Non-linear structure in the Universe
Cosmology on the Beach

Puerto Vallarta
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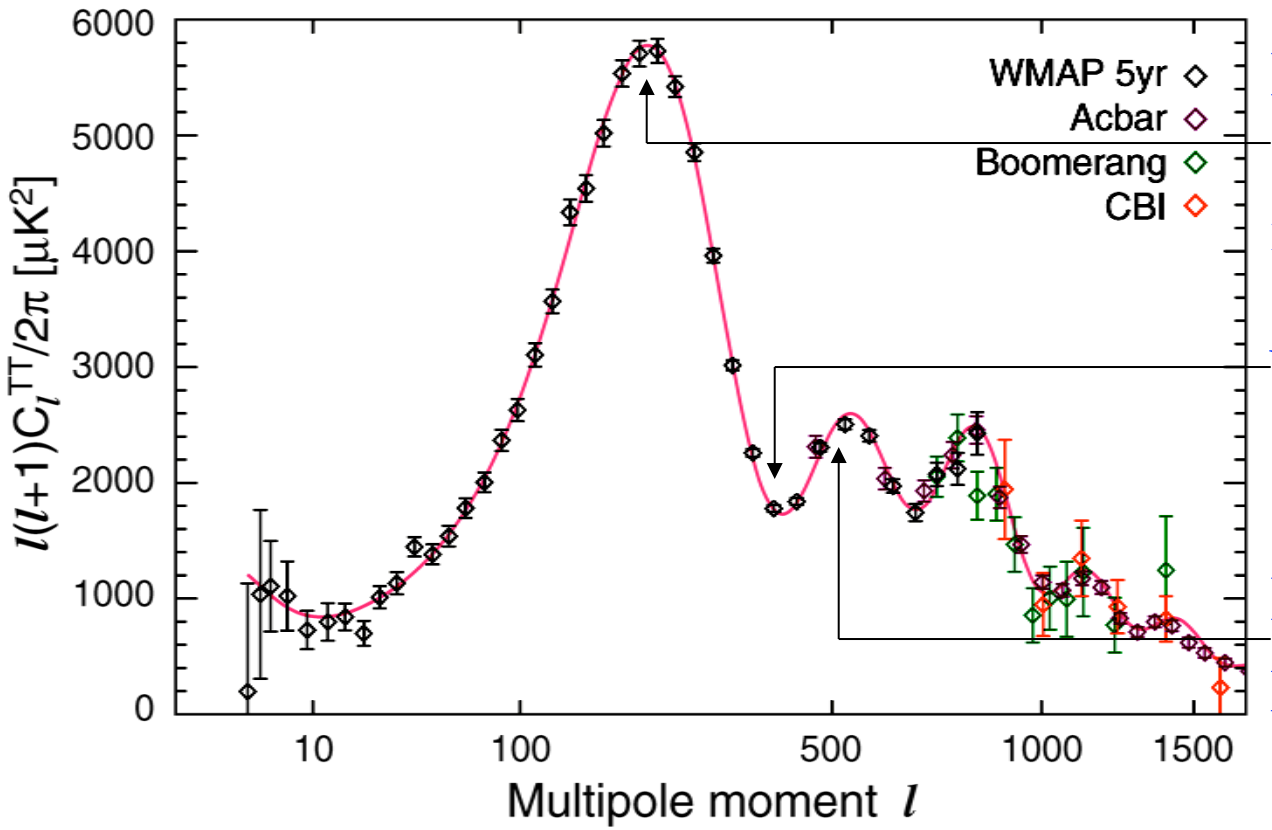
(http://mwhite.berkeley.edu/Talks)

Non-linearities and BAO

Acoustic oscillations



First "compression", at $kc_st_{ls}=\pi$. Density maxm, velocity null.

Velocity maximum

First "rarefaction" peak at $kc_st_{ls}=2\pi$

Acoustic scale is set by the *sound horizon* at last scattering: $s = c_st_{ls}$

CMB calibration

- Not coincidentally the sound horizon is extremely well determined by the structure of the acoustic peaks in the CMB.

$$s = 146.8 \pm 1.8 \text{ Mpc}$$

WMAP 5th yr data

$$= (4.53 \pm 0.06) \times 10^{24} \text{m}$$

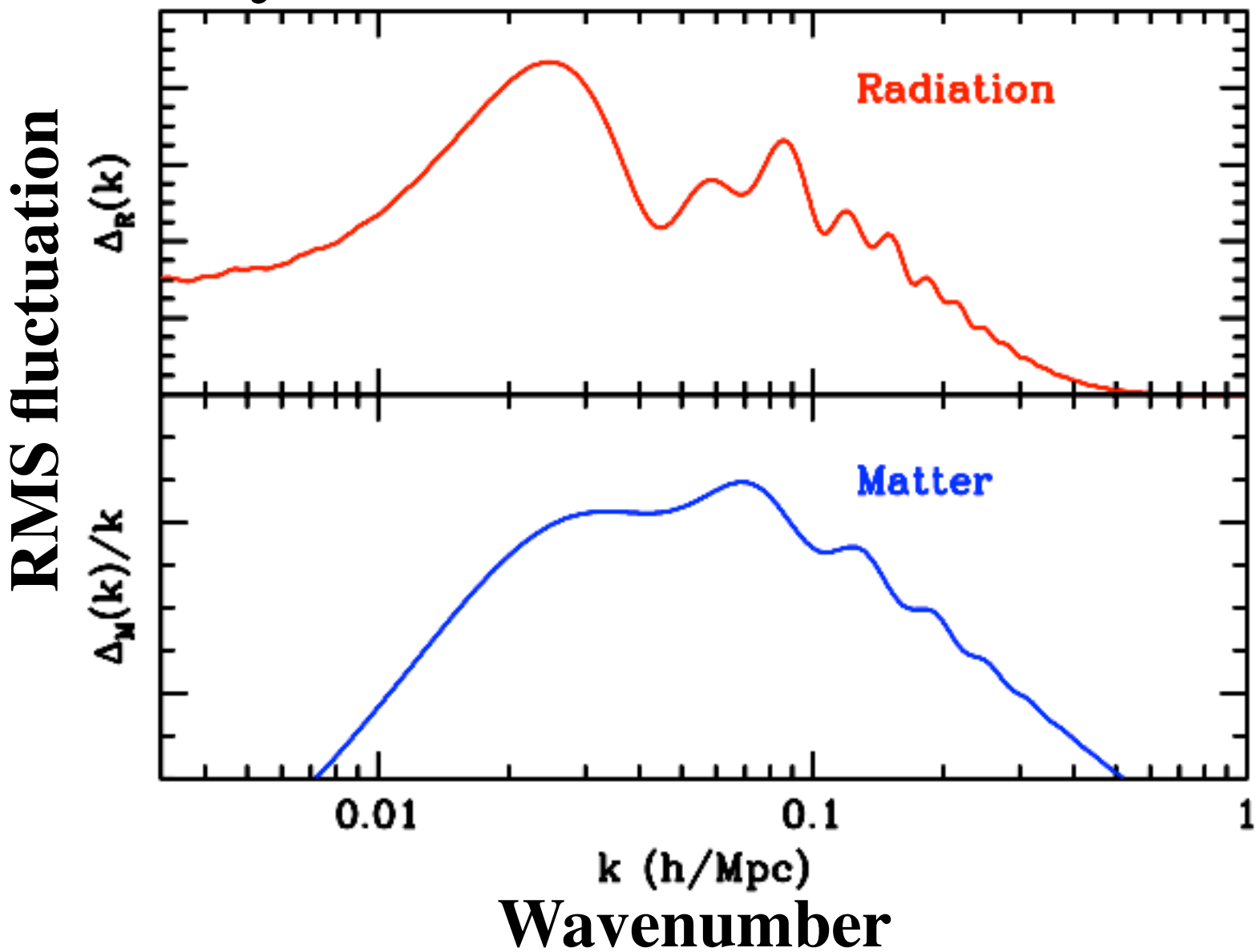
Dominated by uncertainty in ρ_m from poor constraints near 3rd peak in CMB spectrum. (Planck will nail this!)

