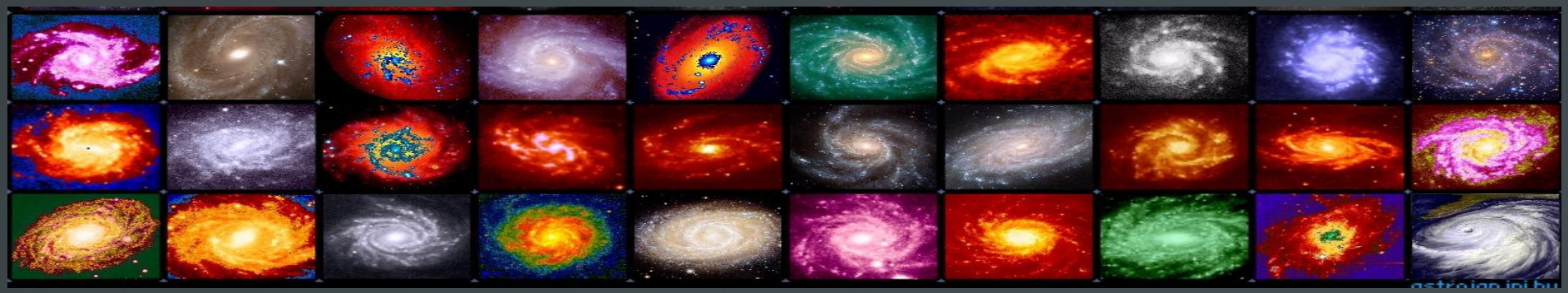
Bosonic gas as a galactic dark matter halo



COSMOLOGÍA EN LA PLAYA

ESSENTIAL COSMOLOGY FOR THE NEXT GENERATION

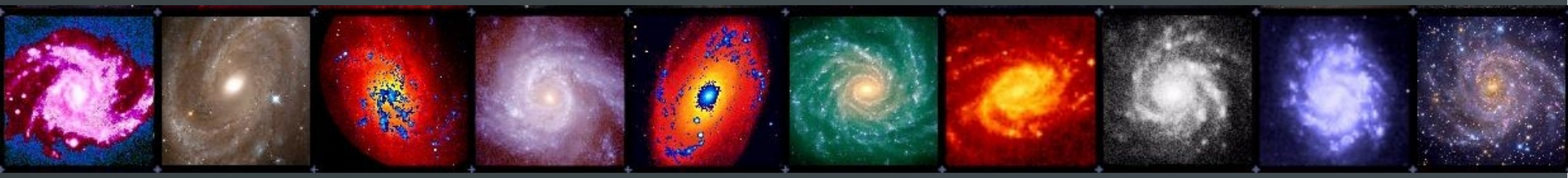
January 10-14 2011, Puerto Vallarta, México

Argelia Bernal, ICN-UNAM

Luis Arturo Ureña López

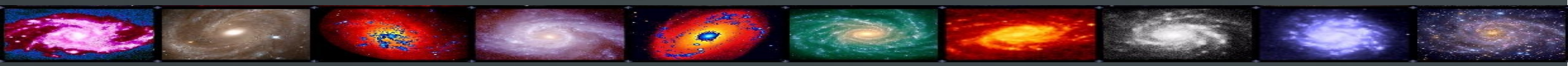
Phys. Rev. D 82, 123535 (2010)

Outline



- Description of a Bose-Einstein Condensate (BEC)
- Dark matter halo as a Newtonian BEC
- ***Multistate configurations***
- Rotation Curves from multistate configurations

Bose-Einstein Condensate



”The many-particle system is described by a second-quantized free field, which in the boson case satisfies the Klein-Gordon equation in general relativity, $\nabla\alpha\nabla\alpha\varphi=\mu 2\varphi$, ... (where $\mu=mc/h$). The coefficients of the metric $g\alpha\beta$ are determined by the Einstein equations with a source term given by the mean value $\langle \varphi|T\mu\nu|\varphi \rangle$ of the energy-momentum tensor operator constructed from the scalar field. The state vector $\langle \varphi|$ corresponds to the ground state of the system of many particles.”

R. Ruffini and S. Bonazzola *Systems of Self-Gravitating Particles in General Relativity and the Concept of an Equation of State* Phys. Rev. 187, 1767 (1969)

Bose-Einstein Condensate

It is assumed that the many boson system is described by a quantized real scalar field

$$\hat{\Phi} = \sum_{nlm} [\hat{b}_{nlm} \Phi_{nlm}(t, \mathbf{x}) + \hat{b}^{\dagger}_{nlm} \Phi^{*}_{nlm}(t, \mathbf{x})],$$

where field coefficients satisfy the Klein Gordon eq. in a curved spacetime

$$(\Box - \mu^2)\Phi_{nlm}(t, \mathbf{x}) = 0,$$

$$\Box = (1/\sqrt{-g})\partial_{\mu}(\sqrt{-g}\partial^{\mu})$$

The most general solution is of the form

$$\Phi_{nlm}(t, \mathbf{x}) = R_{nl}(t, r) Y_{lm}(\theta, \varphi),$$

Assuming that there exist a vacuum state defined by

$$\hat{b}_{nlm} |0, 0, \ldots, 0\rangle = 0, \quad \forall \ (n, l, m)$$

we can construct the orthonormal many-particle states

$$|Q\rangle = |N_{100}, N_{200}, N_{21-1}, N_{210}, \ldots\rangle \equiv \frac{(\hat{b}^{\dagger})^{N_{100}} (\hat{b}^{\dagger})^{N_{200}} \cdots}{N_{100}! N_{200}! \cdots} |0, 0, \ldots 0\rangle,$$

Bose-Einstein Condensate

The dynamics of the gravitational field is described by

$$G_{\alpha\beta} = 8\pi G \langle Q| :\hat{T}_{\alpha\beta}: |Q\rangle,$$

the r.h.s. is the expectation value of the operator

$$\hat{T}_{\alpha\beta} = \partial_\alpha \hat{\Phi} \partial_\beta \hat{\Phi} - \frac{1}{2} g_{\alpha\beta} (\partial^\sigma \hat{\Phi} \partial_\sigma \hat{\Phi} + \mu^2 \hat{\Phi}^2)$$

The orthogonality ensures that the expectation value is given a superposition of the expectation values of the e-m tensor components for each individual state, that is

$$\langle Q| :\hat{T}_{\alpha\beta}: |Q\rangle = \sum_{n=1}^{\infty} \sum_{l=1}^{n-1} \sum_{m=-l}^{l} \langle N_{nlm}| :\hat{T}_{\alpha\beta}: |N_{nlm}\rangle$$

where we are defining the single states

$$|N_{nlm}\rangle \equiv |0, 0, \ldots, N_{nlm}, 0, \ldots, 0\rangle$$

Bose-Einstein Condensate

Hence, the Einstein equations read

$$G_{\alpha\beta} = 8\pi G \sum_{n,l,m} T_{\alpha\beta(nlm)}$$

where

$$T_{\alpha\beta(nlm)} = \partial_\alpha \Phi_{nlm} \partial_\beta \Phi^*_{nlm} - \frac{1}{2} g_{\alpha\beta} (\partial^\sigma \Phi_{nlm} \partial_\sigma \Phi^*_{nlm} + \mu^2 \Phi_{nlm} \Phi^*_{nlm}),$$

Therefore, in the case when particles populate vaious excited levels, the source of the Einstein eq is equivalent to the e-m tensor of many (independent) classical complex scalar fields minimally coupled to gravity.

