$$S_{\text{E.H.}+\text{S.F.}} = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}M_{\text{Pl}}^2 R + M_{\text{Pl}}^2 \dot{H} g^{00} - M_{\text{Pl}}^2(3H^2+\dot{H}) +$$

$$+\frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 +$$

$$-\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^{\mu}{}_{\mu} - \frac{\bar{M}_2(t)^2}{2}\delta K^{\mu}{}_{\mu}{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^{\mu}{}_{\nu}\delta K^{\nu}{}_{\mu} + \ldots\Big]$$

$$g^{00} \quad \rightarrow \quad -1 - 2\dot{\pi} - \dot{\pi}^2 + \frac{1}{a^2}(\partial_i\pi)^2$$

$$S_\pi = \int d^4x\sqrt{-g}\left[-M_{\text{Pl}}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3\right]$$

Parameters: $H, \quad \dot{H}, \quad \ddot{H}, \quad M_2, \quad M_3$

Structure is set by the symmetries, the requirement that everything can be incorporated into the metric by a suitable choice of coordinates. Specific signs and coefficients in front of various terms, requirement of certain interactions, difference between time and space derivatives.

$$S_\pi = \int d^4x\sqrt{-g}\left[-M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3\right]$$

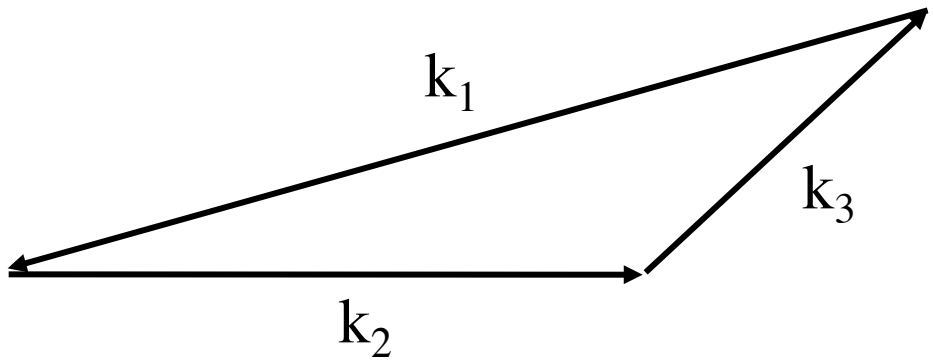
M_2 changes the dispersion relation of modes, introducing a "sound speed". Same term that changes the propagation speed generates interactions.

$$M_2^4 = -\frac{1-c_s^2}{c_s^2}\frac{M_{\rm Pl}^2\dot{H}}{2}$$

$$\omega^2 = c_s^2 k^2$$

$$S_\pi = \int d^4x\,\sqrt{-g}\left[-\frac{M_{\rm Pl}^2\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{M_{\rm Pl}^2\dot{H}}{c_s^2}\dot{\pi}\frac{(\partial_i\pi)^2}{a^2} - \frac{2}{3}\frac{\tilde{c}_3}{c_s^4}M_{\rm Pl}^2\dot{H}\dot{\pi}^3\right]$$

Observational Consequences: 3-pt function



$$k_1^6 \langle \zeta_{k1} \zeta_{k2} \zeta_{k3} \rangle = F(\frac{k_2}{k_1}, \frac{k_3}{k1})$$

Higher order moments, departure from Gaussianity are sensitive to the interactions.
Even after requiring scale invariance and translation invariance the three point function is still an arbitrary function of two continuous variable.

$$S_\pi = \int d^4x\sqrt{-g}\left[-M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3\right]$$

Just two specific shapes for this two shapes are predicted from each of the two possible interactions:

$$\dot{\pi}^3 \quad \dot{\pi}(\partial_i\pi)^2$$

Estimating the 3-pt function

$$S_\pi = \int d^4x \sqrt{-g} \left[-\frac{M_{\rm Pl}^2 \dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{M_{\rm Pl}^2 \dot{H}}{c_s^2} \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} - \frac{2}{3} \frac{\tilde{c}_3}{c_s^4} M_{\rm Pl}^2 \dot{H} \dot{\pi}^3 \right]$$

$$\frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^{3/2}} \sim \left. \frac{L_3}{L_2} \right|_{E \sim H} \sim \frac{1}{c_s^2} \zeta \qquad\qquad f_{NL} = \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2}$$

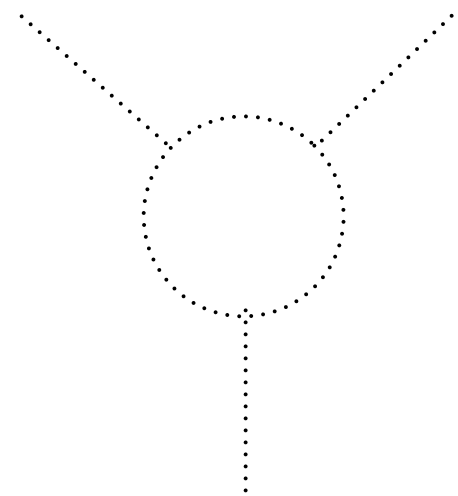
$$f_{NL} \zeta \sim \left. \frac{L_3}{L_2} \right|_{E \sim H} \sim \frac{1}{c_s^2} \zeta \qquad \Rightarrow \qquad f_{\rm NL} \sim \frac{1}{c_s^2} \ .$$