Baryon oscillations in $P(k)$

- Since the baryons contribute ~15% of the total matter density, the total gravitational potential is affected by the acoustic oscillations with scale set by s.
- This leads to small oscillations in the matter power spectrum $P(k)$.
  - No longer order unity, like in the CMB
  - Now suppressed by $\Omega_b/\Omega_m \sim 0.1$

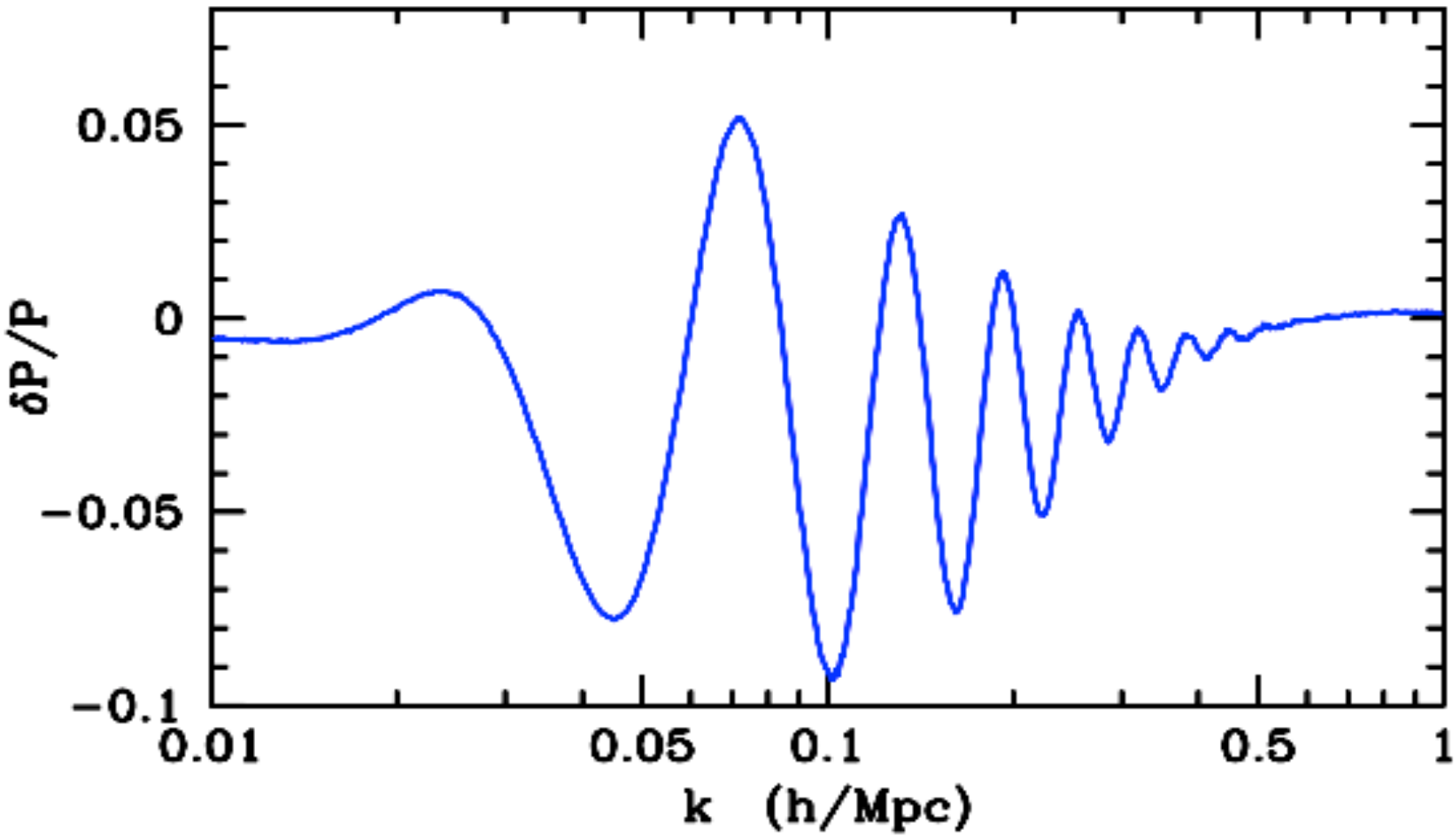
- **Note:** all of the matter sees the acoustic oscillations, not just the baryons.

Baryon (acoustic) oscillations



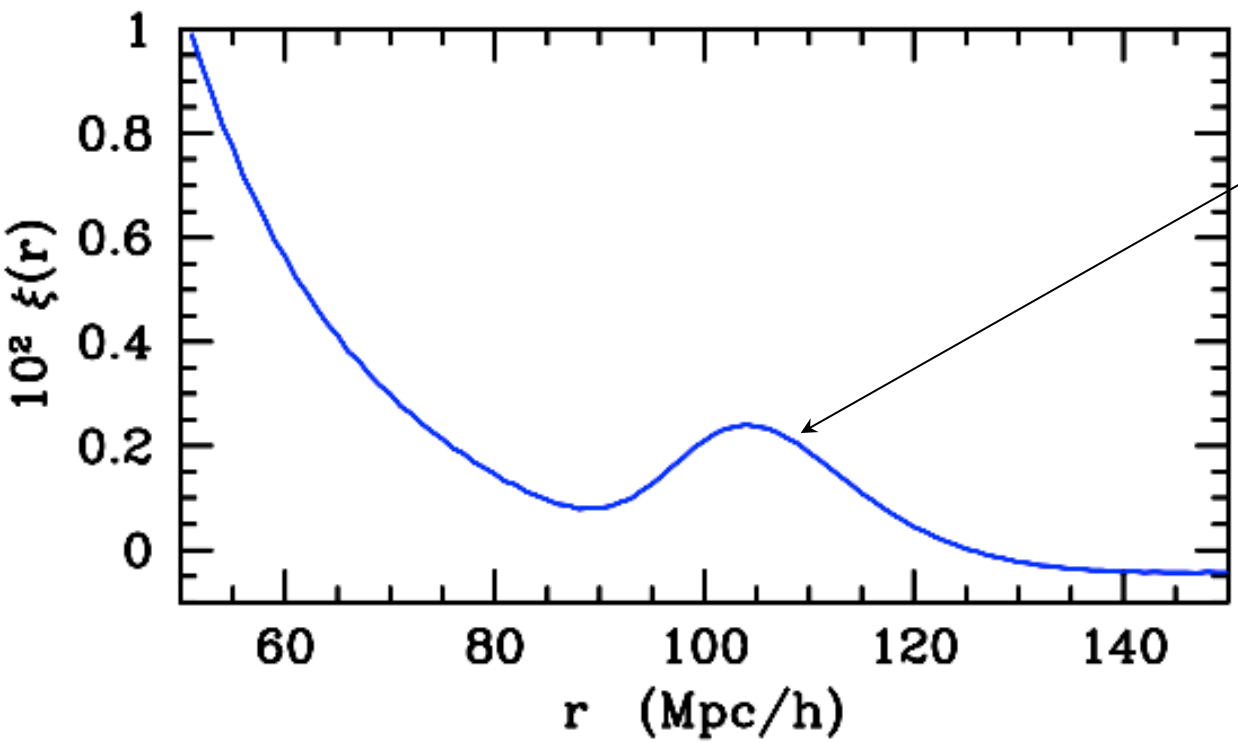
Divide out the gross trend …

A damped, almost harmonic sequence of “wiggles” in the power spectrum of the mass perturbations of amplitude *O*(10%).



In configuration space

- The configuration space picture offers some important insights.
- In configuration space we measure not power spectra but correlation functions: $\xi(r)=\int P(k)e^{ikr}d^3k=\int \Delta^2(k)j_0(kr)\, d\ln k$..
- A harmonic sequence would be a δ-function in r, the shift in frequency and diffusion damping broaden the feature.



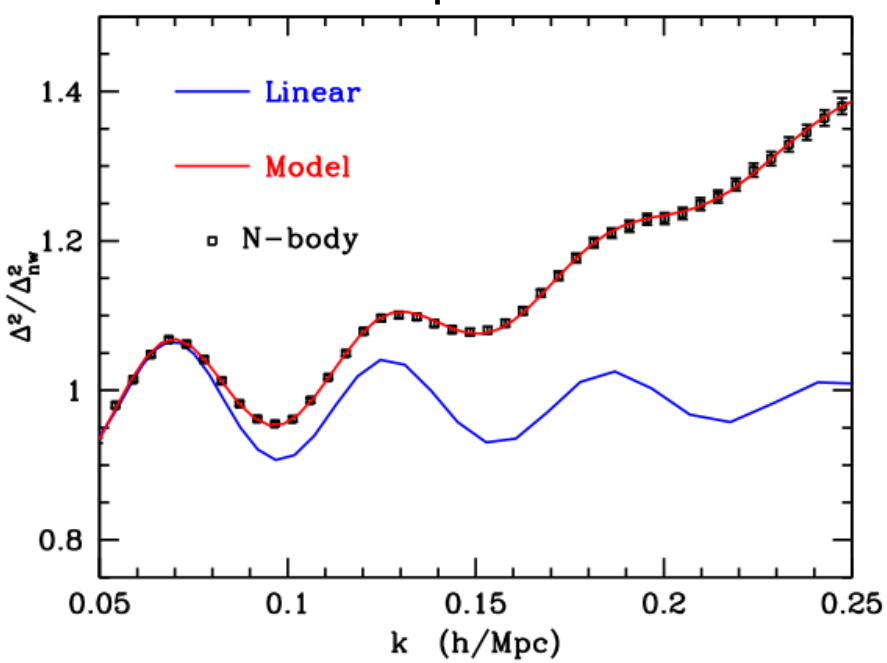
Acoustic feature at ~100 Mpc/h with width ~10Mpc/h (Silk scale)

