Cosmological Scale Tests of Gravity

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References

Caldwell & Kamionkowski 0903.0866
Silvestri & Trodden 0904.0024
Ferreira & Starkman 2009 Science
Uzan 0908.2243
Jain & Khoury 1004.3294
## Cosmological Measurements

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Strongest tests: solar-system scales

In modified gravity, metric outside a spherical mass distribution is usually NOT Schwarzschild

Parametrized Post-Newtonian metric

\[ ds^2 = -(1+2\Phi_N + 2\beta\Phi_N^2)dt^2 + (1-2\gamma_{PPN}\Phi_N)(dx^2 + dy^2 + dz^2) \]

Eddington 1922; Robertson 1962; Thorne and Will 1971.
\[ \Phi_N = -GM/r \]
\[ \text{GR: } \beta = \gamma_{PPN} = 1 \]

Brans-Dicke: \[ \beta = 1, \quad \gamma_{PPN} = (1+\omega)/(2+\omega) \rightarrow 1 - 1/\omega + \ldots \]
Quantitative tests of GR on solar system scales

$|\gamma_{PPN} - 1| < 2 \times 10^{-5}$ on length scales of $10^{-4}$ pc

Cassini spacecraft Shapiro time delay
(Bertotti et al. 2003)

$\omega > 40000$
Quantitative test of GR on galactic scales from gravitational lensing

Schwab et al.
arXiv/0907.4992
Gravitational deflection of light using galaxy-galaxy strong lenses

$\gamma_{PPN} \approx 1$ on length scales of 1 kpc

Figure 1. Constraint on $\gamma$ determined using the method discussed in the text. The gray curves represent the posterior PDF for $\gamma$ from each lens system. The black curve is the joint posterior PDF for all systems, the normalized product of the gray curves. (In this plot the joint PDF is scaled by a factor of one-half so as to relatively enhance the scale of the individual gray PDF curves.) A Gaussian fit to the joint posterior PDF gives $\gamma = 1.01 \pm 0.05$. 
Is this consistent with modified gravity?

Solar-system gravity may depend on Galactic or Cosmological “boundary conditions”


Scalar-tensor and f(R) theories appear viable (e.g. [Hu & Sawicki astro-ph/0705.1158](https://arxiv.org/abs/0705.1158) and many others)
Recap of Lecture 2

Alternative theories of gravity are higher order; they contain at least one additional scalar field.

Generically, gravitational “constant” varies e.g. in $f(R)$ theories

$$G_N \rightarrow \frac{G_N}{1 + f_R}$$

Nonzero “gravitational slip” $\psi - \Phi = \delta \ln (1 + f_R)$

These have consequences for structure formation, gravitational lensing and CMB
Growth of structure

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Gravitational lensing $\psi - \Phi = \delta \ln(1 + f_R)$
Growth of Structure

Weak-field limit, Jordan “frame”

Nonrelativistic dark matter and atoms

\[
\frac{dv}{dt} = -\nabla \Phi, \quad \nabla^2 \Phi = 4\pi G_{\text{eff}} \rho
\]

Growth of structure:

\[
\frac{d^2 \delta_m}{d\tau^2} + \mathcal{H} \frac{d\delta_m}{d\tau} = 4\pi G_{\text{eff}}(k, \tau) a^2 (\delta \rho_m + \delta \rho_{\text{DE}})
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MGCAMB Zhao et al. arXiv:0809.3791
Growth of Structure
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Growth of structure (now with proper t!):
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\frac{d^2 \delta_m}{dt^2} + 2H \frac{d\delta_m}{dt} = 4\pi G_{\text{eff}}(k, t) (\delta \rho_m + \delta \rho_{\text{DE}})
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MG or DE? Growth of structure cannot distinguish
Gravitational Lensing
Weak-field limit, Jordan “frame”

Deflection of light

\[ \frac{dn}{dt} = -\nabla_\perp (\Phi + \Psi), \quad \Psi - \Phi = \epsilon + 4\pi G_{\text{eff}} \Sigma_{\text{DE}} \]

Modified gravity

For small angular displacements

\[ \text{Image}(\vec{\theta}) \approx \text{Source}(M \cdot \vec{\theta}) \]

\[ M_{ij} = \delta_{ij} - \partial_i \partial_j \int (\Phi + \Psi) dz \quad (2\text{-d indices}) \]