Two shapes are possible

$$\vec{x} \quad \longrightarrow \quad \vec{\tilde{x}} = \vec{x}/c_s \qquad\qquad \pi_c = \left(-2M_{\rm Pl}^2 \dot{H} c_s\right)^{1/2} \pi$$

$$S_\pi = \int dt\, d^3\tilde{x}\, \sqrt{-g} \left[\frac{1}{2}\left(\dot{\pi}_c^2 - \frac{(\tilde{\partial}_i \pi_c)^2}{a^2}\right) - \frac{1}{\left(8|\dot{H}|M_{\rm Pl}^2 c_s^5\right)^{1/2}} \dot{\pi}_c \frac{(\tilde{\partial}_i \pi_c)^2}{a^2} - \frac{2}{3} \frac{\tilde{c}_3}{\left(8|\dot{H}|M_{\rm Pl}^2 c_s^5\right)^{1/2}} \dot{\pi}_c^3 \right]$$

$$\Lambda^4 \sim c_s^5 |\dot{H}| M_{\rm Pl}^2 \sim c_s^7 M_2^4$$



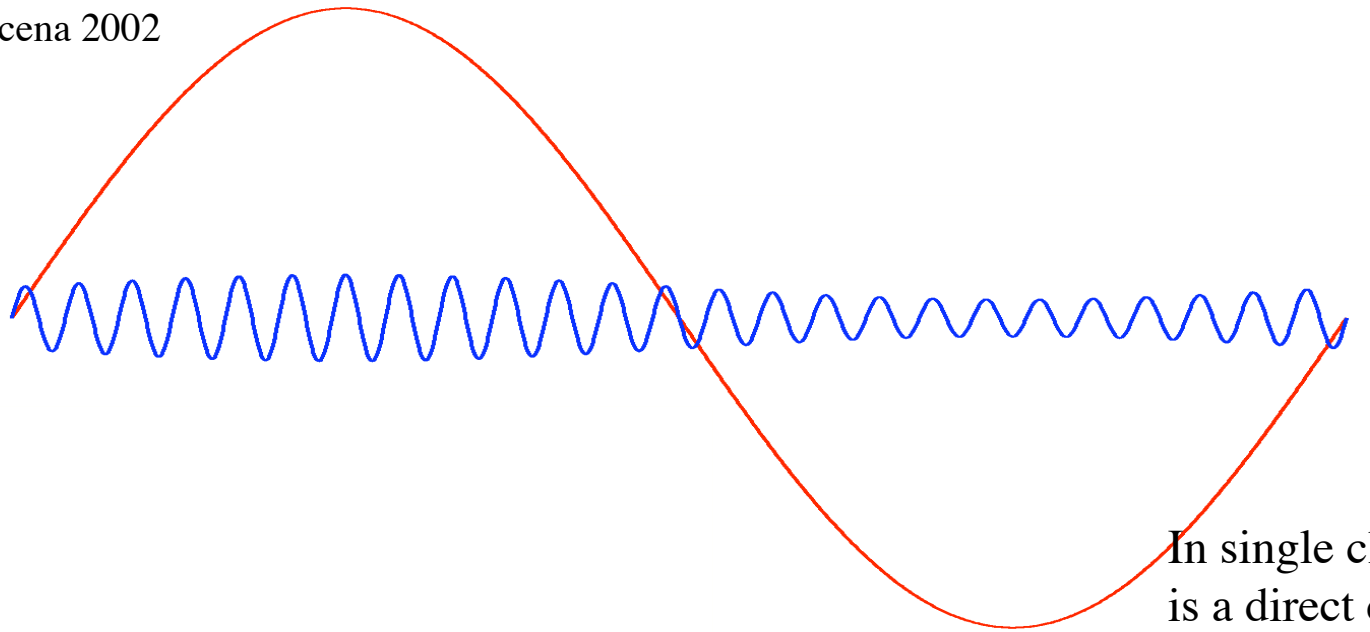
$$\frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^{3/2}} \sim \left(\frac{H}{\Lambda}\right)^2$$

The Squeezed limit

$$k_3 << k_1 \sim k_2$$

$$\zeta(x) = \zeta_g(x) + f_{NL}\zeta_g^2(x) + \cdots$$

Maldacena 2002



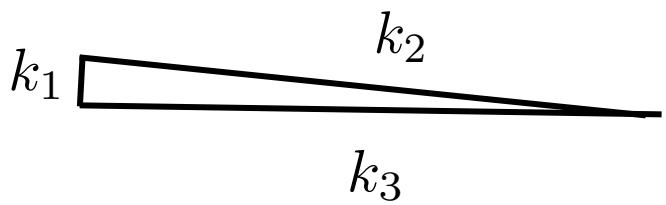
In single clock models there is a direct connection between the departures from scale invariance and the three point function.

Non-Gaussianities $\sim (n-1)10^{-5}$

Consistency relation

$$\lim_{k_1 \to 0} \langle \zeta_{\vec{\kappa}_1} \zeta_{\vec{\kappa}_2} \zeta_{\vec{\kappa}_3} \rangle = -(2\pi)^3 \delta^3 (\sum_i \vec{k}_i)(n_s - 1) P_{k_1} P_{k_3}$$

$$\langle \zeta_{\vec{k}_i} \zeta_{\vec{k}_j} \rangle = (2\pi)^3 \delta^3 (\vec{k}_i + \vec{k}_j) P_{k_i} \qquad \langle \zeta \zeta \rangle \sim k^{-3+(n_s-1)}$$