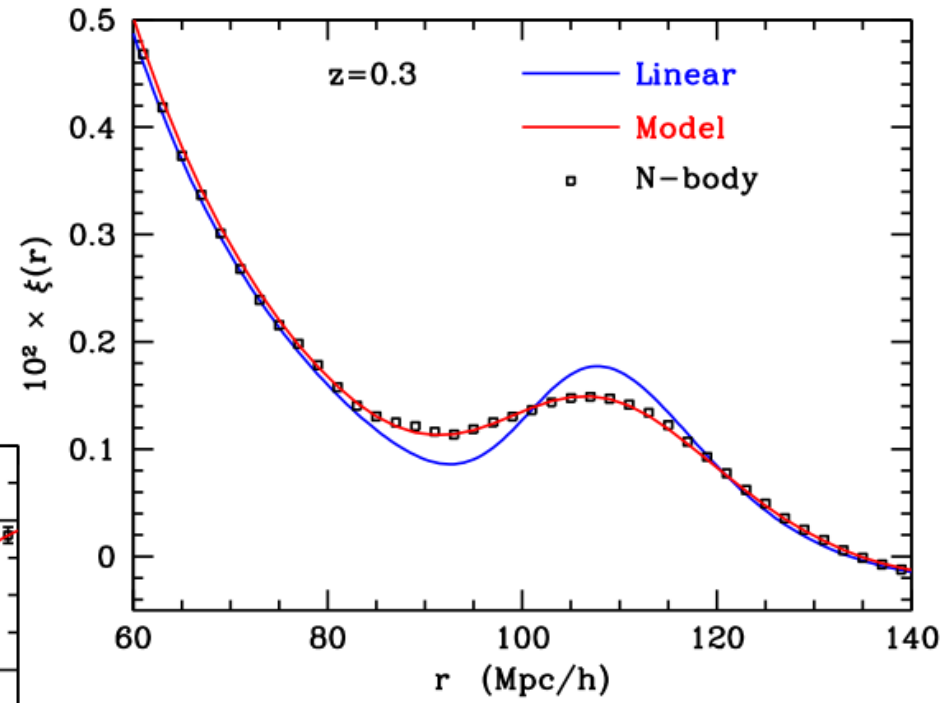
Effects of non-linearity on BAO

- Non-linear evolution has 3 effects on the power spectrum:
  - It generates “excess” high *k* power, reducing the contrast of the wiggles.
  - It damps the oscillations.
  - It generates an out-of-phase component.
- In configuration space:
  - Generates “excess” small-scale power.
  - Broadens the peak.
  - Shifts the peak.

Non-linearities smear the peak

Loss of contrast and excess power from non-linear collapse.





Broadening of feature due to Gaussian smoothing and ~0.5% shift due to mode coupling.

Understanding "shifts"

- We want to fit for the position of the acoustic feature while allowing for variations in the broadband shape (due e.g. to biasing).
  - $P_{fit}(k) = B(k)\, P_w(k,\alpha) + A(k)$
  - $B(k)$ and $A(k)$ are smooth functions.
    - Can take B(k)=const and A(k) as a spline, polynomial, Pade, ...
  - α measures shift relative to "fiducial" cosmology.
  - $P_w(k,\alpha)$ is a template.
    - Numerous arguments suggest $P_w(k,\alpha)=\exp[-k^2\Sigma^2/2]P_L(k/\alpha)$.
    - Take Σ *to be a free parameter, perhaps with a prior.*
- How does this do?

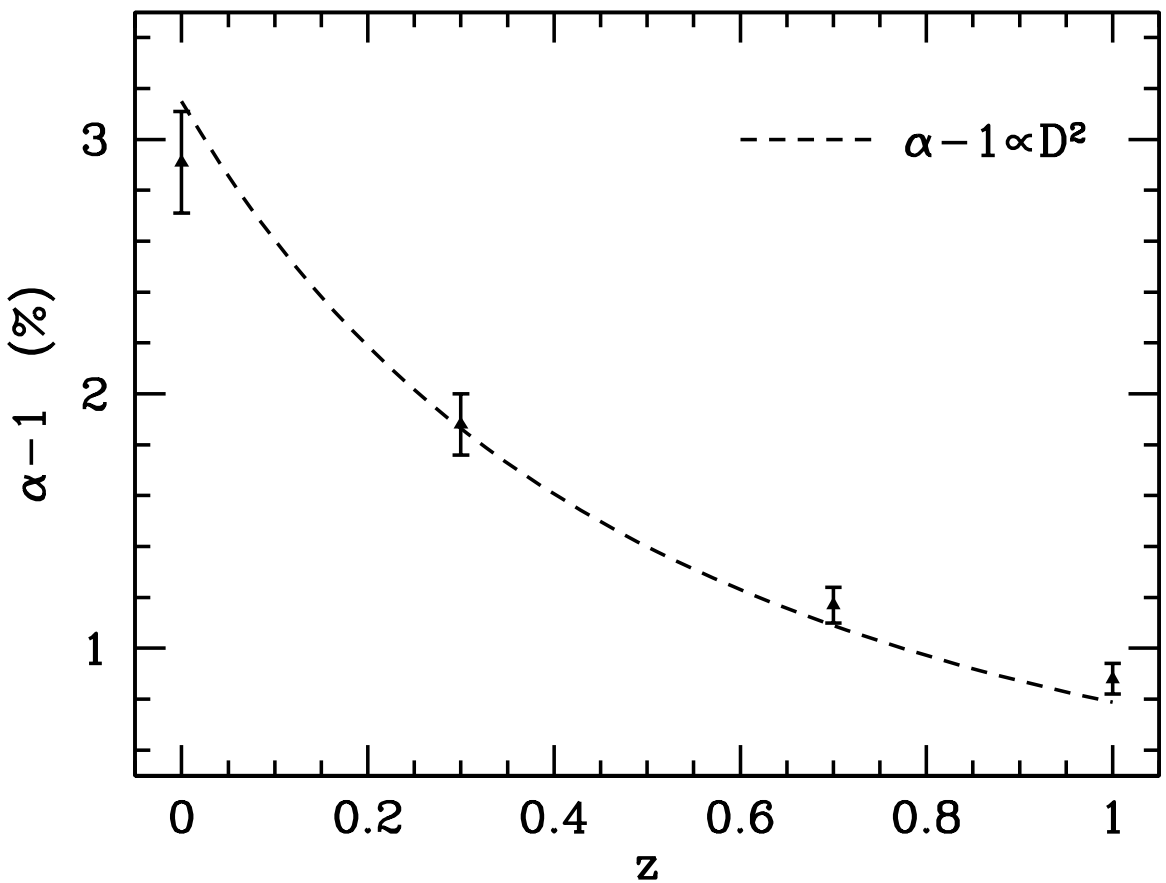
Argument from
Padmanabhan & White (2009);
see also Smith++08.

Measuring shifts in cCDM

- Any "shift" in the acoustic scale is small in ΛCDM, and therefore hard to study.
- Work with a "crazy" cosmology
  - Ω_m=1, Ω_B=0.4, h=0.5, n=1, σ_8=1.
  - Sound horizon 50h^{-1}Mpc, not 100h^{-1}Mpc.
- The fitted shifts are (α-1 in percent):

z	DM	$x\delta_L$	w/P_{22}
0.0	2.91 ± 0.20	-0.2 ±0.1	-0.03 ± 0.16
0.3	1.88 ± 0.12	-0.2 ±0.1	-0.38 ± 0.09
0.7	1.17 ± 0.07	-0.1 ±0.1	-0.12 ± 0.05
1.0	0.88 ± 0.06	-0.1 ±0.1	-0.04 ± 0.04

Shifts vs time



Amplitude of the shift vs. time (redshift) for the mass.

Shifts are consistent with D^2 scaling (dotted) suggesting an origin from 2nd order terms …

Where do the shifts come from?

