fraction of random walks reaching an absorbing barrier by mass *M*.
  - Bias is dependence of mass function on large-scale density (early part of the walk).
  - Assembly bias very hard to explain in this picture.
    - Gao++05, Mo++05, Sandvik++07, Desjacques08, …
- Simulations did not initially shed light on explanation for assembly bias.
- Now understand that assembly bias is a simple consequence of non-linear collapse from Gaussian initial conditions.
  - Dalal++08.

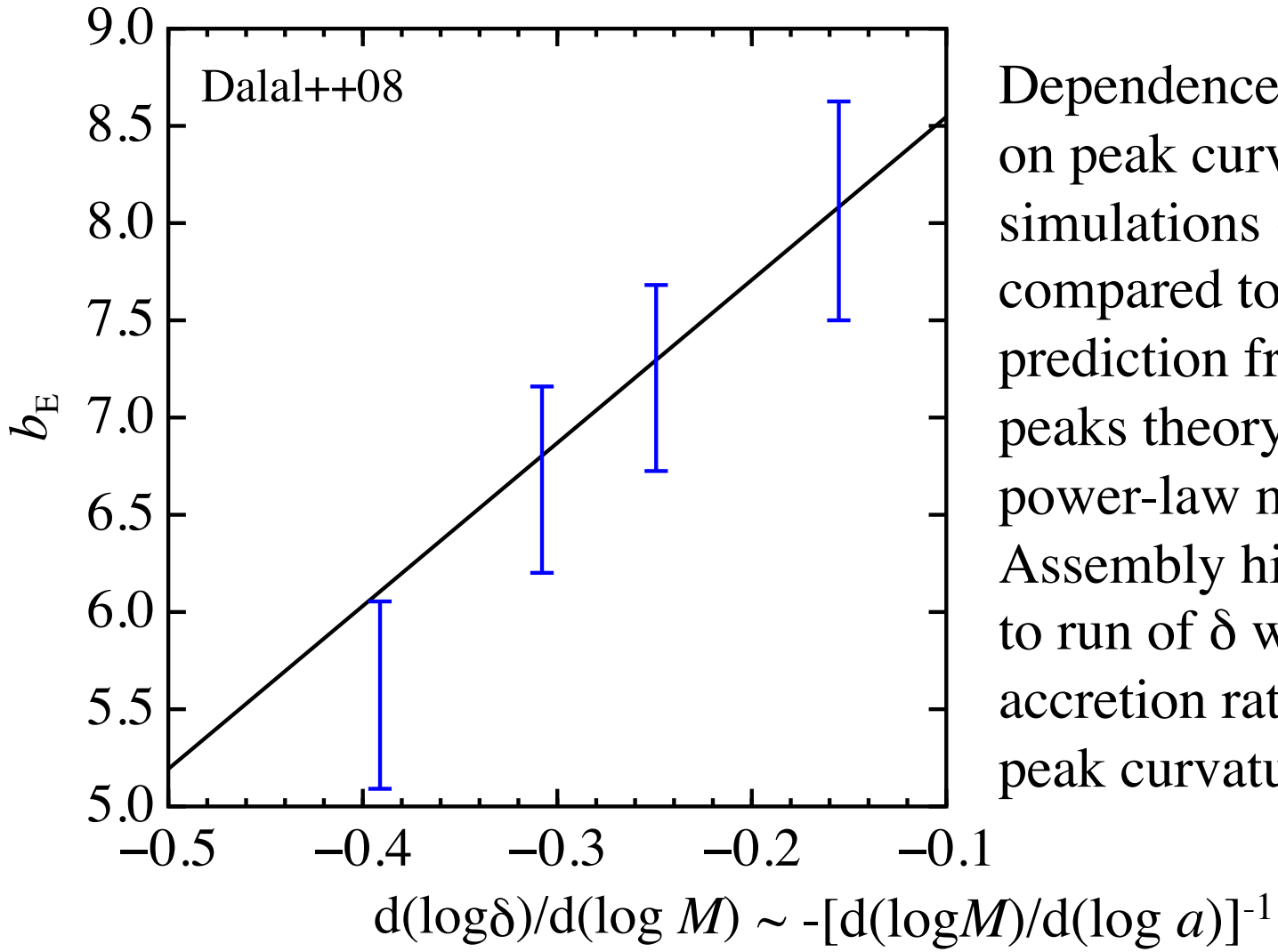
Assembly bias: high mass.