The condensate is obtained when

$$|Q\rangle = |N_{100}, N_{200}, N_{21-1}, N_{210}, \ldots\rangle$$

$$|N_{nlm}\rangle \equiv |0, 0, \ldots, N_{nlm}, 0, \ldots, 0\rangle$$

Bose-Einstein Condensate

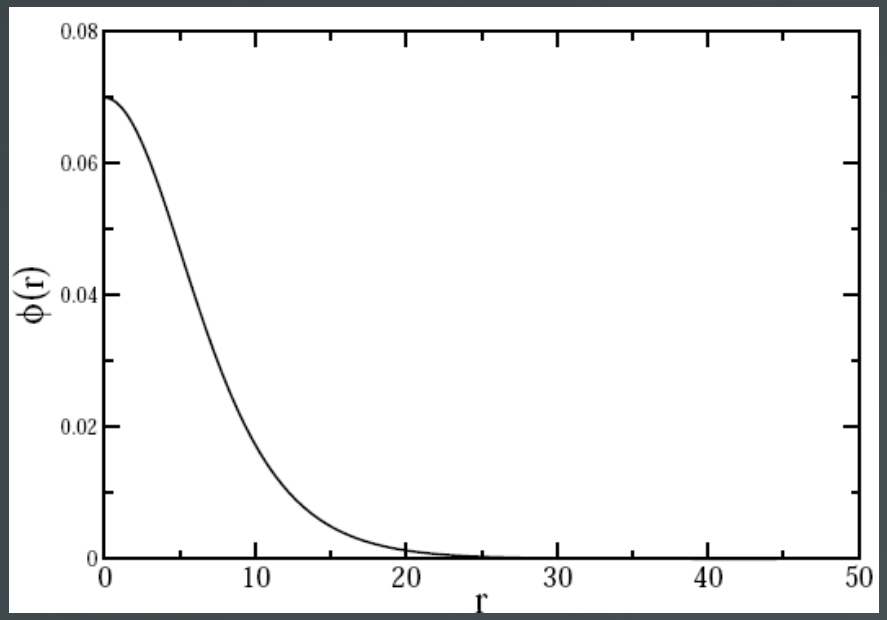
For the condensate, assuming a spherically symmetric metric

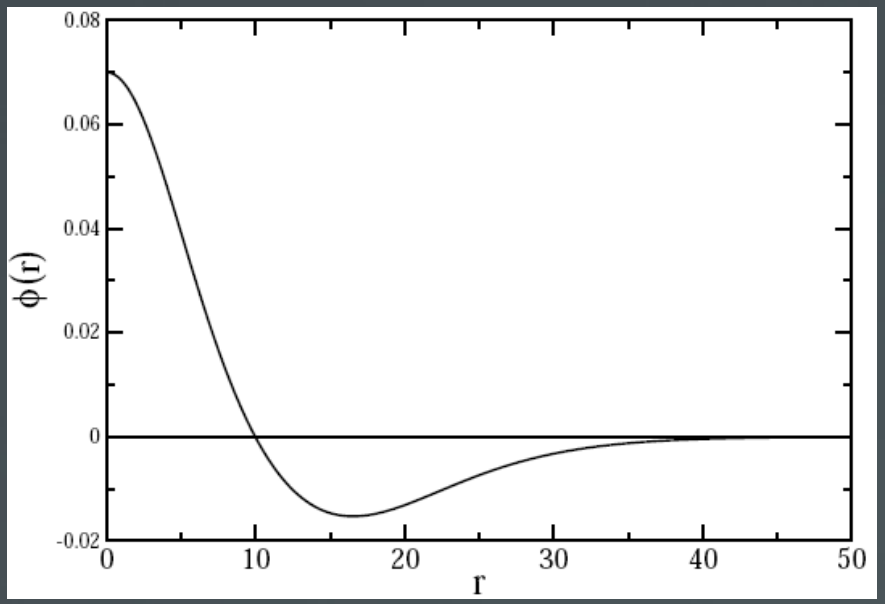
$$ds^2 = -\alpha^2(t,r)dt^2 + a^2(t,r)dr^2 + r^2 d\Omega$$

l=m=0 and harmonic time dependence

$$\sqrt{8\pi G}\,\Phi_{nlm}(t,\mathbf{x}) \rightarrow e^{-i\gamma t}\phi(r)$$

demanding non singular and bounded configurations, radial profiles obtained are





Newtonian BEC

In order to model galactic halos it is considered the Newtonian limit of the EKG equations which results in the so-called SP system

$$\nabla^2 U = \sum_{nlm} |\Psi_{nlm}|^2$$

$$i\partial_t \Psi_{nlm} = -\frac{1}{2}\nabla^2 \Psi_{nlm} + U\Psi_{nlm}$$

where Ψ_{nlm} is related to Φ_{nlm} by

$$\sqrt{8\pi G}\,\Phi_{nlm}(t, \mathbf{x}) = e^{-i\mu t}\Psi_{nlm}(t, \mathbf{x})$$

$$\Psi_n = e^{-i\gamma_n t}\phi_n(r)$$

The Newtonian version of the EKG equations describes the dynamics of nonrelativistic wave functions which are coupled among themselves through the Newtonian gravitational potential U. Once in this regime, we can define

$$K = -\frac{1}{2}\sum_{n,l,m}\int(\Psi^*_{nlm}\nabla^2\Psi_{nlm} + \Psi_{nlm}\nabla^2\Psi^*_{nlm})dv$$