Recall in PT we can write $\delta=\delta^{(1)}+\delta^{(2)}+\ldots$ or
$P = \{P_{11} + P_{13} + P_{15} + \ldots\} + \{P_{22} + \ldots\} = P_{1n} + P_{mn}$.
We can isolate these two types of terms by considering the cross-spectrum of the final with the initial field, which doesn't contain P_{mn}.

z	DM	$x\delta_L$	w/P_{22}
0.0	2.91 ± 0.20	-0.2 ±0.1	-0.03 ± 0.16
0.3	1.88 ± 0.12	-0.2 ±0.1	-0.38 ± 0.09
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1.0	0.88 ± 0.06	-0.1 ±0.1	-0.04 ± 0.04

Shifts in the cross-spectrum are an order of magnitude smaller than shifts in the auto-spectrum!

$$P_{1n}(k) \sim P_L(k) \int \prod_k \left[d^3 q_k P_L(q_k)\right] F_n(\cdots)$$

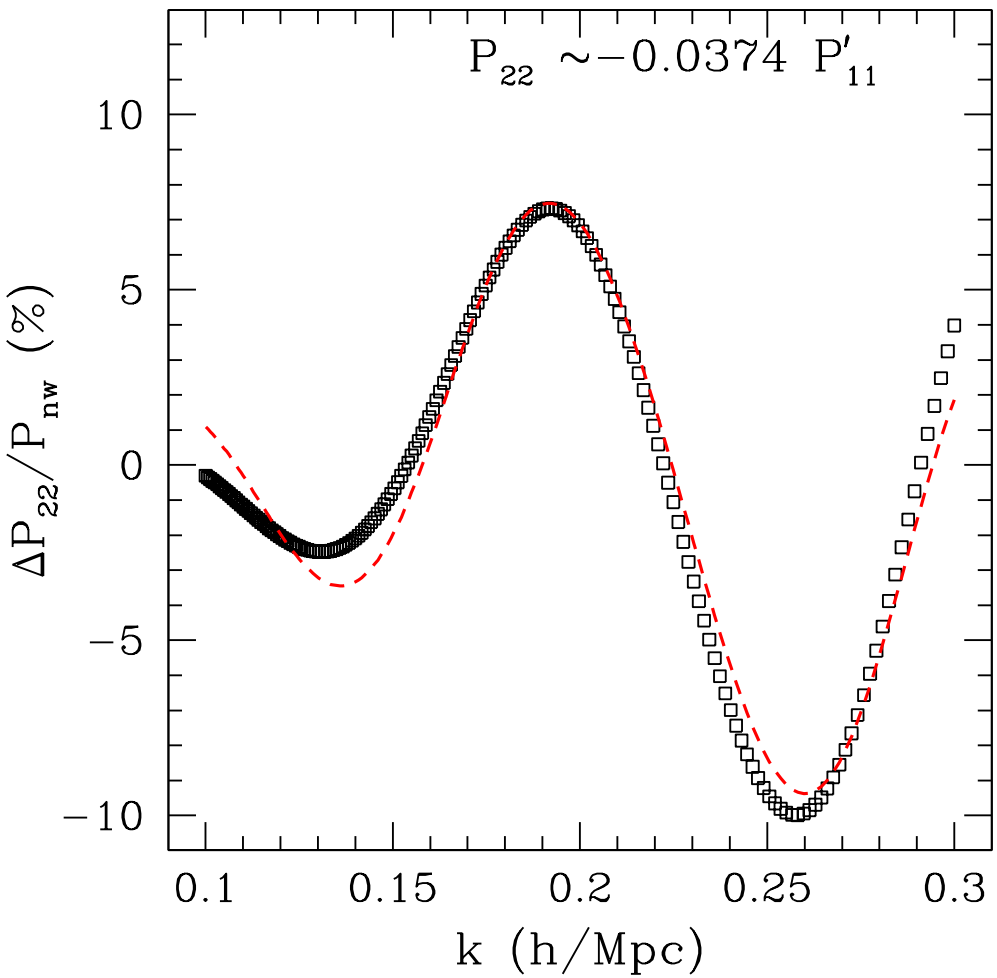
Broad kernel suppresses oscillations.

Mode-coupling terms

Recall in PT we can write $\delta=\delta^{(1)}+\delta^{(2)}+\ldots$ or
$P = \{P_{11} + P_{13} + P_{15} + \ldots\} + \{P_{22} + \ldots\} = P_{1n} + P_{mn}$.

- The P_{1n} terms are benign.
- By contrast the P_{mn} terms involve integrals of products of P_Ls times peaked kernels.
- Example: $P_{22} \sim \int P_L P_L\, F_2$ and F_2 is sharply peaked around $q_1 \approx q_2 \approx k/2$.
- Thus the $\int P_L P_L$ term contains an out-of-phase oscillation
  - $P_L \sim \ldots + \sin(kr)$: $P_L P_L F_2 \sim \sin^2(kr/2) \sim 1+\cos(kr)$
- Since $\cos(x) \sim d/dx \sin(x)$ this gives a "shift" in the peak
  - $P(k/\alpha) \sim P(k) - (\alpha-1)\, dP/d\ln k + \ldots$

Mode-coupling approximates derivative



Up to an overall factor the mode-coupling term, P_{22}, is well approximated by $dP_L/d\ln k$.

Modified template

- This discussion suggests a modified template, which has just as many free parameters as our old template:

$$P_{\rm w}(k,\alpha) = \exp\left(-\frac{k^2\Sigma^2}{2}\right) P_L(k/\alpha) + \exp\left(-\frac{k^2\Sigma_1^2}{2}\right) P_{22}(k/\alpha)\,.$$

- This removes most of the shift.

z	DM	$x\delta_L$	w/P_{22}
0.0	2.91 ± 0.20	-0.2 ±0.1	-0.03 ± 0.16
0.3	1.88 ± 0.12	-0.2 ±0.1	-0.38 ± 0.09
0.7	1.17 ± 0.07	-0.1 ±0.1	-0.12 ± 0.05
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Padmanabhan & White (2009)

Biased tracers?

- In order to remove the shift we needed to know the relative amplitude of P_{11} and P_{22}.
  - For the mass, this is known.
- What do we do for biased tracers?
  - Eulerian bias

$$P_h = \left(b_1^E\right)^2 (P_{11} + P_{22}) + b_1^E b_2^E \left(\frac{3}{7} Q_8 + Q_9\right) + \frac{(b_2^E)^2}{2} Q_{13} + \cdots$$

  - Lagrangian bias

$$\begin{aligned} P_h &= \exp\left[-\frac{k^2\Sigma^2}{2}\right] \left\{ \left(1 + b_1^L\right)^2 P_{11} + P_{22} + b_1^L \left[\frac{6}{7} Q_5 + 2Q_7\right] + b_2^L \left[\frac{3}{7} Q_8 + Q_9\right] \right. \\ &+ \left. \left(b_1^L\right)^2 [Q_9 + Q_{11}] + 2 b_1^L b_2^L Q_{12} + \frac{1}{2} \left(b_2^L\right)^2 Q_{13} \right\} + \cdots \end{aligned}$$

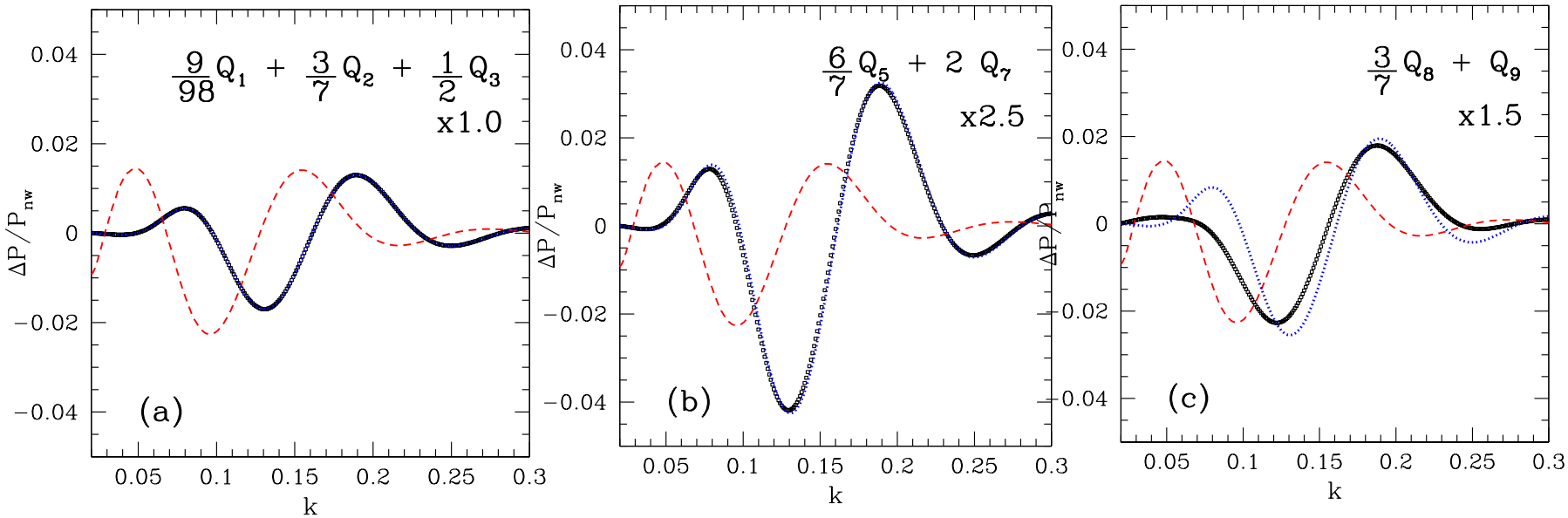
Mode-coupling integrals

$$Q_n(k) = \frac{k^3}{4\pi^2}\int_0^\infty dr\, P_L(kr)\int_{-1}^{1} dx\, P_L(k\sqrt{1+r^2-2rx})\widetilde{Q}_n(r,x)$$

$$\widetilde{Q}_1 = \frac{r^2(1-x^2)^2}{y^2}, \quad \widetilde{Q}_2 = \frac{(1-x^2)rx(1-rx)}{y^2},$$
$$\widetilde{Q}_3 = \frac{x^2(1-rx)^2}{y^2}, \quad \widetilde{Q}_4 = \frac{1-x^2}{y^2},$$
$$\widetilde{Q}_5 = \frac{rx(1-x^2)}{y}, \quad \widetilde{Q}_6 = \frac{(1-3rx)(1-x^2)}{y},$$
$$\widetilde{Q}_7 = \frac{x^2(1-rx)}{y}, \quad \widetilde{Q}_8 = \frac{r^2(1-x^2)}{y},$$
$$\widetilde{Q}_9 = \frac{rx(1-rx)}{y}, \quad \widetilde{Q}_{10} = 1-x^2,$$
$$\widetilde{Q}_{11} = x^2,\ \widetilde{Q}_{12} = rx,\ \widetilde{Q}_{13} = r^2$$

(Matsubara 2008)

Out-of-phase?