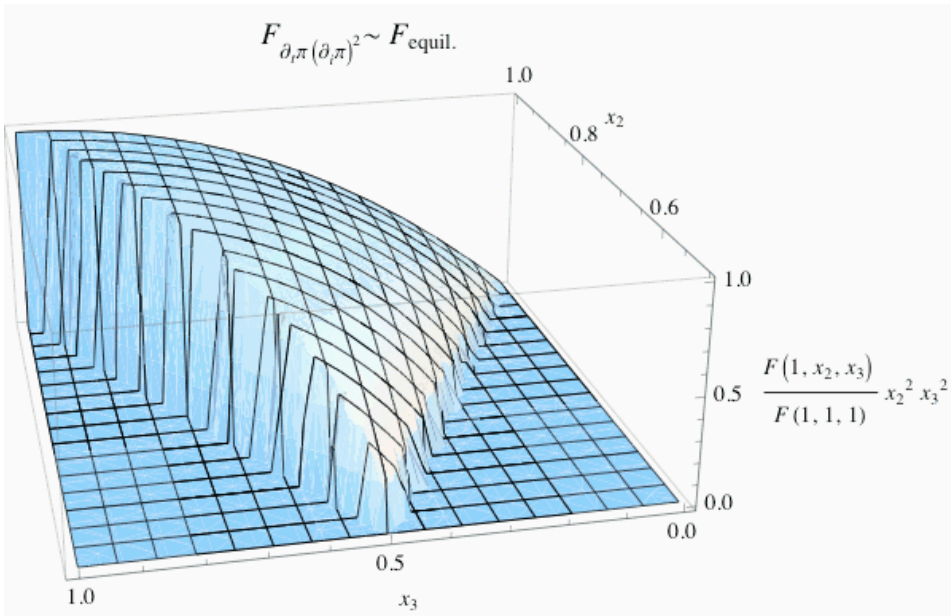


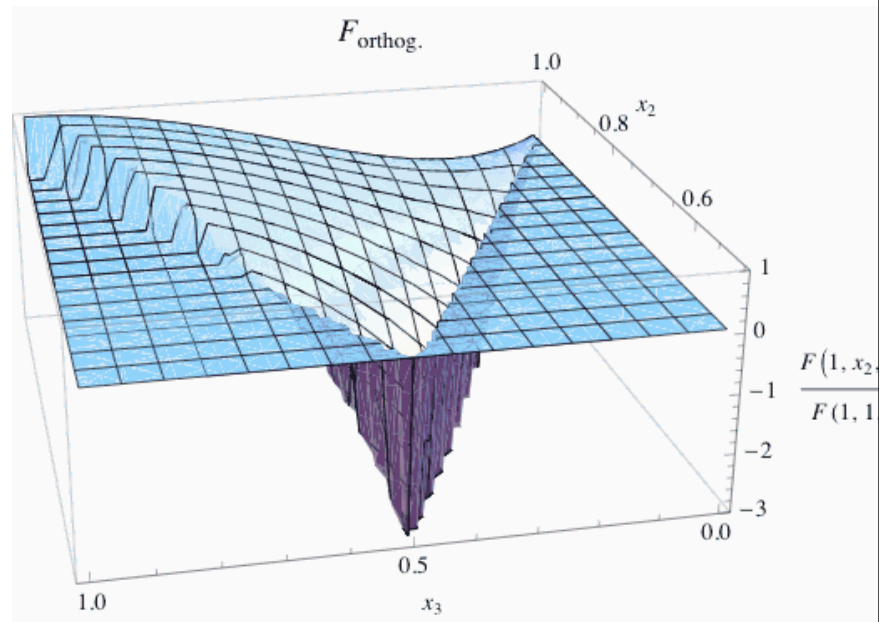
$$ds^2 = -dt^2 + e^{2\zeta(x)} a(t)^2 dx_i dx_i$$

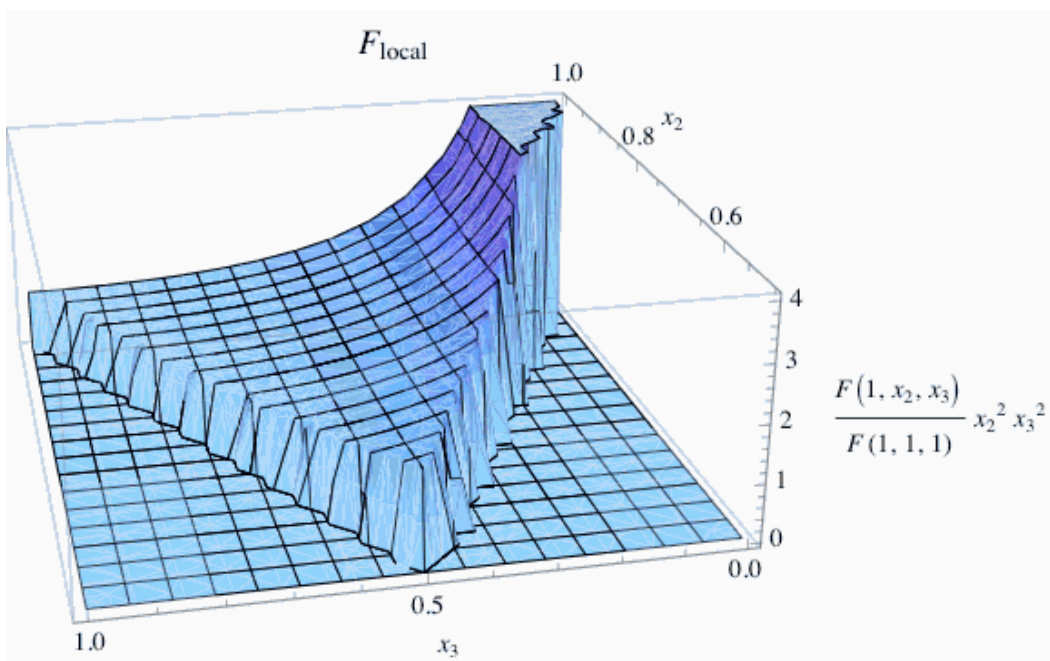
For modes outside the horizon

$$\frac{\partial}{\partial \zeta_1} \langle \zeta(x) \zeta(0) \rangle \cdot \zeta_1 = x \frac{d}{dx} \langle \zeta(x) \zeta(0) \rangle \cdot \zeta_1$$

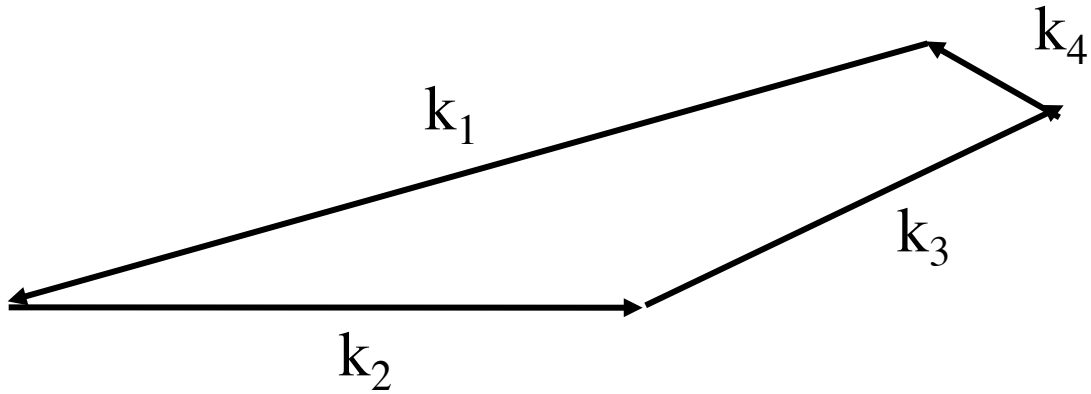
The shapes in pictures







Observational Consequences: 4-pt function



$$k_1^9 \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = F(\frac{k_2}{k_1}, \frac{k_3}{k_1}, \frac{k_4}{k_1}, \theta, \phi)$$

Higher order moments have even more freedom.
Even after requiring scale invariance and translation invariance the four point function is still an arbitrary function of five continuous variable.