- Later forming, high mass halos are more clustered than typical halos of the same mass.
  - Also dependence on concentration.
- Massive halos collapse almost spherically from rare peaks in ICs.
  - Collapse reasonably explained by STHC.
- For Gaussian field, bias depends on curvature, $s=d\langle\delta\rangle/d\ln M$, of peak (as well as height).
  - Peak curvature is "environment": $\delta_b=\delta_{pk} + s\, d\ln M + \ldots$
  - Peaks with smaller $|s|$ have larger background densities.

$$b - 1 \approx \frac{1}{\sigma}\,\frac{\nu - \langle\nu x\rangle\, x}{1 - \langle\nu x\rangle^2} \quad , \quad \nu \equiv \frac{\delta}{\sigma_\delta}\ ;\ x \equiv \frac{s}{\sigma_s}$$

(Cross-correlation coefficient)

Bias: high mass

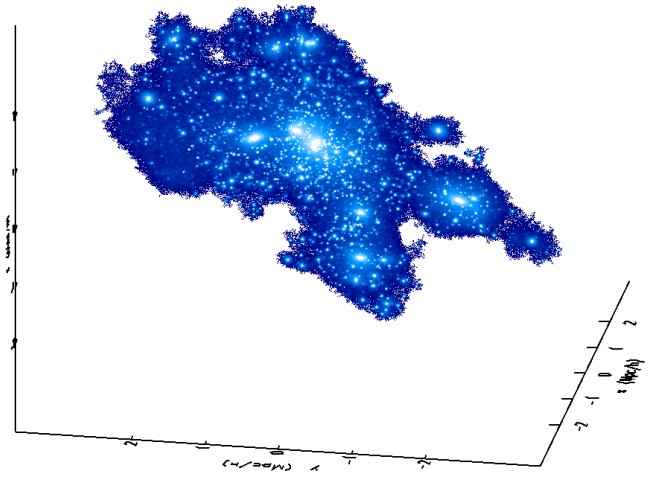


Dependence of halo bias on peak curvature from simulations (points) compared to the prediction from Gaussian peaks theory (line) for a power-law model. Assembly history related to run of δ with M – accretion rate related to peak curvature!