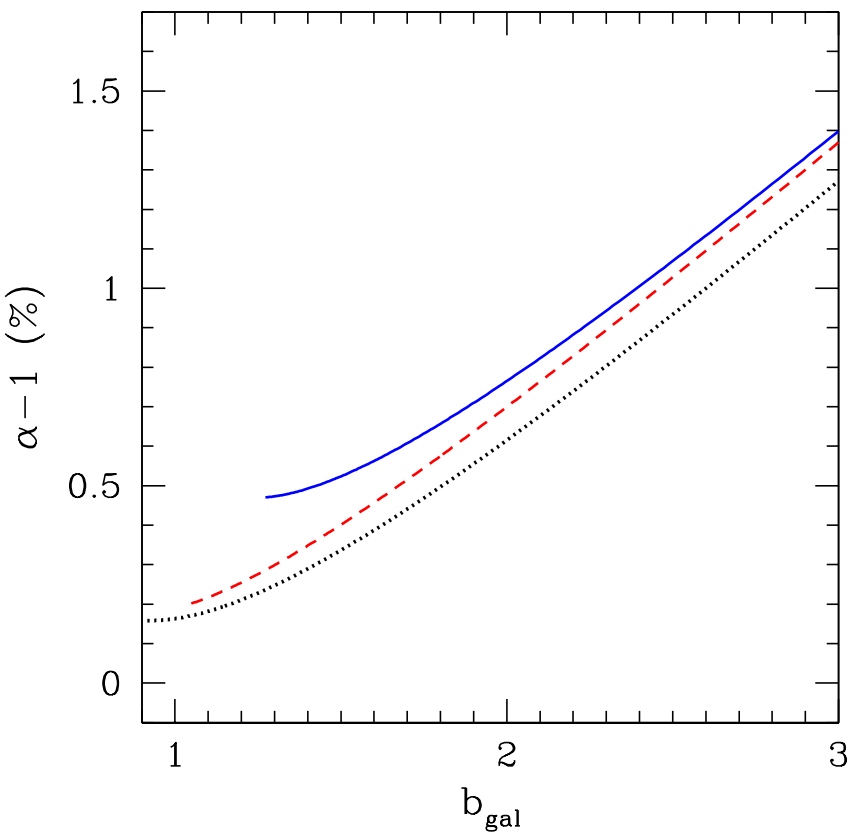


The numerous combinations that come in are also well approximated by the (log-)derivative of P_{11}! All of these terms can be effectively written as:

$$P_h = \exp\left(-\frac{k^2\Sigma^2}{2}\right)\left[\mathcal{B}_1 P_L + \mathcal{B}_2 P_{22}\right].$$

Size of the shifts?

- Simple model explains B_1-B_2 relation.
  - True for a variety of cosmologies, including ΛCDM.
  - Can also be measured from simulations (using some tricks).
- For ΛCDM the shifts are:
  - α-1~0.5% x D^2 x B_2/B_1



Shifts at z=0 for

Halos of mass M
Halos above M
N~[1+M/M_1]

At higher z the shift decreases as D^2.

Recall, the final error in BAO scale is the *uncertainty* in this correction, not the size of the correction itself!

Redshift space

- In resummed LPT we can also consider the redshift space power spectrum for biased tracers.
- For the isotropic P(k) find a similar story though now the scaling coefficients depend on $f \sim dD/d\ln a$.
  - Expressions become more complex, but the structure is unchanged.
- The amplitude of the shift increases slightly.

Perturbation theory & BAO

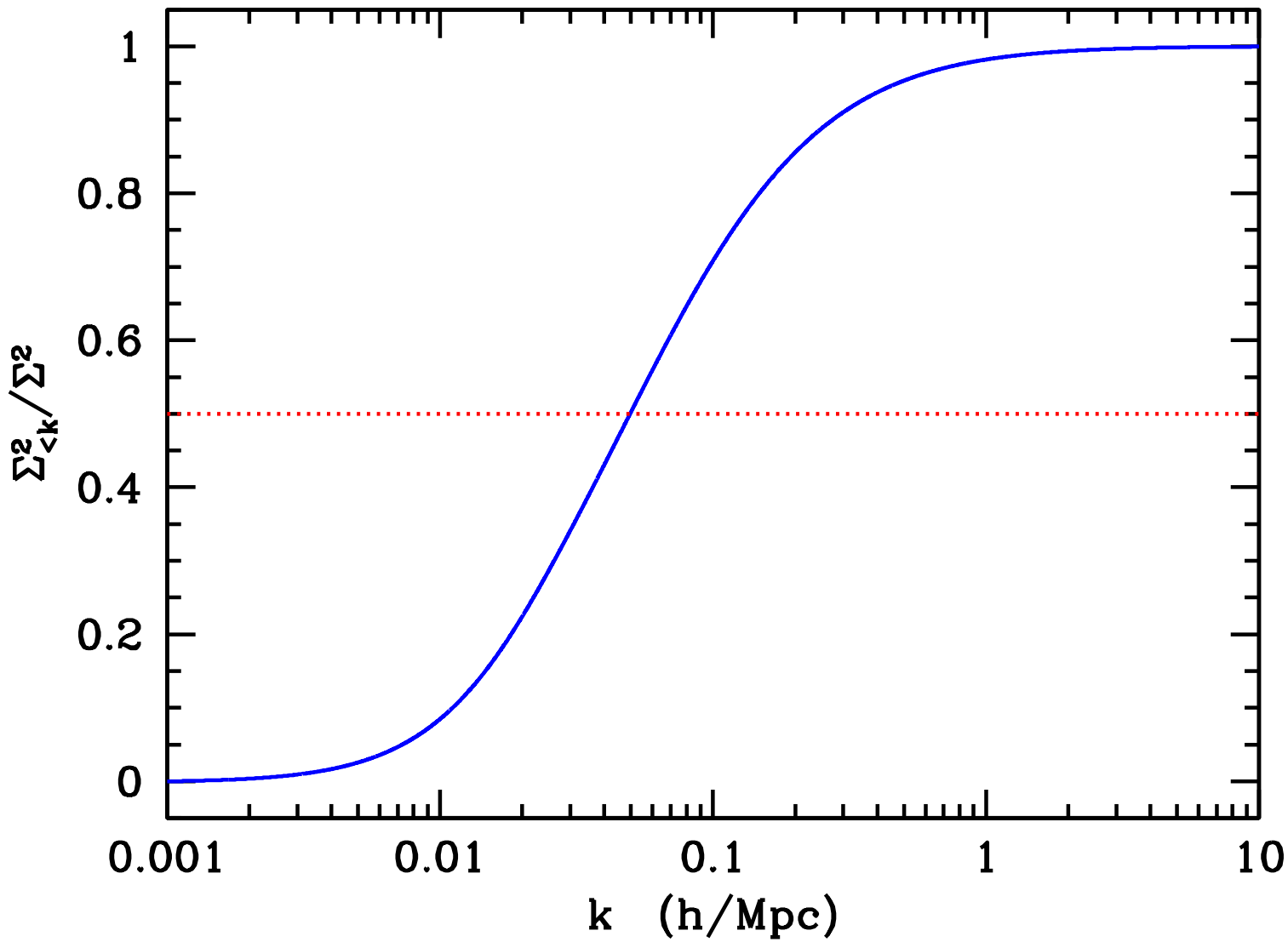
- Meiksin, White & Peacock, 1999
  - Baryonic signatures in large-scale structure (SPT)
- Crocce & Scoccimarro, 2007
  - Nonlinear Evolution of Baryon Acoustic Oscillations
- Matsubara, 2008ab
- Jeong & Komatsu, 2006, 2009
  - Perturbation theory reloaded I & II
- Pietroni, 2008; Lesgourgues et al. 2009; Anselmi et al. 2010; Elia et al. 2010
  - Flowing with time, resummation schemes.
- Padmanabhan & White 2009; Padmanabhan et al., 2009; Noh et al. 2009
  - Calibrating the baryon oscillation ruler for matter and halos
  - Reconstructing baryon oscillations: A Lagrangian theory perspective
  - Reconstructing baryon oscillations.
- Nishimichi et al., 2007, 2010; Taruya et al., 2009, 2010.
  - Characteristic scales of BAO from perturbation theory
  - Non-linear Evolution of Baryon Acoustic Oscillations from Improved Perturbation Theory in Real and Redshift Spaces

Reconstruction
an analytic understanding?