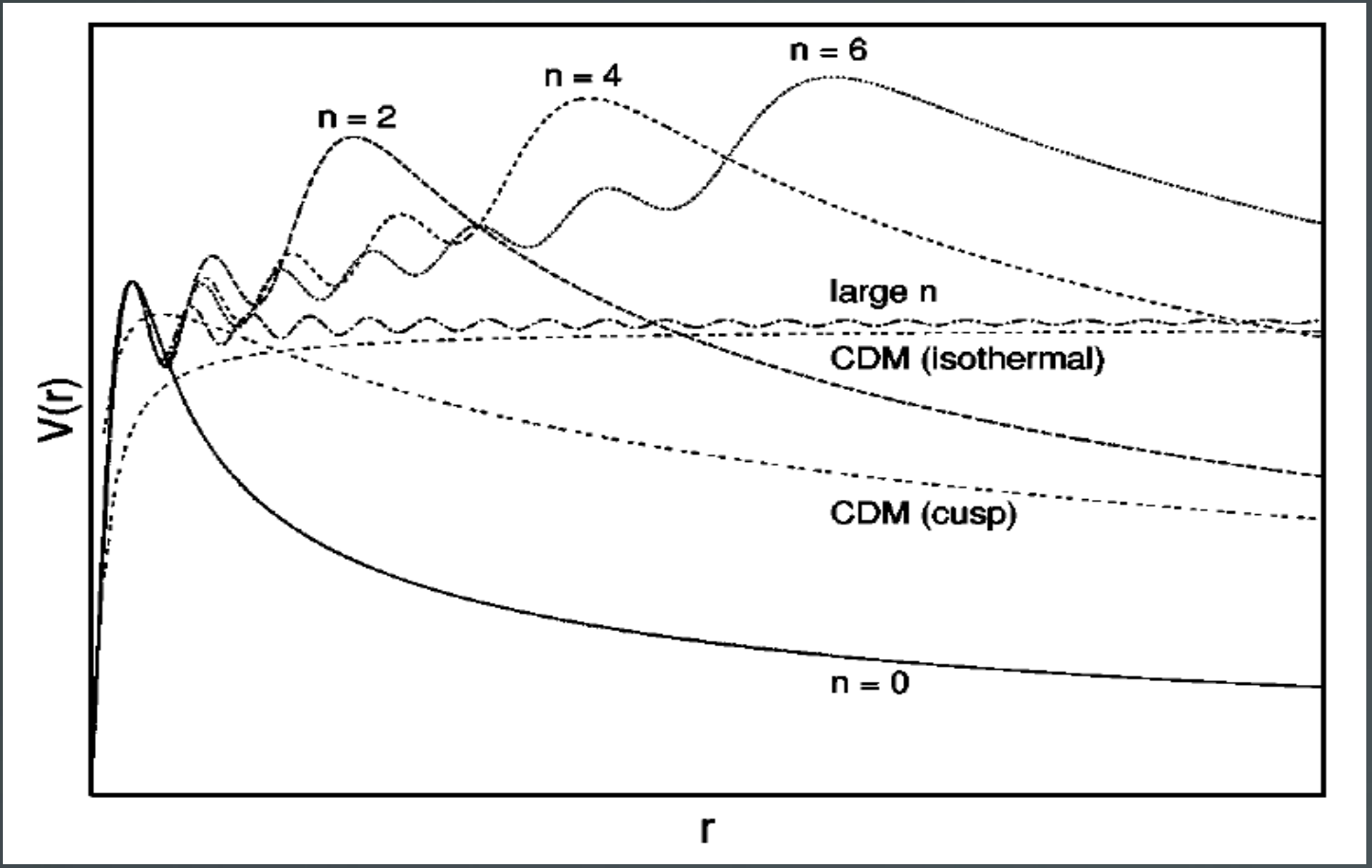
$$W = \sum_{n,l,m}\int U|\Psi_{nlm}|^2 dv$$