Senatore & MZ in preparation.

Could one have an observable 4-pt function without a much larger 3-pt?

$$S_{\text{E.H. + S.F.}} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}M_{\text{Pl}}^2 R + M_{\text{Pl}}^2 \dot{H} g^{00} - M_{\text{Pl}}^2(3H^2 + \dot{H}) +$$
$$+\frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 +$$
$$-\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^{\mu}{}_{\mu} - \frac{\bar{M}_2(t)^2}{2}\delta K^{\mu}{}_{\mu}{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^{\mu}{}_{\nu}\delta K^{\nu}{}_{\mu} + ...\Big]$$

$$g^{00} \quad \rightarrow \quad -1 - 2\dot{\pi} - \dot{\pi}^2 + \frac{1}{a^2}(\partial_i \pi)^2$$

Although there are quartic interactions inside $(g_{00}+1)^2$ and $(g_{00}+1)^3$ quartic terms, those are very small given current constraints on the three point function.

Could one have a large 4-pt function

$$g^{00} \quad \rightarrow \quad -1 - 2\dot{\pi} - \dot{\pi}^2 + \frac{1}{a^2}(\partial_i \pi)^2 \qquad\qquad (g^{00}+1)^2$$ contains quartic terms

$$M_2^4 (\partial_i \pi)^4 \ . \qquad\qquad \frac{\langle \zeta^4 \rangle}{\langle \zeta^2 \rangle^2} \sim \left. \frac{L_4}{L_2} \right|_{E \sim H} \sim \frac{1}{c_s^4} \zeta^2$$

$$\tau_{\rm NL} = \langle \zeta^4 \rangle / \langle \zeta^2 \rangle^3$$

$$\tau_{\rm NL} \zeta^2 \sim \left. \frac{L_4}{L_2} \right|_{E \sim H} \sim \frac{1}{c_s^4} \zeta^2 \qquad \Rightarrow \qquad \tau_{\rm NL} \sim \frac{1}{c_s^4}$$

$$\Delta[f_{NL} \zeta] \sim \frac{1}{N_{\rm pix}^{1/2}} \ , \qquad \Delta[\tau_{NL} \zeta^2] \sim \frac{1}{N_{\rm pix}^{1/2}}$$

Errors on the level of non-G are similar for 3 and 4 pt functions. The signal generated by these operators is very small.

Could one have a large 4-pt function

What about starting with an operator that does not have a cubic interaction?

$$M_4^4(\delta g^{00})^4 \to M_4^4\left(16\dot{\pi}^4 - 32\dot{\pi}^3(\partial_\mu\pi)^2 + 24\dot{\pi}^2(\partial_\mu\pi)^4 - 8\dot{\pi}(\partial_\mu\pi)^6 + (\partial_\mu\pi)^8\right)$$

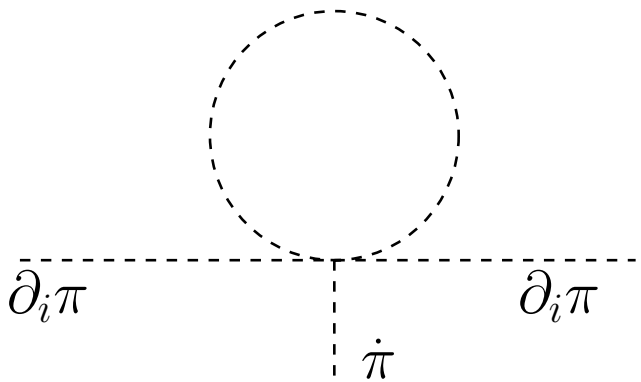
$$\frac{\langle\zeta^4\rangle}{\langle\zeta^2\rangle^2} \sim \tau_{\rm NL}\zeta^2 \sim \left.\frac{L_4}{L_2}\right|_{E\sim H} \sim \frac{M_4^4}{\dot{H}M_{\rm Pl}^2}\zeta^2 \qquad \Rightarrow \qquad \tau_{\rm NL} \sim \frac{M_4^4}{\dot{H}M_{\rm Pl}^2}$$

$$\Lambda_U^4 \sim \frac{(\dot{H}M_{\rm Pl}^2)^2}{M_4^4} \sim \frac{(\dot{H}M_{\rm Pl}^2)}{\tau_{NL}}$$

$$\tau_{NL}\zeta^2 \sim \left.\frac{L_4}{L_2}\right|_{E\sim H} \sim \frac{H^4}{\Lambda_U^4}$$

Could one have an observable 4-pt function without a much larger 3-pt?

Could one have a $(g_{00}+1)^4$ terms without $(g_{00}+1)^2$ and $(g_{00}+1)^3$ ones?



$$M_4^4(\delta g^{00})^4 \to M_4^4\left(16\dot{\pi}^4 - 32\dot{\pi}^3(\partial_\mu\pi)^2 + 24\dot{\pi}^2(\partial_\mu\pi)^4 - 8\dot{\pi}(\partial_\mu\pi)^6 + (\partial_\mu\pi)^8\right)$$

Although loop corrections generate 3-pt interactions they can be consistently small.

$$M_4^4(\delta g^{00})^4 \qquad \to \qquad \dot{H}M_{\rm Pl}^2(\delta g^{00})^2\ ,\dot{H}M_{\rm Pl}^2(\delta g^{00})^3 \qquad f_{NL} \sim 1$$

$$k_1^9 \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = F(\frac{k_2}{k_1}, \frac{k_3}{k_1}, \frac{k_4}{k_1}, \theta, \phi)$$

Although an arbitrary function of five continuous variable only one possibly large shape from single clock models, created by the interaction

$$\dot{\pi}^4$$

Searching for the signal

The best data set is the one with the largest number of high signal to noise measurements (pixels, Fourier modes). Constraints go as

$$N_{\rm pix}^{-1/2}$$

WMAP and future all sky CMB experiments are the most promising.