Reconstruction and LPT

- Recall that the effect of non-linearity was to broaden (and slightly shift) the acoustic peak.
- The broadening was equal to the Zel'dovich displacement.
  - Much of the broadening comes from large scales.
- Since those scales are measured by the survey, one could hope to "reconstruct" the initial, unbroadened feature.
  - Eisenstein, Seo, Sirko & Spergel (2007).
- What does this procedure do?
  - Lagrangian perturbation theory is almost perfectly suited to studying reconstruction.

Contributions to the displacement

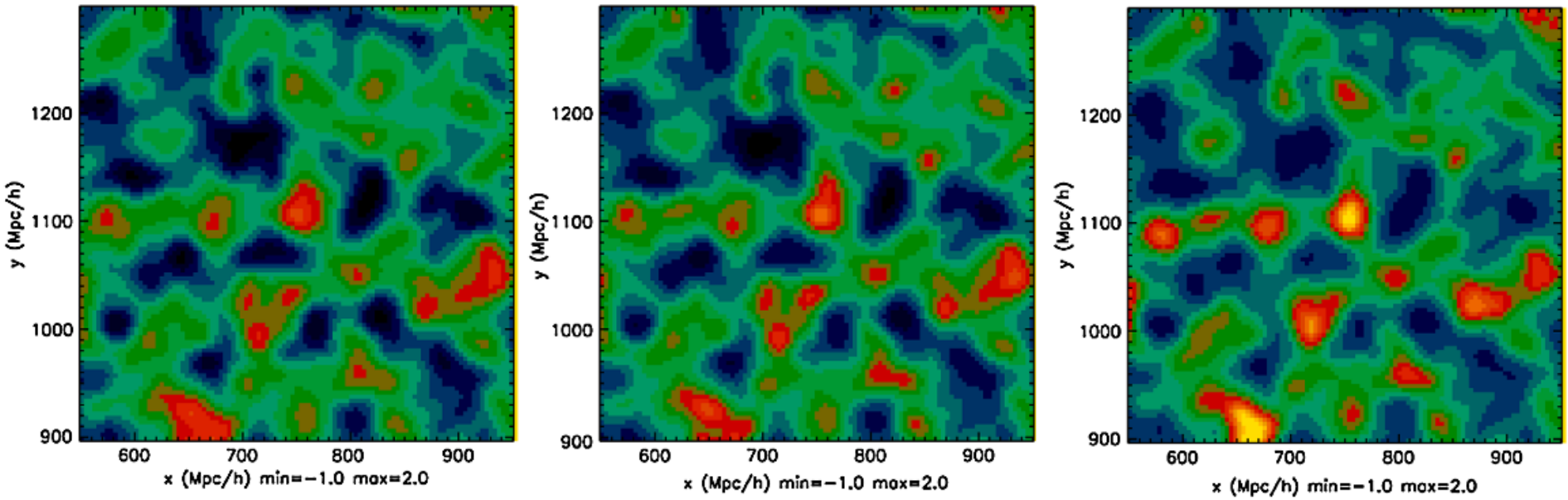


Reconstruction procedure

1. Smooth the density field
   - $\delta(k) \rightarrow \delta(k)\, S(k)$
2. Compute the negative Zel'dovich displacement, s, from the smooth field.
   - $\boldsymbol{s}(k) = (-i\boldsymbol{k}/k^2)\, S(k)\, \delta(k)$
3. Shift particles by s to generate "displaced" field, δ_d.
   - In linear theory $\delta_d=0$.
4. Shift spatially uniform grid of points by s to give "shifted" field, δ_s.
   - In linear theory $\delta_s=-\delta_d$.
5. Define $\delta_r=\delta_d-\delta_s$ (equals δ in linear theory).
6. Note: $S \rightarrow 0$ is equivalent to no reconstruction.

In pictures

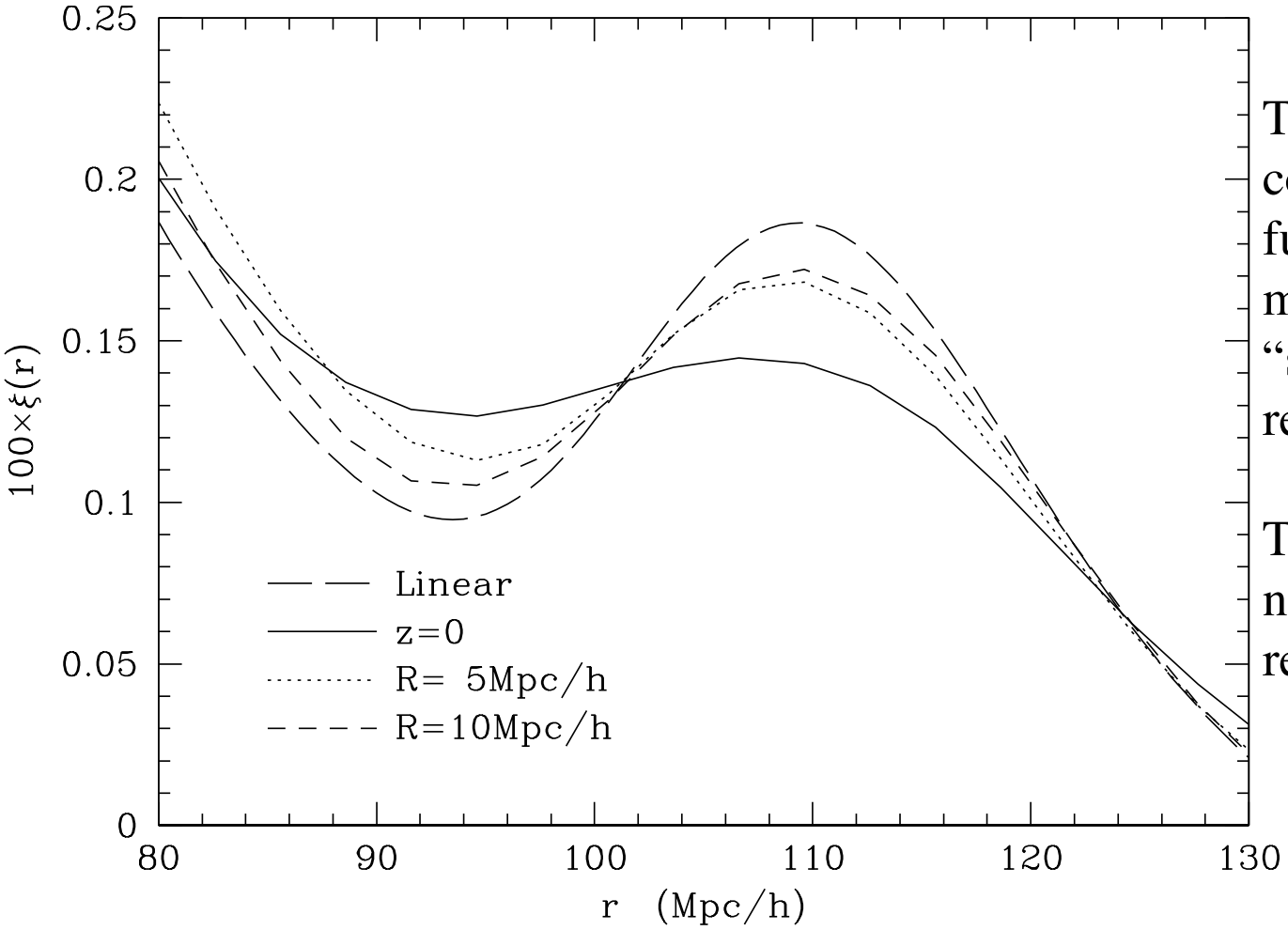
Noh++09



Initial Recon Final/NL

Note: the final field has sharper, more pronounced peaks than either the initial or reconstructed density fields.

Sharpens the peak



The z=0 correlation function of the mass in ΛCDM is "sharpened" by reconstruction.

The linear field is not fully recovered.