Assembly bias: low mass

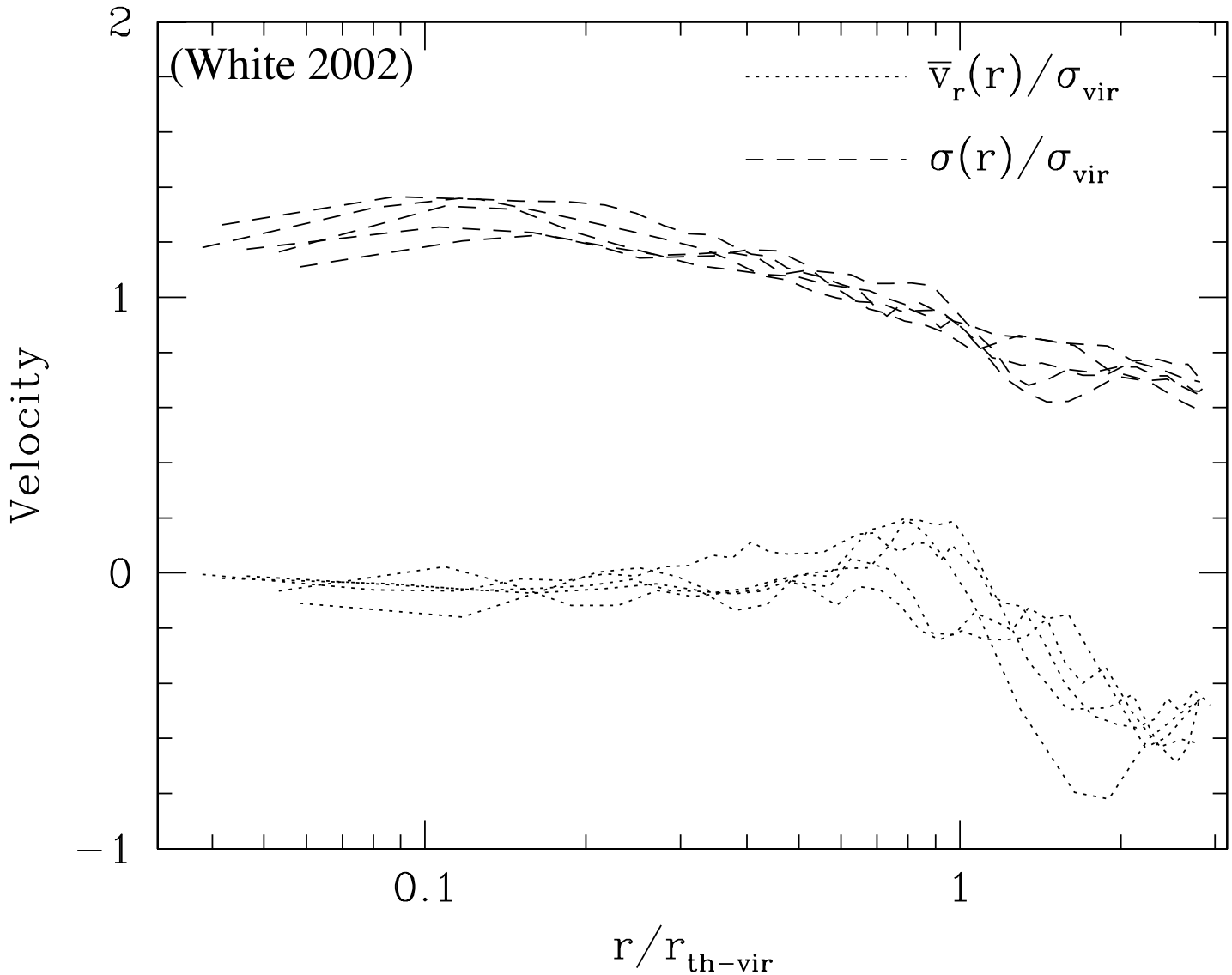
- Oldest, most concentrated, low mass halos are more than twice as clustered as the youngest halos of the same mass.
- Youngest ~80% of halos have
  - $b \sim 1-\delta_c^{-1} \sim 0.4$ (as expected).
- Oldest 20% of low mass halos act like test particles ($b \rightarrow 1$)
  - Most of these are associated with nearby, high-mass halos.
  - Early formers who's growth is stunted by "hot" environments of massive neighbors.