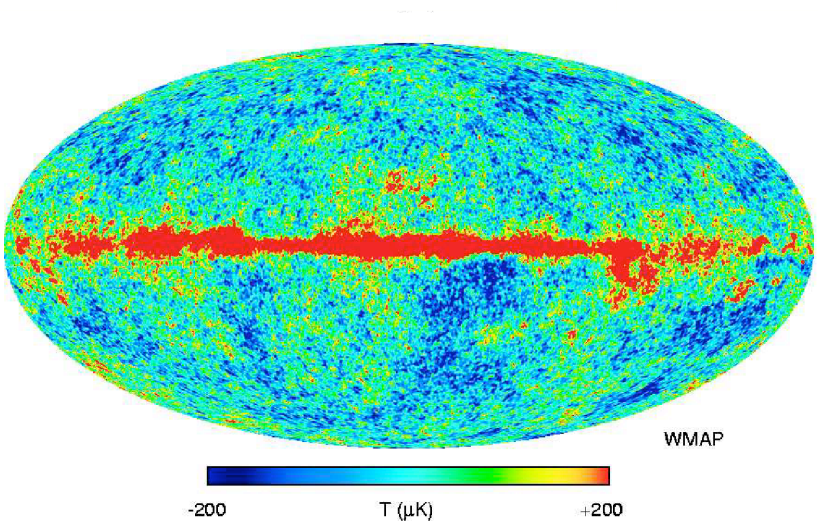
Surveys of hydrogen at high redshift using its 21 cm line could potentially do better.

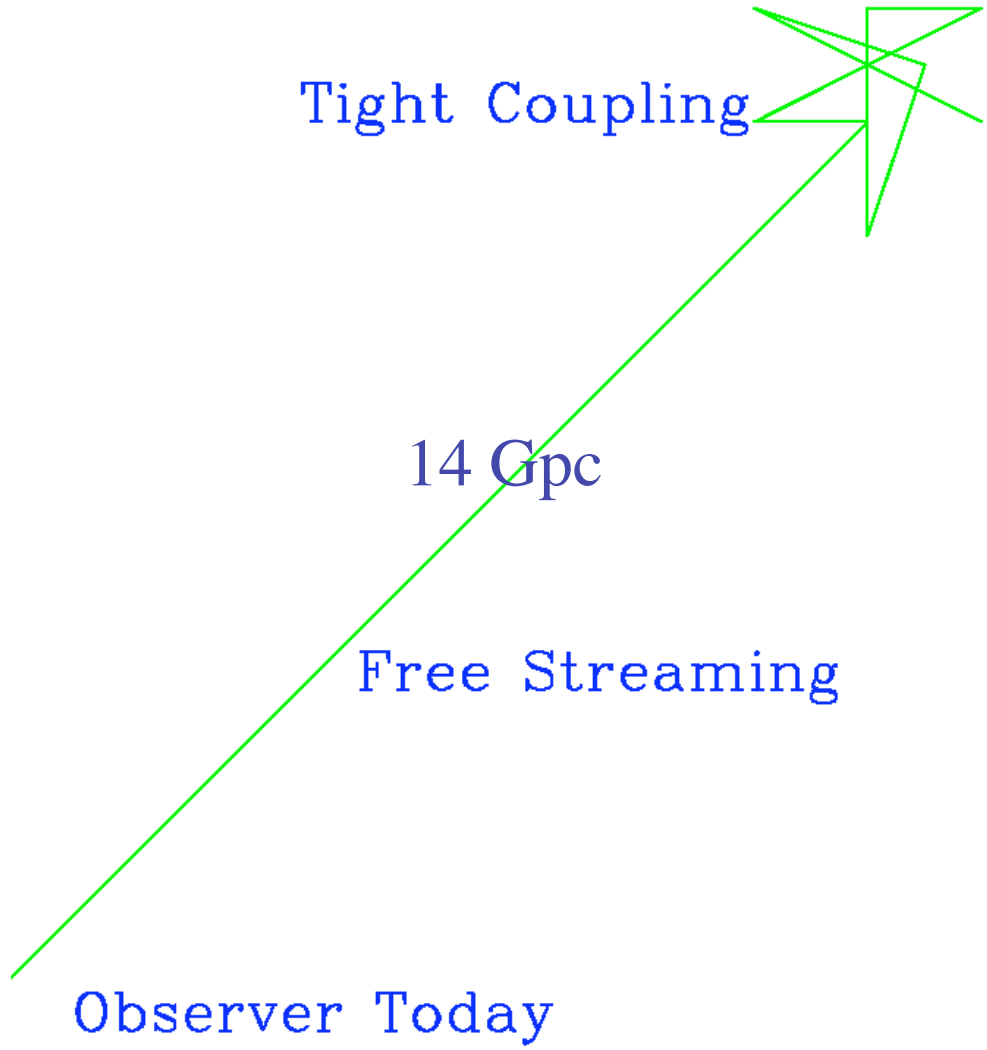
The basics of CMB Anisotropies

$$\frac{\delta T}{T} = \phi + \frac{\delta_\gamma}{4} + \frac{v_r}{c}$$

All 3 effects have the same origin

$$P \sim \lambda_T \nabla v$$





Analysis

Optimal weight

Three point function

$$\mathcal{E} = \frac{1}{N} \cdot \sum_{l_i m_i} \frac{\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle}{C_{l_1} C_{l_2} C_{l_3}} a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}$$

Amplitude of primordial fluctuations

$$a_{lm} = 4\pi i^l \int \frac{d^3k}{(2\pi)^3} \Delta_l^T(k) \Phi(k) Y_{lm}^*(\hat{k})$$

Primordial 3 point function

$$\langle \Phi(\mathbf{k}_1) \Phi(\mathbf{k}_2) \Phi(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) F(k_1, k_2, k_3)$$