$$N = \sum_{n,l,m}\int |\Psi_{nlm}|^2 dv$$

Rotation Curves from Newtonian BEC

RC from single states

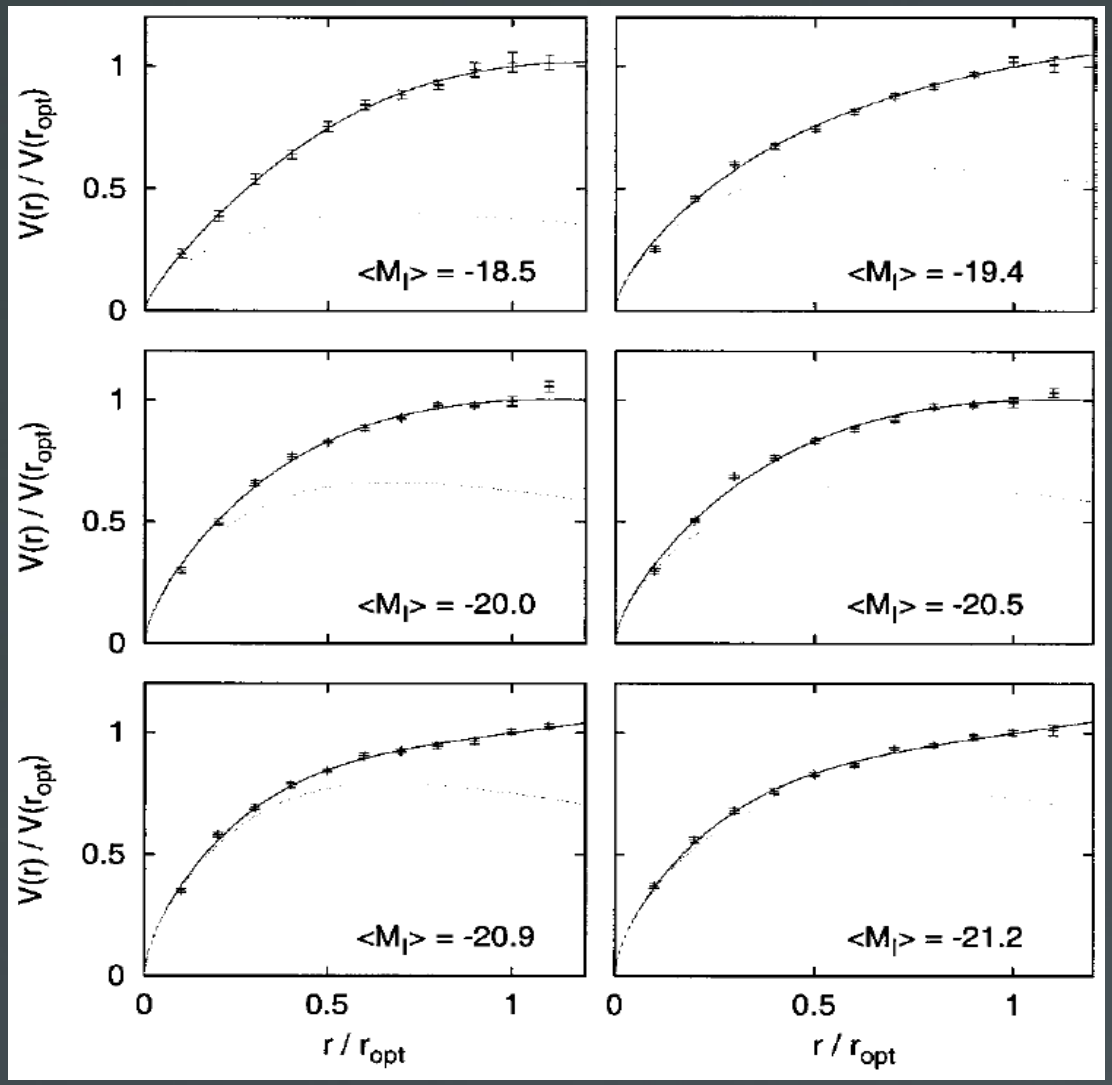
$$v(r) = \sqrt{N(t, r)/r}$$



A. Arbey et. al. *Quintessential halos around galaxies* Phys. Rev. D 64, 123528

Fits of RC with Newtonian BEC

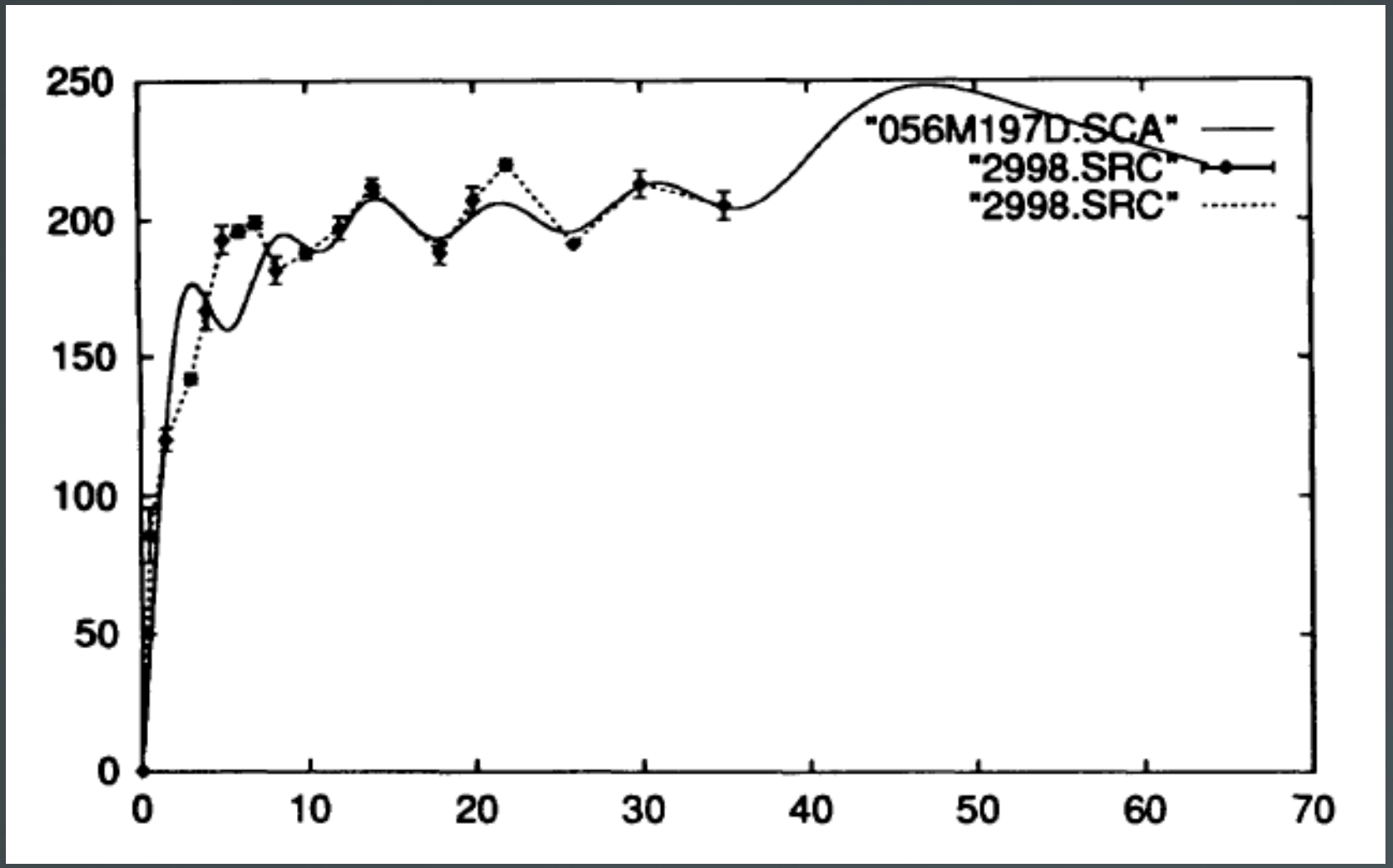
RC fits with ground states



A. Arbey et. al. *Quintessential halos around galaxies* Phys. Rev. D 64, 123528

Fits of RC with Newtonian BEC

RC fits with excited states



S. Sin. *Late-time phase transition and the galactic halo as a Bose liquid* Phys. Rev. D 50, 3650

Shortcomings

- The ground state is the only stable solution of the SP system against gravitational perturbations; other excited configurations are intrinsically unstable

  P. Jetzer, Phys. Rep. 220, 163 (1992)

  J-W. Lee, J. Korean Phys. Soc. 54, 2622 (2009)

  F.S. Guzman and L.A. Urena-Lopez, Phys. Rev. D 69,023511 (2003)

  F.S. Guzman and L.A. Urena-Lopez, Astrophys. J. 645, 814 (2006)

- "Because the mass m and the coupling λ define a unique scale of ~2.3 kpc, the Bose condensate does not extend enough to account for the dark matter inside large systems. A single self-interacting bosonic halo fails to reproduce at the same time the dark matter inside light and massive spirals."

  A. Arbey et. al. *Galactic Halos of Fluid Dark Matter* Phys. Rev. D 68,023511 (2003)

Multistate configurations

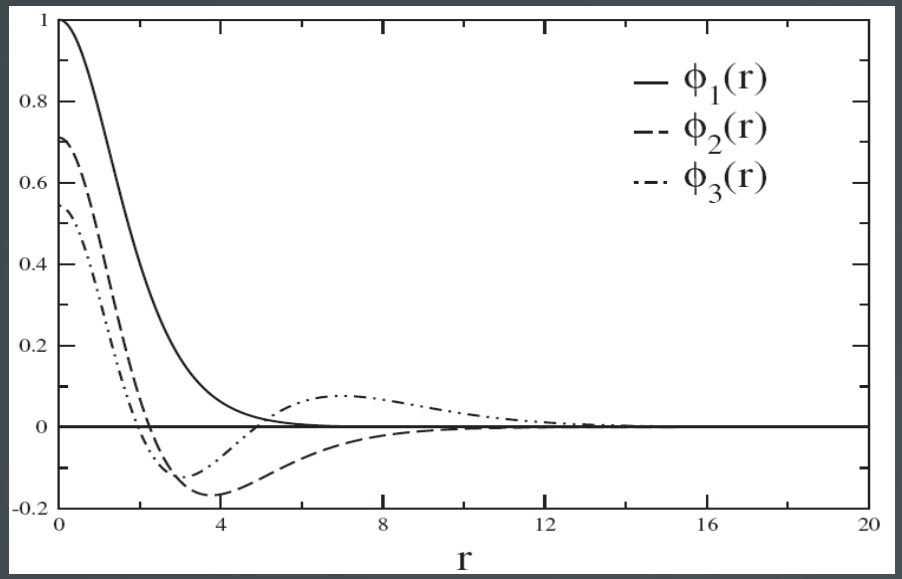
Equilibrium configurations of the SP system, in which many-particles states coexist simultaneously can be constructed, so that the whole system is stable under small radial perturbations