DM halos



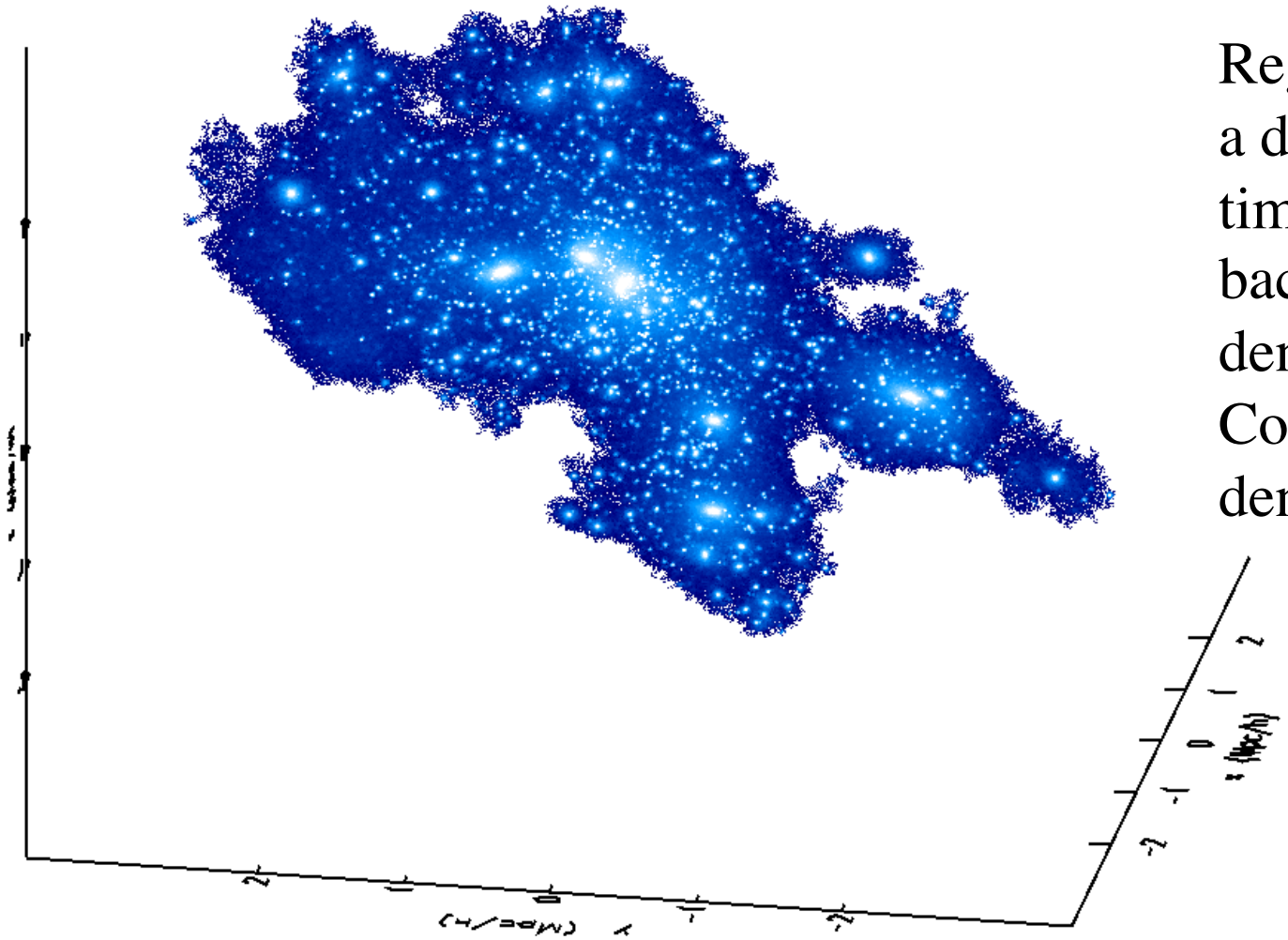
- Generally triaxial spheroids.
- More elongated at
  - Smaller radii.
  - Larger redshifts.
  - Higher mass.
- Approximately in virial equilibrium.
- Aligned with the filamentary, cosmic web which feeds halo growth.
- Average mass accretion exponential.
  - In EPS formalism $dM=-f(M)M\, d\delta_c$, with $f(M)$~constant.
- Spin parameter, λ, grows significantly in major mergers, slowly declines in accretion.

Dynamical state



Roughly isothermal, roughly virialized, self-bound objects.

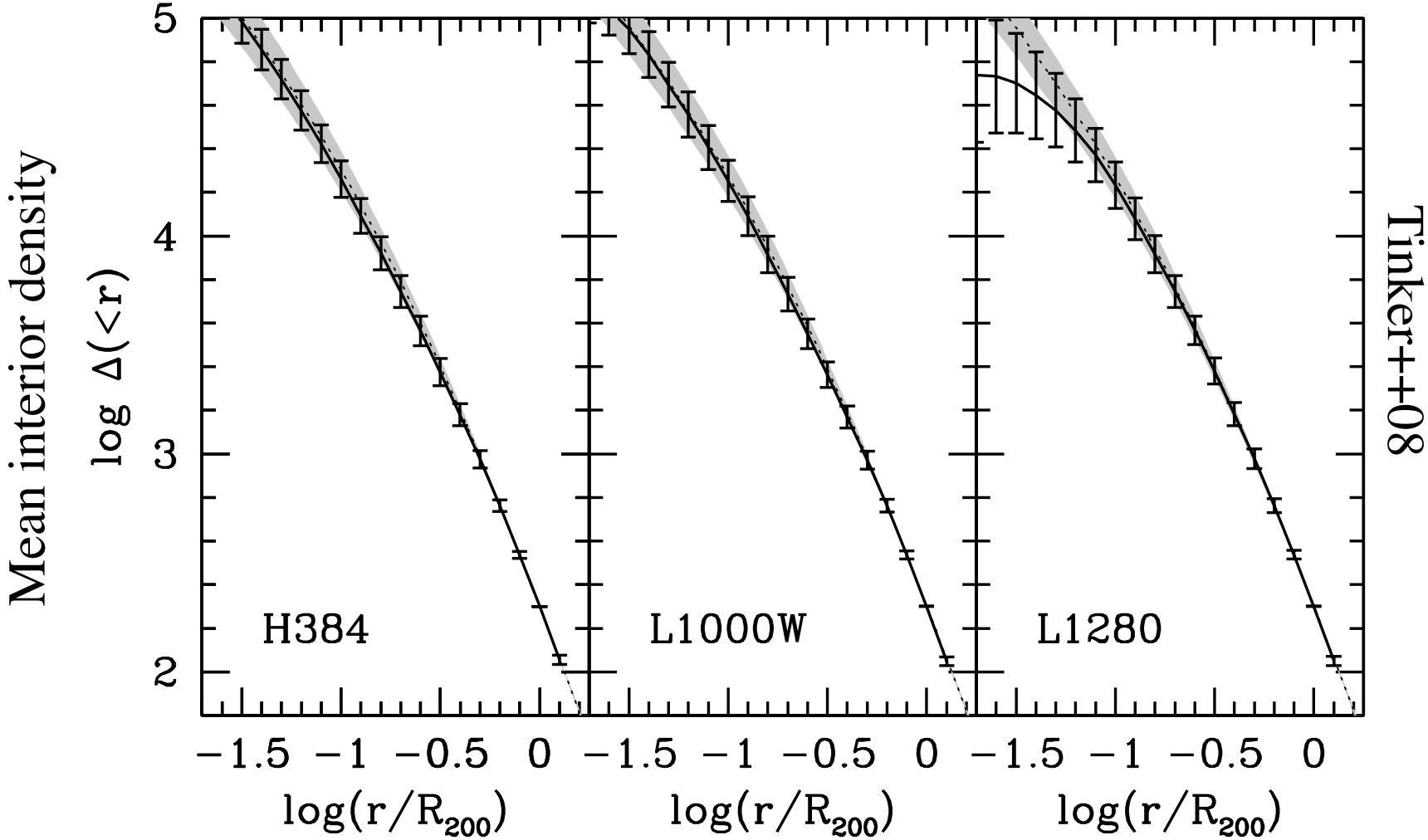
DM halos are aspherical and have significant substructure