LPT

- Recall in LPT $\delta(\mathbf{k}) = \int d^3q\ e^{-i\mathbf{k}\cdot\mathbf{q}} \left(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} - 1\right)$
- The displaced field is generated by **Ψ+s**
- The shifted field is generated by **s**.
- To lowest order $\delta_r = \delta_L$.
- To next order

$$\begin{aligned}
\delta_r^{(2)} &= \delta^{(2)} - \frac{1}{2}\int \frac{d^3k_1 d^3k_2}{(2\pi)^3}\delta^{(D)}\left(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}\right) \\
&\times \quad \delta_l(\mathbf{k}_1)\delta_l(\mathbf{k}_2)\ \mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_1)\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_2) \\
&\times \quad [\mathcal{S}(\mathbf{k}_1) + \mathcal{S}(\mathbf{k}_2)]
\end{aligned}$$

Why does reconstruction help?

A toy model

- Imagine $\Psi=\Psi_L+\Psi_H$ both Gaussian and uncorrelated.
  - Ψ_L is generated by δ_{lin},
  - Ψ_H contains no BAO.

$$P(k) = \int d^3q e^{-i\mathbf{k}\cdot\mathbf{q}} \left(\left\langle e^{-ik_i \Delta\Psi_i(\mathbf{q})} \right\rangle - 1 \right)$$

$$\left\langle e^{-i\mathbf{k}\cdot\Delta\mathbf{\Psi}(q)} \right\rangle = \exp\left[-\frac{1}{2} k_i k_j \left\langle \Delta\Psi_i(\mathbf{q}) \Delta\Psi_j(\mathbf{q}) \right\rangle \right]$$

$$k_i k_j \left\langle \Delta\Psi_i(\mathbf{q}) \Delta\Psi_j(\mathbf{q}) \right\rangle = 2k_i^2 \langle \Psi_i^2(\mathbf{0}) \rangle - 2k_i k_j \xi_{ij}(\mathbf{q})$$

A toy model

- $\xi_{ij}(0)=(\delta_{ij}/2)\,\Sigma^2$, and $\Sigma^2 \approx \Sigma_L^{\,2}$

$$P(k) = e^{-k^2\Sigma_L^2/2} \int d^3q\; e^{-ik_iq_i}\;\; e^{k_ik_j\xi_{ij}(q)}\,.$$

- Leave zero-lag piece exponentiated:

$$P_{\rm obs}(k) = e^{-\frac{1}{2}k^2\Sigma_L^2} P_L(k) + P_{\rm mc}(k) + \cdots$$

$O(\Psi_{\rm H}^{\,2})$ and $O(\Psi_{\rm L}^{\,4})$

- Now $\mathbf{s}(k)=-S(k)\Psi_L(k)$, so the displaced and shifted fields are generated by $[1-S]\Psi_L+\Psi_H$ and $-S\Psi_L$.

A toy model

- The reconstructed power spectrum is
  - $P_r=(\delta_s-\delta_d)^2=P_{ss}+P_{dd}-2P_{sd}$
- with:
  - $P_{ss}=\exp[-k^2\Sigma_{ss}^2/2]S^2(k)\,P_L(k)+\ldots$
  - $P_{dd}=\exp[-k^2\Sigma_{dd}^2/2][1-S(k)\,]^2P_L(k)+\ldots$
  - etc.
- And modified damping terms (e.g.):

$$\Sigma_{ss}^2 = \frac{1}{3\pi^2}\int dp\ \mathcal{S}^2(p)P_L(p)$$

- The effect of the S and [1-S] terms and the structure of the damping is to "effectively" reduce Σ to ~0.5 Σ.

LPT

- A very similar calculation carries through in the full LPT, except you have to keep more terms in the exponential if things aren't all Gaussian.
- The damping turns out to be the same.
  - We were working to lowest order in Σ, so this is not surprising.
- You additionally get the mode-coupling terms.
  - Slightly painful since you need to redo all of Matsubara with 3 different spectra.
- Find that the mode-coupling term is suppressed.

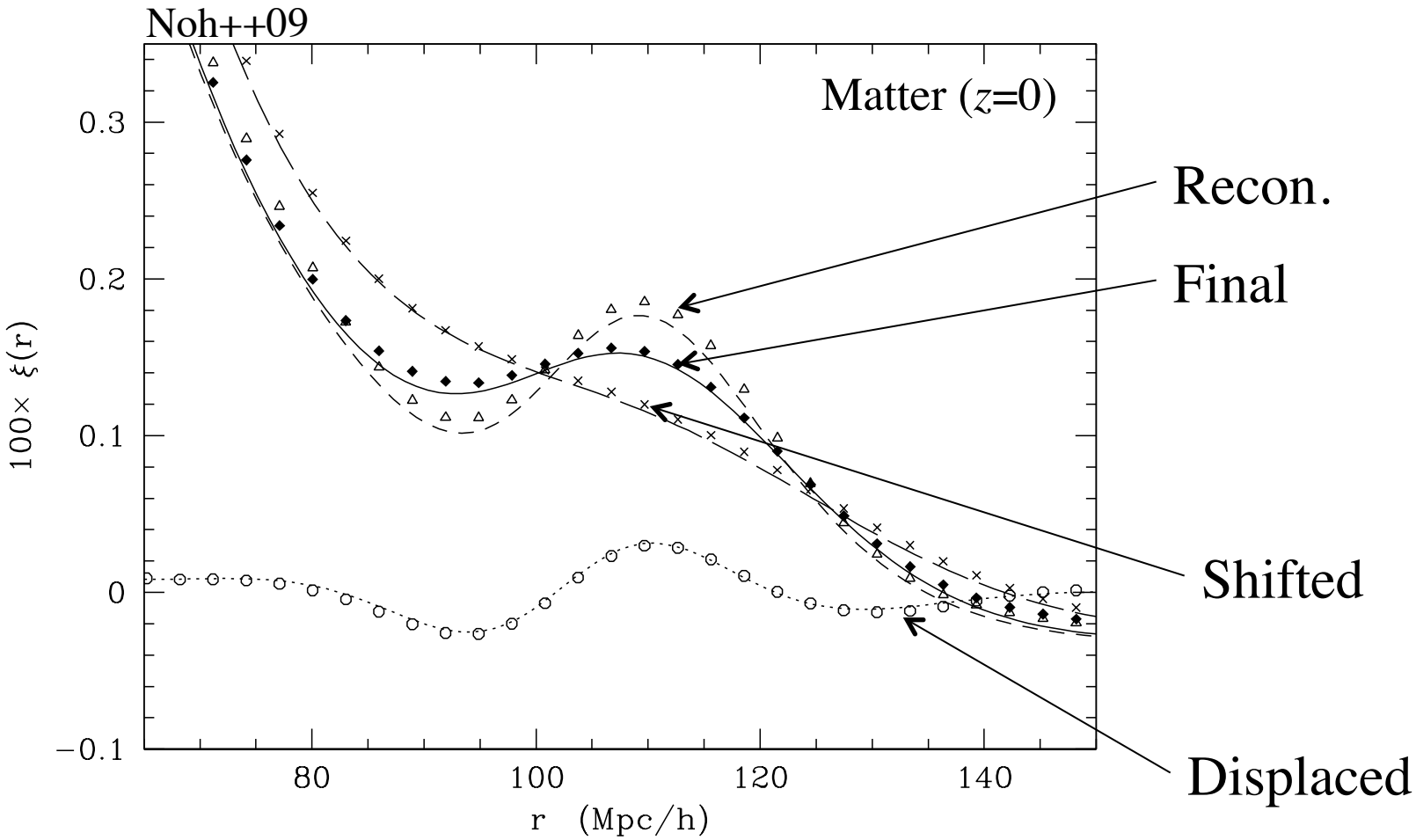
The details

$$
\begin{aligned}
P^{dd} \propto\ & P_L\bar{\mathcal{S}}^2 + \frac{9}{98}Q_1 + \frac{3}{7}Q_2^{(1d1d)} + \frac{1}{2}Q_3^{(dddd)} \\
+\ & \bar{\mathcal{S}}\left[\frac{10}{21}R_1 + \frac{6}{7}R_2^{(d)}\right] \\
+\ & \langle F'\rangle\left[2P_L\bar{\mathcal{S}} + \frac{6}{7}Q_5^{(1d11)} + \boxed{2Q_7^{(1ddd)}} + \right. \\
& \left. \frac{10}{21}R_1 + \frac{6}{7}R_2^{(d)} + \frac{6}{7}\bar{\mathcal{S}}(R_1 + R_2)\right] \\
+\ & \langle F''\rangle\left[\frac{3}{7}Q_8 + Q_9^{(1d1d)}\right] \\
+\ & \langle F'\rangle^2\left[P_L + \frac{6}{7}(R_1 + R_2) + Q_9^{(1d1d)} + Q_{11}^{(11dd)}\right] \\
+\ & 2\langle F'\rangle\langle F''\rangle Q_{12}^{(111d)} + \frac{1}{2}\langle F''\rangle^2 Q_{13} \qquad (1)
\end{aligned}
$$