$$\Psi_n = e^{-i\gamma_n t}\phi_n(r)$$

$$\frac{1}{r^2}\frac{d^2(r^2U)}{dr^2} = \sum_{n=1}^{I}|\phi_n|^2,$$

$$\frac{1}{r^2}\frac{d^2(r^2\phi_n)}{dr^2} = 2(U-\gamma_n)\phi_n,$$

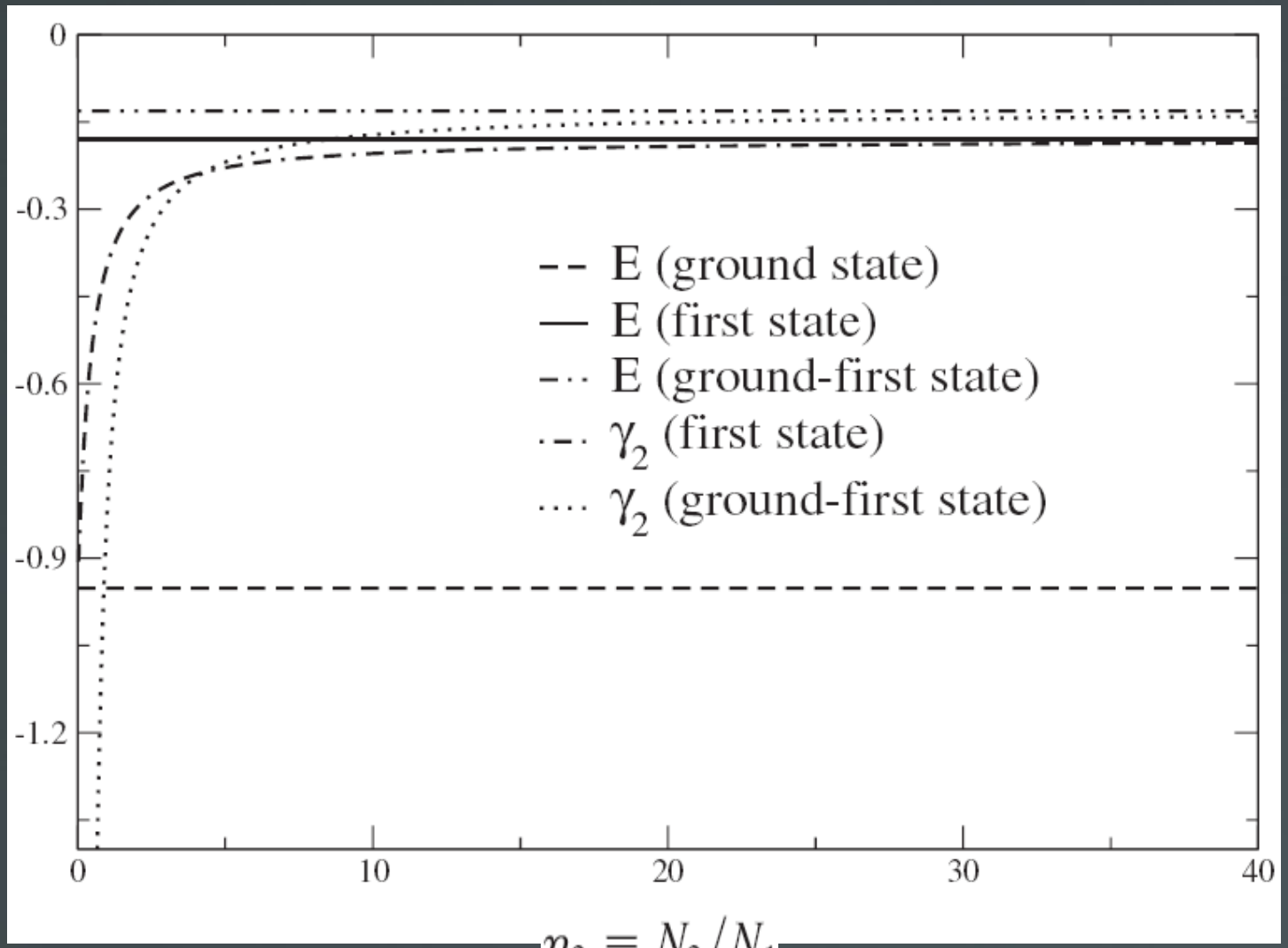
$$n = 1, \ldots, I$$



$|N_1, 0.96, 0.91\rangle$ $\quad \phi_2(0) = 0.710,\ \phi_3(0) = 0.543$ $\quad \gamma_1 = -1.185,\ \gamma_2 = -0.712,\ \gamma_3 = -0.471$

Multistate configurations

Multistates are generalizations of single states. If the number of particles N of a single state is fixed, the total energy E=K+W takes a fixed value. This behaviour of E changes radically if the same number of particles N are distributed in a mixed configuration



Stability of Multistate configurations

If the system is considered open (the number of particles is not strictly conserved), an excited state under small radial perturbations, emit particles, looses its nodes, and eventually settles down on to a ground state. ***It is then expected that the excited state of a ground-first configuration should remain stable if there are very few particles in the excited state***.

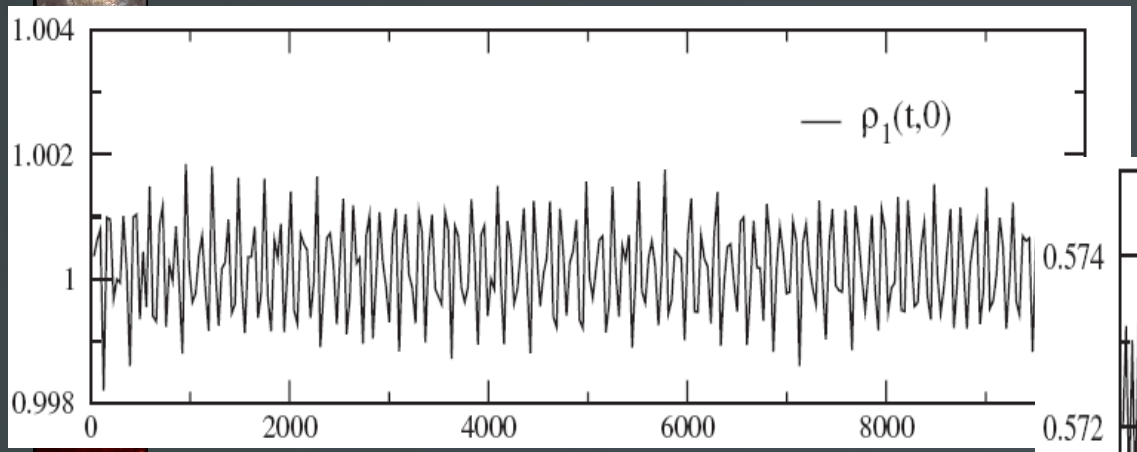
We found that, by adding particles to the ground state, it is possible to construct ground-first configurations for which, under open conditions, the mixed state is stable under radial perturbations.