Region above a density 10^2 times the background density.
Color: log-density.

Spherical "NFW" profile

$$\rho(x = r/r_s) \propto x^{-1}(1+x)^{-2}$$



A 1-parameter family

- Find $c=r_{vir}/r_s$ is a function of M.
  - More massive halos less concentrated.
  - c, like M, depends on definitions!
  - $c \sim M^{-0.15}$
  - Large, log-normal scatter in c.
- The inner, r^{-1}, part of the halo forms early and r_s stays ~constant.
  - Subsequent accretion kept away by angular momentum barrier.
  - Concentration, $c=r_{vir}/r_s \sim (1+z)^{-1}$.

Other forms

- A generalized NFW makes “-1” and “-2” variable.
- Einasto profile:

$$\rho \propto \exp\left\{-d_n\left[\left(\frac{r}{r_e}\right)^{1/n} - 1\right]\right\} \quad , \quad n \approx 5 - 10$$

- Note no cusp!
- Important new insights in Lithwick & Dalal (2010).
  - Building on earlier work by Fillmore & Goldreich and Bertschinger.
- The NFW profile is “transitional”.
  - r^{-3} slope comes from continued accretion of material. This stops in DE-domination.
  - Busha, Evrard & Adams (2007).
    - Exponential truncation of NFW profile at large radius.

Subhalos

- A generic prediction of hierarchical theories, such as CDM, is that the virialized regions of DM halos contain *subhalos.*
  - Self-gravitating, bound clumps of mass.
- Subhalos account for $O(10\%)$ of halo mass.
- Luminous galaxies form via the cooling and condensation of gas in subhalos.

Subhalos

- Density profiles of subhalos similar to that of halos, but they can be truncated.
- Subhalos track DM closely in terms of density and velocity.
  - Trends of central concentration and velocity bias with ratio of subhalo to host halo mass.
  - Depends on *how* subhalos are selected.
- Beyond a certain point, the number of subhalos above a given mass grows linearly with host halo mass.
  - Length of “plateau” set by dynamical friction and mean density of collapsed structures.
  - Subhalo mass function and halo mass function are “scaled” versions of each other.

$$\frac{dn_{\rm sat}}{dM_{\rm sat}} \sim \left(\frac{M_{\rm host}}{M_{\rm sat}}\right)^2 \qquad , \qquad M_{\rm sat} \ll M_{\rm host}$$

Thank you!

- I would like to thank
  - The participants.
  - The other lecturers.
  - The organizers.
  - The staff.
- for making this a pleasant, informative and productive meeting.

The End