Noh++09; based on Matsubara 07 & 08

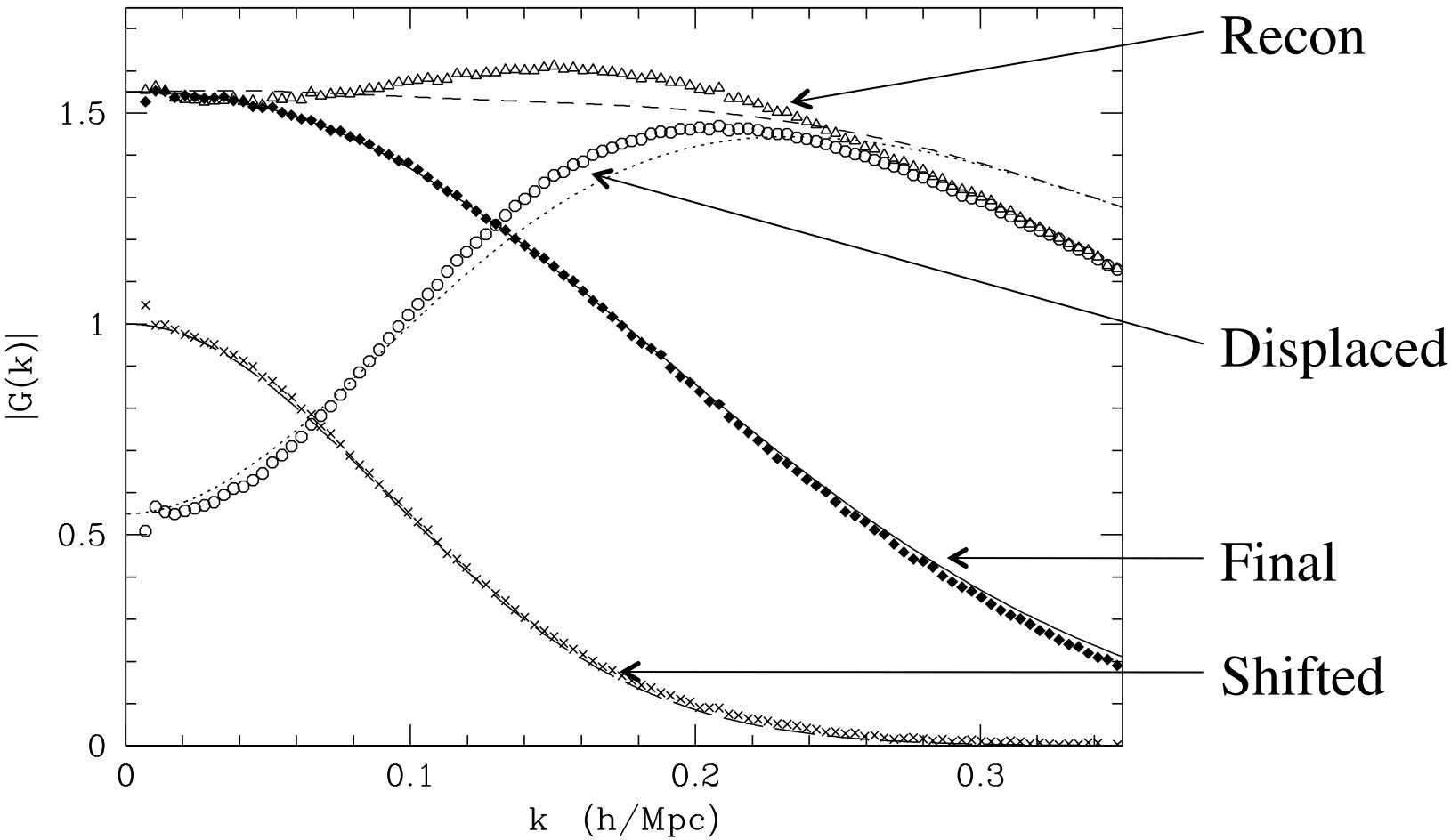
$$
Q_7^{(1ddd)}(k) = \frac{k^3}{(2\pi)^2}\int_0^\infty dr\ P_L(kr)\bar{\mathcal{S}}(kr)\int_{-1}^{+1} d\mu\ P_L(ky)\bar{\mathcal{S}}(ky)\bar{\mathcal{S}}(ky)\widetilde{Q}_7(r,\mu)
$$

LPT agrees with simulations



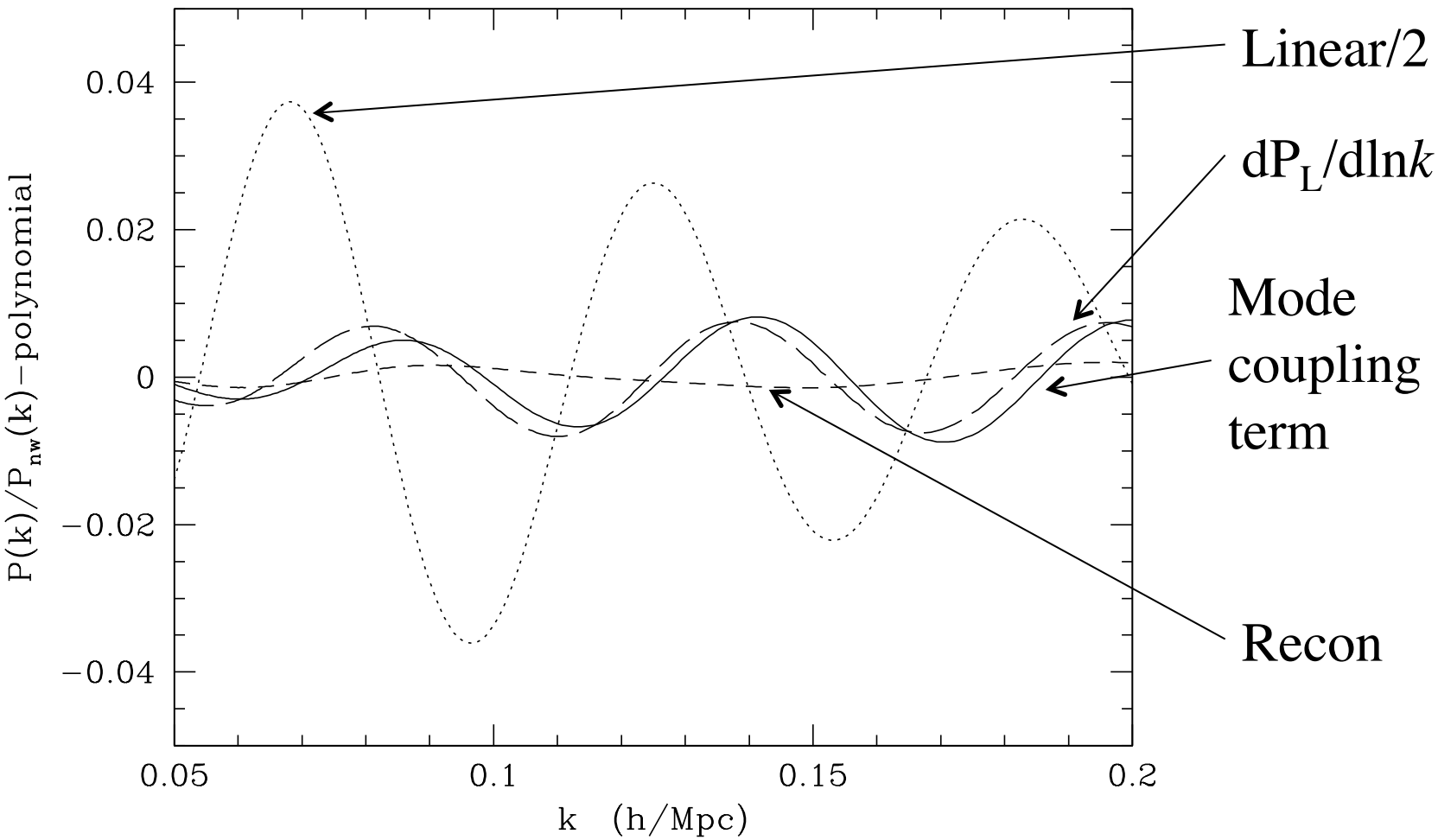
Coherence regained

Noh++09



The cross-correlation between the initial field and the other fields for halos above 10^{13}.

Out-of-phase term reduced



Out-of-phase terms in P(k) for halos more massive than 10^{13}.

Effects of shot-noise

- Within the LPT formalism the effects of shot-noise from finite galaxy number density are easy to include.
- The largest effect is a change in the damping scale:

$$\Sigma^2_{ss} \quad \rightarrow \quad \frac{1}{3\pi^2}\int dp\ \mathcal{S}^2(p)\left[P_L(p)+P_N(p)\right]$$

$$\Sigma^2_{dd} \quad \rightarrow \quad \frac{1}{3\pi^2}\int dp\ \left[1-\mathcal{S}(p)\right]^2 P_L(p)+\mathcal{S}^2(p)P_N(p)\,,$$

- where $P_N=1/(b^2 n)$ is the shot-noise power.
- Gains saturate around $n\sim10^{-4}\,(h/\mathrm{Mpc})^3$.

White (2010)