$$\Psi_1 = \phi_1(r)e^{-i\gamma_1 t}$$

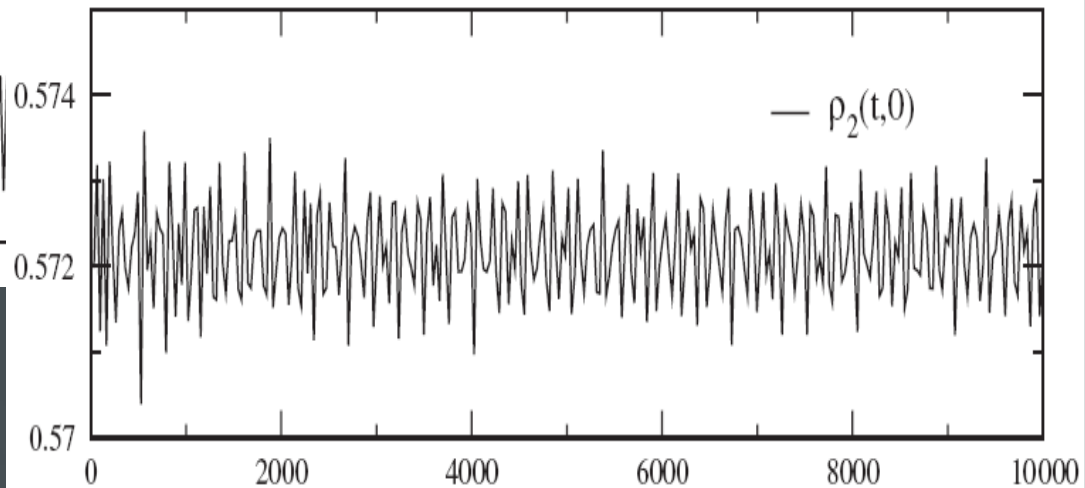
$$\rho_1 = |\Psi_1|^2$$

$$\Psi_{\text{pert}} = \Psi_1 + \delta\Psi_1$$

$$\rho_{pert} = \rho_1(r) + \delta\rho(r)\cos(2\pi\sigma t)$$

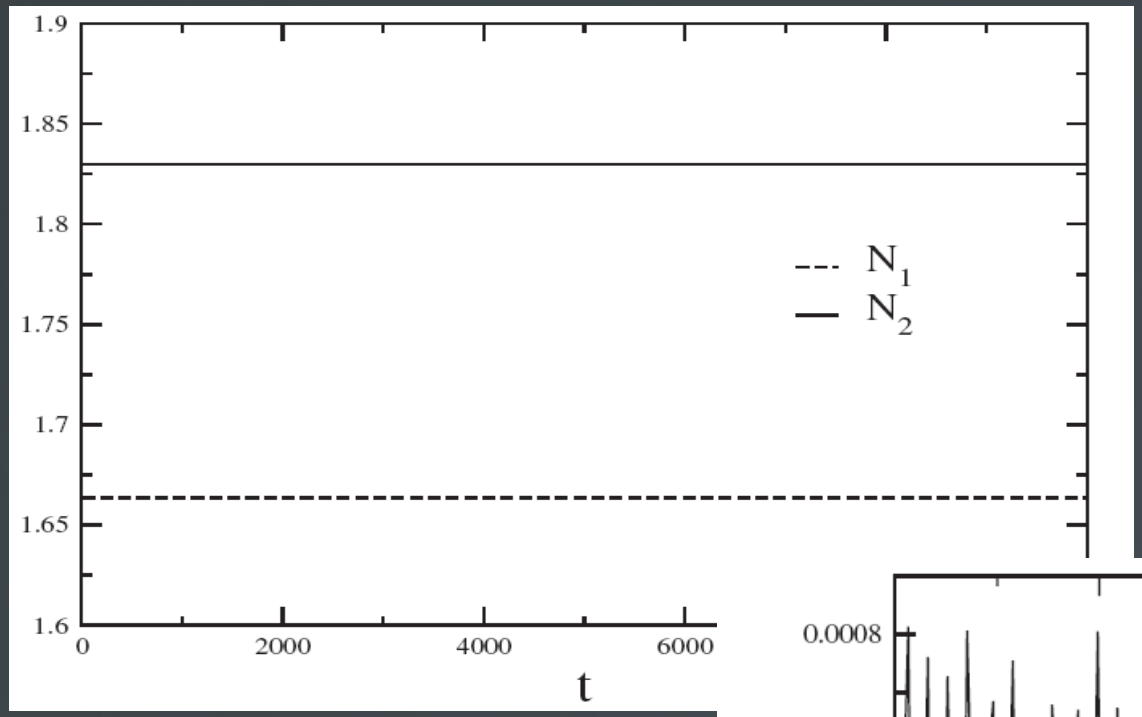


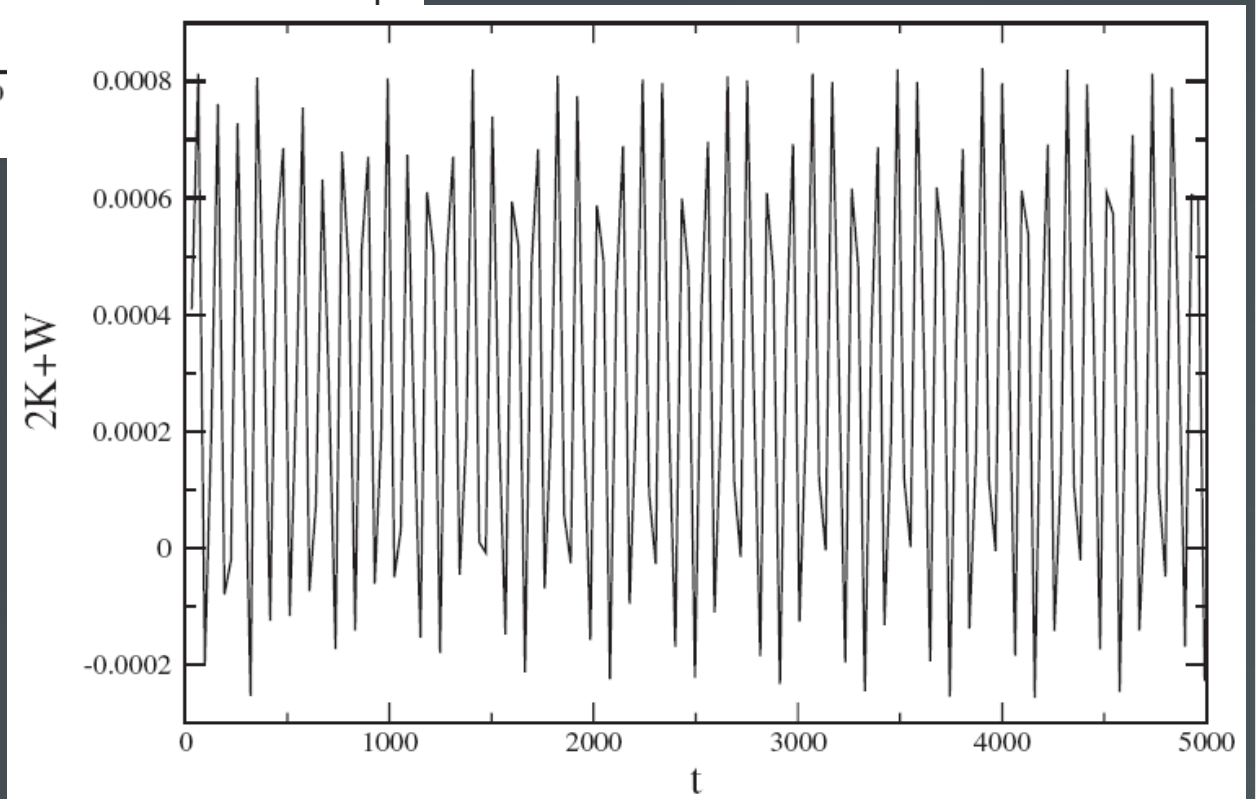
$$\eta_2 = 1.1$$



Stability of Multistate configurations

