Gravity on Cosmic Scales #2

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Recap of Lecture 1

Goals of observational cosmology:

1. Determine $\rho(a)$ by measuring $H(a)$ and using Friedmann equation

3. Test General Relativity on distance scales from kpc to Gpc

To do both, one must measure perturbations, not just Hubble expansion

Linear CDM perturbation evolution

Commonly used: $\ddot{\delta}_m + \mathcal{H}\dot{\delta}_m = 4\pi G a^2 \bar{\rho}_m \delta_m$

Is it correct?

Yes, *if*:

1. δ_m = physical density perturbation in matter rest frame
2. Length scales >> Jeans, nonlinear
3. Λ only; dark energy is non-clustering

Exact solution by quadrature

Grav. Potential

$$\Psi_+(\tau) = \frac{\mathcal{H}}{a^2}\int^{\tau} \frac{\alpha(\tau')a^2(\tau')}{\mathcal{H}^2(\tau')}\,d\tau' \quad k^2 \ll k_{\mathrm{J}}^2$$

Total density perturbation in comoving gauge

$$\nu \propto D_+ = \frac{H}{a^3(\bar{\rho}+\bar{p})}\int^{a} \frac{(\bar{\rho}+\bar{p})\,da}{H^3}$$

Can $H(a) + D_+(a)$ test GR?

Determine 3 dark energy functions:

1. $\rho_{DE}(a)$
2. Entropy potential $\sigma_{DE}(a)$
3. Shear stress potential $\pi_{DE}(a)$

This is an underdetermined problem.

What about modified gravity? Give up 2 functions, acquire how many?

Cosmological Perturbations in Other Metric Theories

General references

f(R)	Sotiriou & Faraoni 0805.1726
	De Felice & Tsujikawa 1002.4928
Scalar-Tensor	Tsujikawa 1101.0191
TeVeS	Skordis 0903.3602

f(R) theories

with Alessandra Silvestri

Simplest generalization of GR

$$S[g_{\mu\nu},\psi] = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,[R+f(R)] + \int d^4x\,\sqrt{-g}\,\mathcal{L}_M(g_{\mu\nu},\psi)$$

Matter action unchanged so DM dynamics still driven by gravitational Potential Φ

Field equations from $\delta S/\delta g_{\mu\nu}=0$

Modified Einstein equations

$$(1+f_R)R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}(R+f)+(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)f_R = 8\pi G T_{\mu\nu}$$

where

$$f = -2\Lambda + c_1 R + \frac{1}{2}c_2 R^2 + \cdots \ , \quad f_R \equiv \frac{df}{dR}$$

Modified Friedmann equation (if K=0)

$$(1+f_R)H^2 + \frac{1}{6}f - \frac{1}{a^3}\frac{d^2a}{dt^2}f_R + \frac{H}{a}\frac{df_R}{dt} = \frac{8\pi}{3}G\bar{\rho}$$

Field equations are 4th order in time

$$(1+f_R)R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}(R+f)+(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)f_R=8\pi G T_{\mu\nu}$$

where f_R is a function of

$$\frac{1}{6}a^2R=\dot{\mathcal{H}}+\mathcal{H}^2-f_{RR}(\ddot{\Psi}+\cdots)$$

where $f_{RR}=d^2f/dR^2$.

New propagating degree of freedom:

Scalaron f_R (Starobinsky 1980)

Perturbations: Φ, Ψ, ε

Perturb Ricci and f_R about Robertson-Walker background

$$R^{\mu}{}_{\nu} \to R^{\mu}{}_{\nu}(\tau) + \delta R^{\mu}{}_{\nu}(\tau, x)$$

$$f_R \to f_R(\tau) + f_{RR}(\tau)\delta R \equiv f_R + \epsilon(\tau, x)$$

Perturbed Einstein equations (half of them)

Changing Newton constant

$$(1+f_R)(\Delta+3K)\Psi - \frac{3}{2}\dot{f}_R(\dot{\Psi}+\mathcal{H}\Phi)$$

$$-\frac{1}{2}\left(\Delta+3\dot{H}-3\mathcal{H}^2\right)\epsilon = \alpha\nu_{\rm tot}$$

GR

$$(1+f_R)(\Psi-\Phi) - \epsilon = 3\alpha\pi_{\rm tot}$$

Modified Poisson equation (time-varying G, scalaron terms)

Scalaron drives “gravitational slip” Ψ–Φ

Technical challenge

Derive all four modes for a w-fluid

Unsolved

Short of this, derive corrections when $\varepsilon << \Psi$

Calculations underway

WKB limit: two massive scalaron modes

$$\Psi \propto e^{i(kx-\omega t)}$$

$$\omega^2 = k^2 + a^2 m_s^2 \ , \quad m_s^2 = \frac{1+f_R}{3f_{RR}}$$

Singular perturbations

GR limit: $f_{RR} \rightarrow 0$, $\omega \rightarrow m_s \rightarrow \infty$

Similar to singular boundary layer in zero-viscosity limit of Navier-Stokes

Key role of initial conditions: Is the GR solution changed as $f_{RR} \rightarrow 0$?

Currently under investigation

Conclusions on f(R)

Nonzero gravitational slip induced by scalaron

Modified growth of structure, possibly even as $f \rightarrow 0$

Can get time-varying G, similar to scalar-tensor theories

Additional quantities to constrain besides exotic dark energy (entropy, shear stress)

Another view of f(R)

Conformal transformation of the metric can put the gravitational action in Einstein form

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = (1 + f_R) g_{\mu\nu}$$

$$\phi \equiv \left(\frac{3}{16\pi G}\right)^{1/2} \ln(1 + f_R)$$

$$V(\phi) \equiv \frac{R(1 + f_R)^{1/2} - (R + f)}{16\pi G(1 + f_R)}$$

Action for transformed fields (“Einstein frame”)

$$
\begin{aligned}
S_E[\tilde{g}_{\mu\nu}, \phi, \psi] \;=\; & \frac{1}{16\pi G} \int d^4x \sqrt{-\tilde{g}}\, \tilde{R} \\
& - \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right] \\
& + \int d^4x \sqrt{-\tilde{g}}\, \mathcal{L}_M \left(\frac{\tilde{g}_{\mu\nu}}{1 + f_R}, \psi \right)
\end{aligned}
$$

Gravity is now GR but matter no longer follows geodesics: “fifth force”

Conformal “frames”

“Frame” is a misnomer – they are not the same as frames of special relativity

Change of “frame” = Redefinition of fields (not coordinates)

Predictions for physical observables are independent of “frame”

Scalar-Tensor theories

Gravity: Einstein+scalar field

Matter couplings: minimal (geodesic)

Classic case: Brans-Dicke

$$S[g_{\mu\nu},\phi,\psi] = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,\left[\phi R - \frac{\omega}{\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] + \int d^4x\,\sqrt{-g}\,\mathcal{L}_M(g_{\mu\nu},\psi)$$

Field equations for Brans-Dicke

$$R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = \frac{8\pi G}{\phi} \left[T^{\mu\nu} + T^{\mu\nu}_{(\phi)} \right]$$

$$\Box \phi = \frac{8\pi G T}{3 + 2\omega} = \frac{8\pi G (3p - \rho)}{3 + 2\omega}$$

Variable Newton constant!

Solar-system tests: ω > 40,000 (f becomes constant): Deviations from GR < 10^{-5}

Next time:

~~Birkhoff theorem~~ in f(R) and scalar-tensor theories: trouble from small scales?

Cosmological scale tests of gravity: growth of structure and gravitational slip