Computationally very difficult unless F is factorizable:

$$F = g_1(k_1) g_2(k_2) g_3(k_3) + \cdots$$

$$F_{\dot\pi(\partial_i\pi)^2}(k_1,k_2,k_3) = -\frac{27}{17}\, f_{NL}^{\dot\pi(\partial_i\pi)^2}\Delta_\Phi^2$$
$$\times\frac{(24{K_3}^6 - 8{K_2}^2{K_3}^3K_1 - 8{K_2}^4{K_1}^2 + 22{K_3}^3{K_1}^3 - 6{K_2}^2{K_1}^4 + 2{K_1}^6)}{{K_3}^9{K_1}^3}$$
$$F_{\dot\pi^3}(k_1,k_2,k_3) = 162\, f_{NL}^{\dot\pi^3}\Delta_\Phi^2 \cdot \frac{1}{{K_3}^3{K_1}^3}\ .$$

$$f_{NL}^{\dot\pi(\partial_i\pi)^2} = \frac{85}{324}\left(1-\frac{1}{c_s^2}\right)\ ,$$
$$f_{NL}^{\dot\pi^3} = \frac{10}{243}\left(1-\frac{1}{c_s^2}\right)\left(\tilde c_3 + \frac{3}{2}c_s^2\right)$$

$$F(k,k,k) = f_{NL}\cdot\frac{6\Delta_\Phi^2}{k^6}$$

$$\begin{aligned} K_1 &= k_1+k_2+k_3\ , \\ K_2 &= (k_1k_2+k_2k_3+k_3k_1)^{1/2} \\ K_3 &= (k_1k_2k_3)^{1/3}\ . \end{aligned}$$

$$F_{(1)} \cdot F_{(2)} = \sum_{k_i^{\text{physical}}} F_{(1)}(k_1, k_2, k_3) F_{(2)}(k_1, k_2, k_3) / \left(P_{k_1} P_{k_2} P_{k_3}\right)$$

$$\cos(F_{(1)}, F_{(2)}) = \frac{F_{(1)} \cdot F_{(2)}}{(F_{(1)} \cdot F_{(1)})^{1/2} (F_{(2)} \cdot F_{(2)})^{1/2}}$$

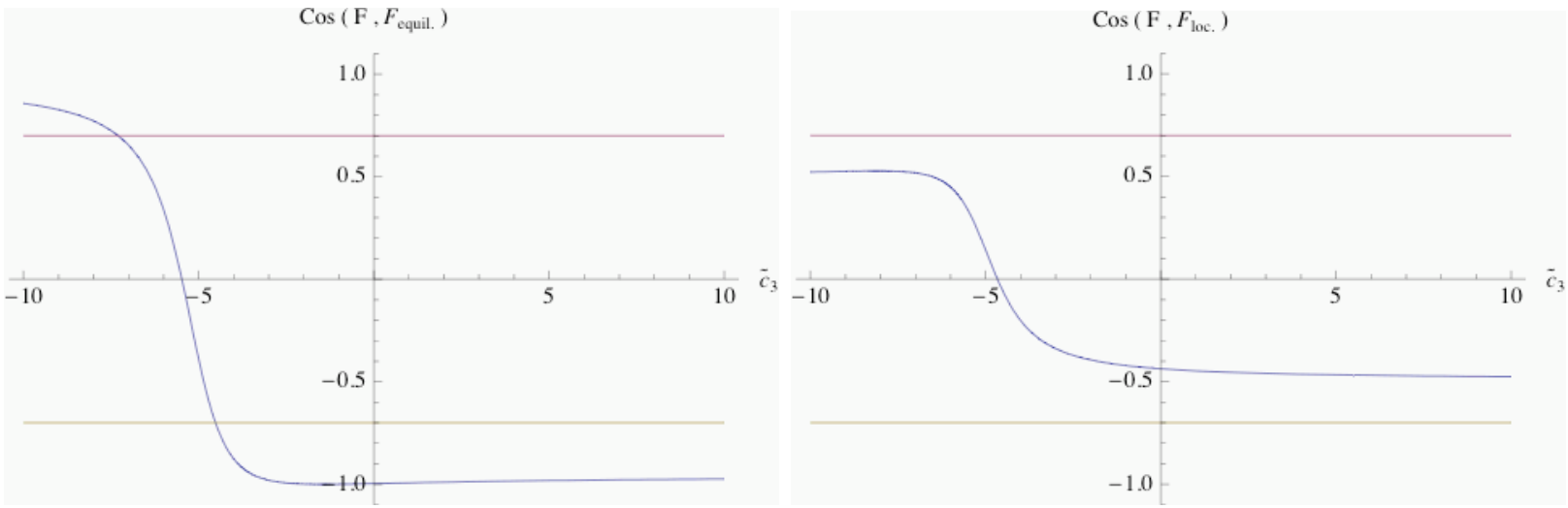


Figure 2: *Left:* Cosine of single-field shape with the equilateral shape as we vary $\tilde{c}_3$ with $c_s \ll 1$, the regime in which it is independent of c_s. The two horizontal lines represent when the scalar product is equal to ± 0.7, to give a rough measure of when the cosine becomes small. *Right:* Cosine with the local shape.