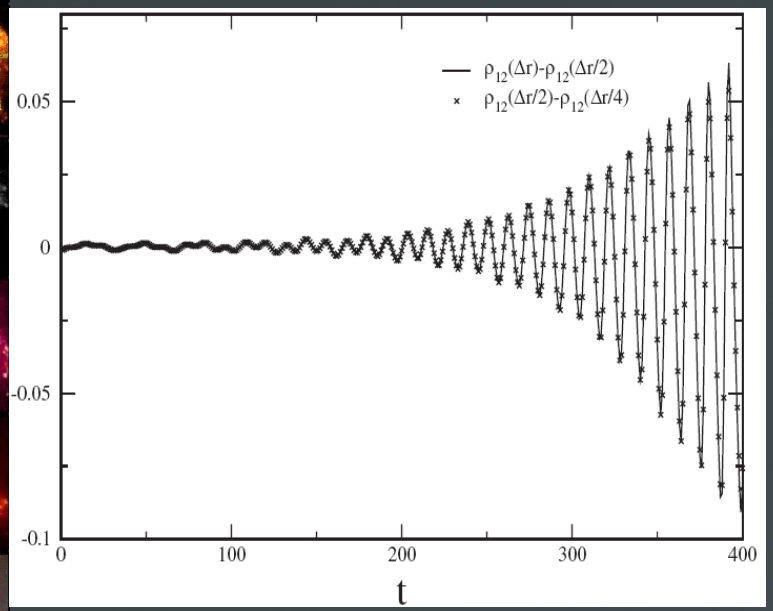
Evolution of the number of particles and the virialization relation .

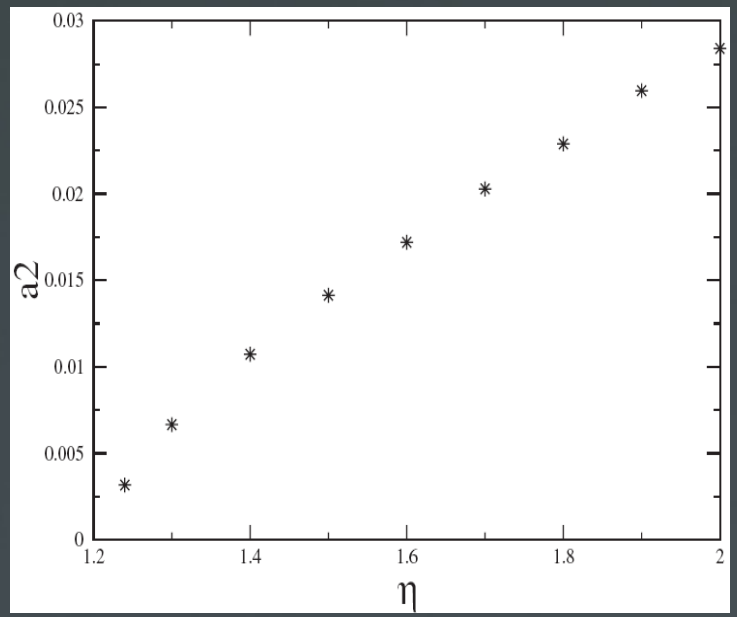


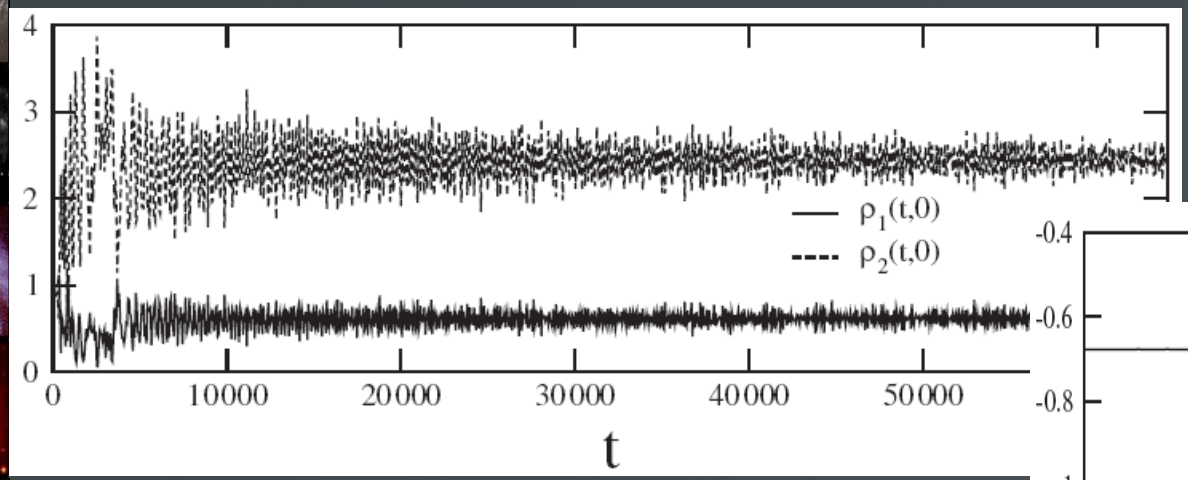


Late time behavior of unstable Multistate configurations

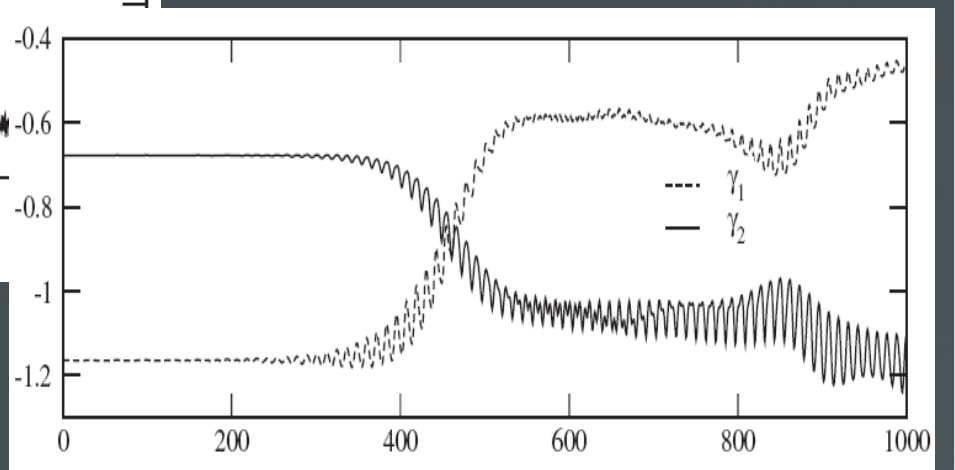
Evolution of a multistate with $\eta_2 = 1.6$