Hoekstra & Jain arXiv:0805.0139
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Weak-field limit, Jordan “frame”

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Hoekstra & Jain arXiv:0805.0139
Testing gravity on cosmic scales

Key observables:

1. $H(a) = \frac{\mathcal{H}}{a}$ from SNe, BAO
2. $\delta_m(k,a)$ from redshift surveys, weak lensing
3. $\theta_m(k,a) = k^2 u_m$ from redshift-space distortion
4. $(\Phi+\Psi)(k,a)$ from weak lensing, ISW
Testing gravity on cosmic scales

Use peculiar velocities to determine $\Phi$:

$$\Phi(k, a) = \mathcal{H} \frac{\partial}{\partial a} (a u_m)$$

This does not assume GR is valid.

Check Poisson (DE or MG)
Calculate Slip $\Psi - \Phi$ (DE or MG)
An approach to testing GR

Assumption #1: DE = 0, only DM clusters

\[ \frac{d^2 \delta_m}{dt^2} + 2H \frac{d \delta_m}{dt} = 4\pi G_{\text{eff}}(k, t) \delta \rho_m \]

Null hypothesis: \( G_{\text{eff}}(k, t) = G_N \)

1. Measure \( D_+(a) \) on linear scales via power spectrum from galaxy surveys, weak lensing
2. Compare with Linder growth exponent in GR

\[ \beta(a) \equiv \frac{d \ln D_+}{d \ln a} = \Omega_m(a)^\gamma(a) \]

Linder astro-ph/0507263
An approach to testing GR

Assumption #1: DE = 0, only DM clusters

\[
\frac{dn}{dt} = -\nabla_\perp (\Phi + \Psi), \quad \Psi - \Phi = \varepsilon
\]

Null hypothesis: \(\varepsilon = 0\)

1. Measure weak-lensing shear correlation function
2. Compare with density perturbation growth or peculiar velocities

Zhang et al. 0704.1932
Comparison of observational constraints with predictions from general relativity and viable modified theories of gravity.


Zhang et al. 2007

\[
\langle E_G \rangle = \frac{\nabla^2 (\Phi + \Psi)}{3H_0^2 \beta \delta/a} = \Omega_m^{1-\gamma} \quad \text{in GR}
\]
Zhang et al statistic

$\Phi + \Psi$ from weak lensing shear

Density perturbations indirectly from divergence of peculiar velocity; peculiar velocities from “redshift-space distortion”

$$\dot{\delta} = -\theta = H \delta \Omega_m(a) \gamma$$

Important consideration: cancellation of systematic errors, galaxy bias
Other measurements

Integrated Sachs-Wolfe \( \frac{\Delta T}{T}(n) = \int (\Phi + \Psi)_{\text{ret}} \, dl \)

This ISW modification is smaller than cosmic variance but can be measured by cross-correlating CMB with galaxies.
Further tests of gravity on Gpc scales

Daniel et al. 1002.1962
Zhao et al. 1003.0001 add ISW-galaxy corr.
Daniel & Linder 1008.0397

These are extensive analyses of weak lensing, H(z) from supernovae, WMAP-5, ++

Future tests by Euclid, LSST, etc. important!
Open issues

What is the best parametrization of $G_{\text{eff}}(k,\tau)$ and gravitational slip $\Psi - \Phi$?

BZ 0801.2431 parametrized $(\Psi/\Phi)(k,a)$

See discussion by Pogosian et al 1002.2382
For more information

1. H. Motohashi poster: Tests of specific models, e.g. parametrized f(R) models

2. A. Silvestri talk and A. Hojjati poster: Data compression using Fisher matrix and Principal Component Analysis

3. Apologies to those I’ve missed!
Conclusions

The #2 goal of observational cosmology is to determine $\rho(a)$ by measuring $H(a)$ in the Friedmann equation.

The #3 goal of observational cosmology is to test general relativity on distance scales from kpc to Gpc.
Conclusions

The #1 goal is to learn and enjoy sharing the learning with each other, thanks to the hospitality of our wonderful conference organizers.

Thank you!