Current constraints on single clock models

From WMAP 5 yr data release

$$-125 \leq f_{NL}^{\rm equil} \leq 435 \quad 95\% \ \ {\rm CL}$$

$$-369 \leq f_{NL}^{\rm orthog} \leq 71 \quad 95\% \ \ {\rm CL}$$

$$\Longrightarrow \quad c_s \geq 0.011 \quad 95\% \ \ {\rm CL}$$

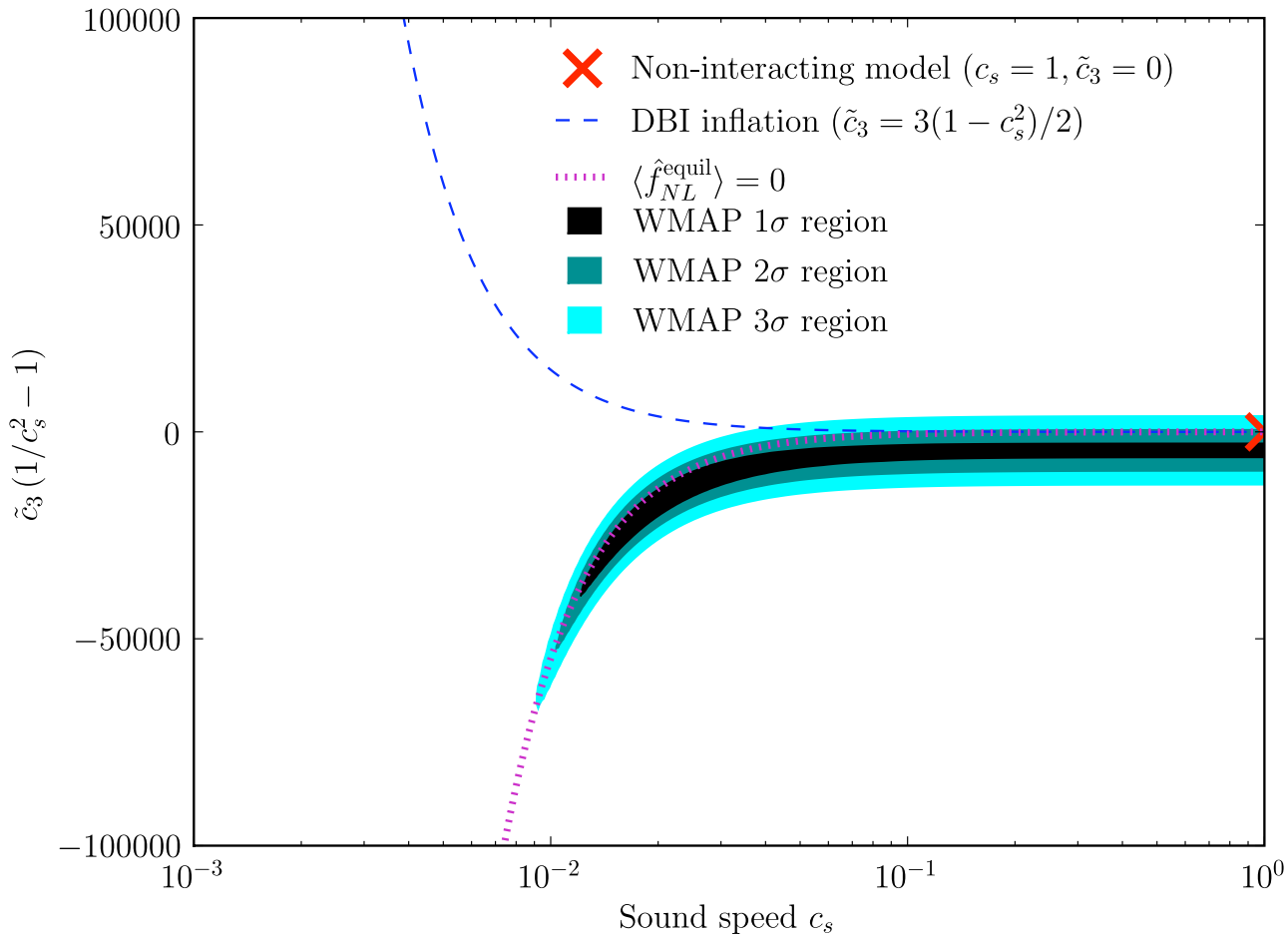
Power spectrum

$$\langle \Phi_{\vec{k}_1} \Phi_{\vec{k}_2} \Phi_{\vec{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\sum_i \vec{k}_i) F(k_1, k_2, k_3) \qquad F(k,k,k) = f_{NL} \cdot \frac{6\Delta_\Phi^2}{k^6} \ ,$$

$$f_{NL}^{\dot{\pi}(\partial_i \pi)^2} = \frac{85}{324}\left(1 - \frac{1}{c_s^2}\right) \ ,$$

$$f_{NL}^{\dot{\pi}^3} = \frac{10}{243}\left(1 - \frac{1}{c_s^2}\right)\left(\tilde{c}_3 + \frac{3}{2}c_s^2\right)$$

Smith, Senatore, MZ 0905.3746



$$c_s \geq 0.011 \qquad \text{at } 95\% \text{ CL}$$

$$-125 \leq f_{NL}^{\rm equil.} \leq 435 \quad \text{at } 95\% \text{ CL}$$

$$-369 \leq f_{NL}^{\rm orthog.} \leq 71 \quad \text{at } 95\% \text{ CL}$$

One cannot put a bound on cs if only one shape is measured.

Current constraints on the local shape

$$-4 \leq f_{NL}^{\rm local} \leq 80 \quad 95\% \;\; {\rm CL}$$

Our error bars are roughly 40% smaller than previous analysis.

We outperform the previous analysis on both the large and small scales.

Our results are robust to doing different cuts on the data (3 yr vs 5yr, details of the mask, range of l used). All the differences we see are consistent with being statistical.

We see no evidence of foreground contamination and we are robust to the procedure used to subtract point sources.