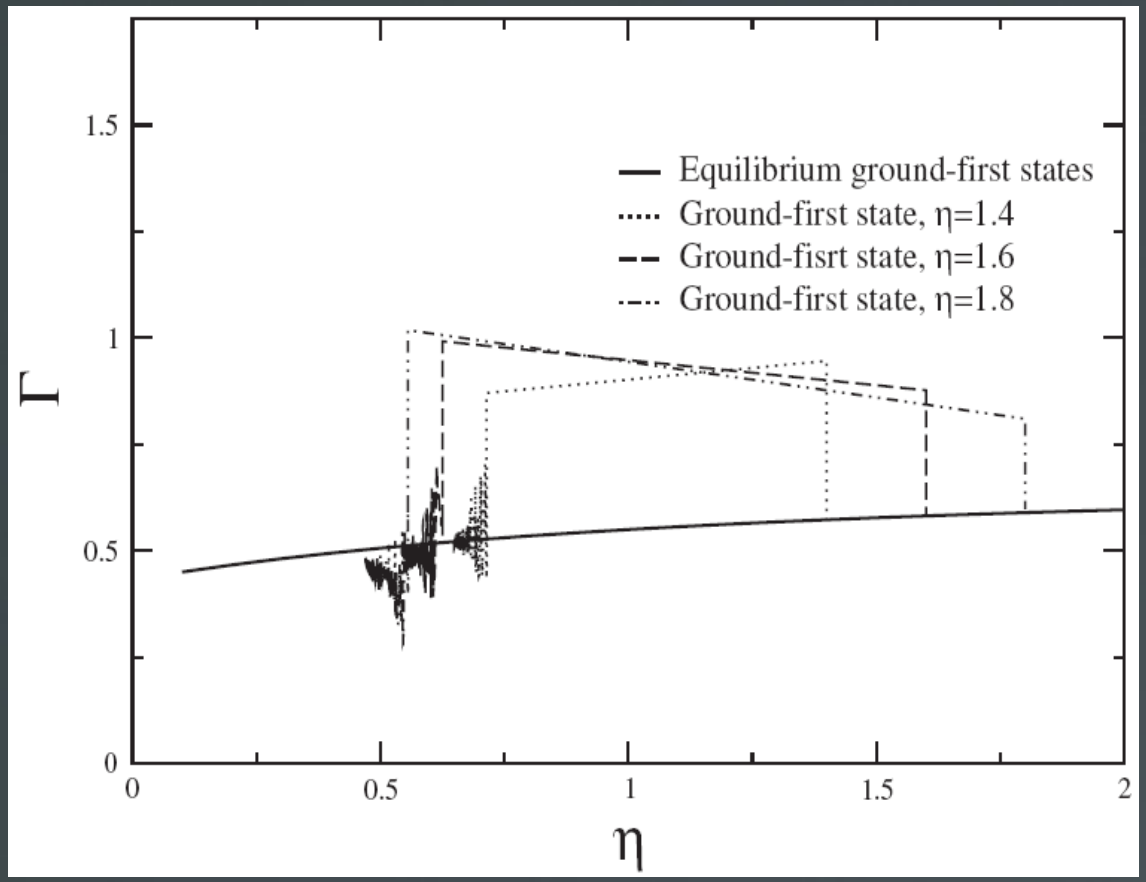


$$\rho_n \simeq a_1 e^{a_2 t} \cos(a_3 t + a_4),$$



Late time behavior of unstable Multistate configurations

Fate of unstable configurations. Configurations oscillate around the line representing stationary equilibrium configurations on the stable branch $\eta_2^{\text{tresh}} \sim 1.13$

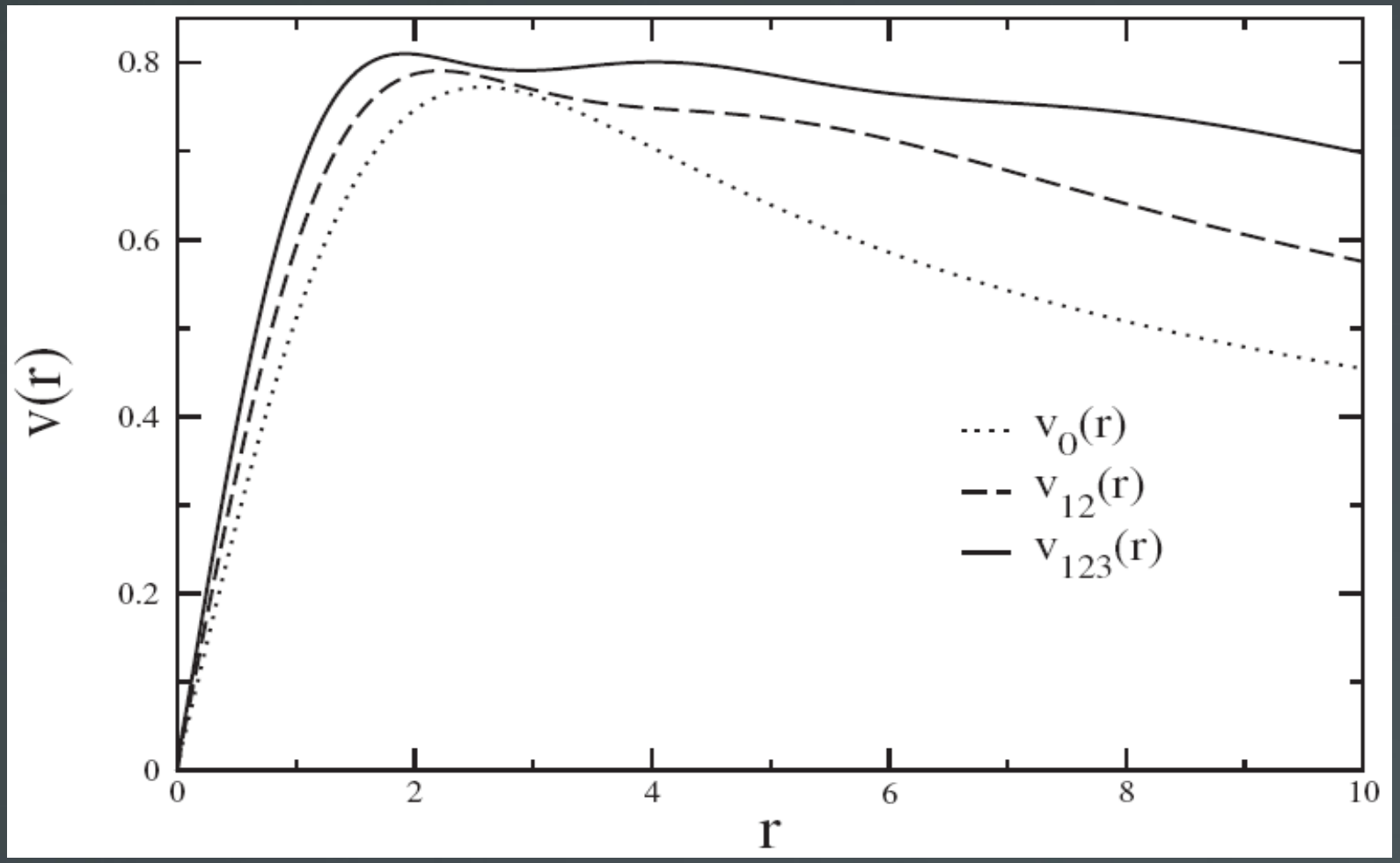


$$\Gamma_n \equiv \gamma_n / \gamma_1$$

RC from Multistate configurations

Rotation curve for single and multistate configurations.

Notice that the flatness of the curve is improved if more excited states are taken into account



Conclusion

Multistate configurations could improve fits or realistic RC as they are stable and they introduce extra parameters, namely N1, N2, etc., that can avoid the “characteristic scale” restriction