Smith, Senatore & MZ 0901.2572

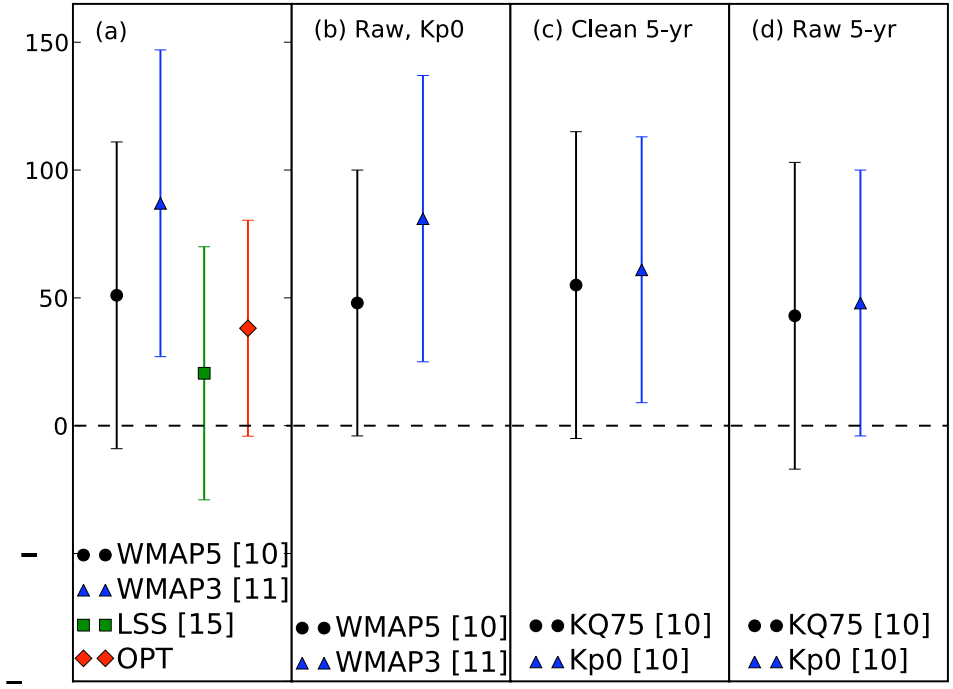


Figure 1: Current constraints on $f_{NL}^{\rm local}$. Errors in this figure and throughout the paper are 2-σ. Panel (a) best results from WMAP 5 years from the WMAP team [10] and WMAP 3 years from Yadav & Wandelt [11] together with the large scale structure results from Slosar et al [15] and the results from this paper using our optimal method (OPT). Panel (b) comparison of [10] and [11] for the same choice of analysis parameters ($l_{max} = 500$, raw maps and the Kp0 mask). Panels (c) and (d) show the effect of the mask for cleaned and raw maps respectively (from [10]).

$$\widehat{\mathcal{E}} = \frac{1}{\mathcal{N}} B_{\ell_1\ell_2\ell_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left[(C^{-1}\hat{a})_{\ell_1 m_1} (C^{-1}\hat{a})_{\ell_2 m_2} (C^{-1}\hat{a})_{\ell_3 m_3} - 3 C^{-1}_{\ell_1 m_1, \ell_2 m_2} (C^{-1}\hat{a})_{\ell_3 m_3} \right]$$

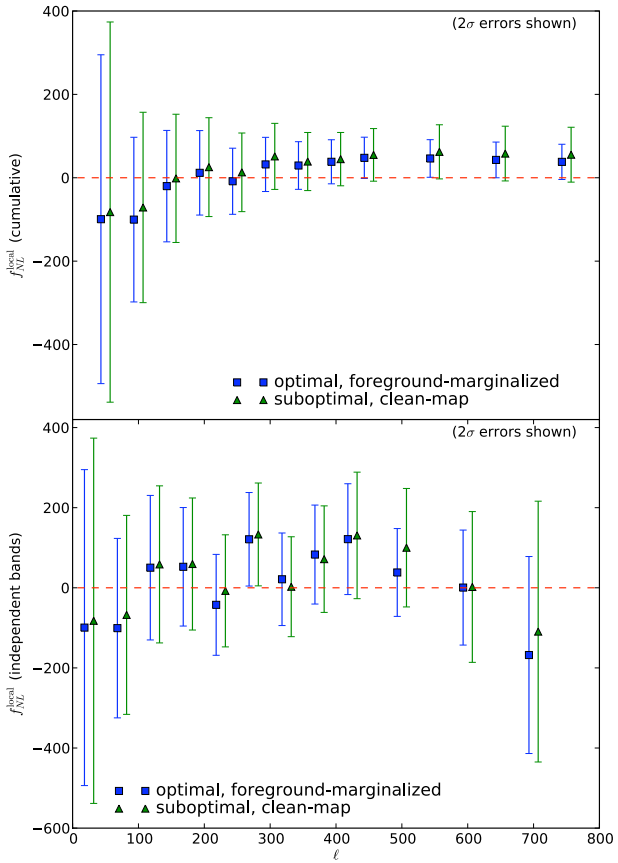


Figure 2: Constraints on $f_{NL}^{\rm local}$ using 5-year data and KQ75 mask, using both the optimal estimator (squares) and the old estimator applied to clean maps (triangles). The top panel shows cumulative results (constraints using all the information up to a given ℓ) while the bottom one shows contributions from separate ℓ bins. Our overall $f_{NL}^{\rm local}$ estimate, taking $\ell_{\rm max} = 750$, is (38 ± 21) for the optimal estimator and (55 ± 33) for suboptimal.

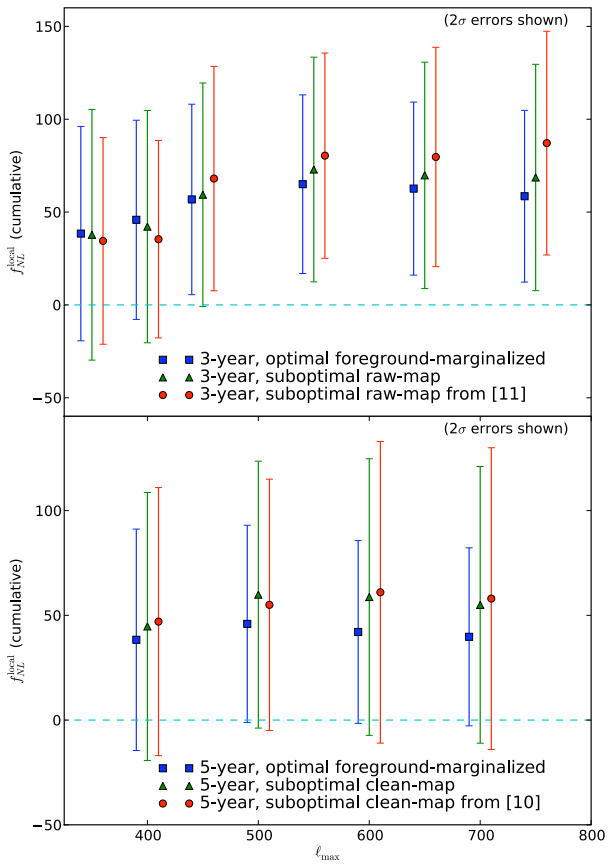


Figure 4: *Top panel:* Comparison between 3-year results reported in [11] and results obtained from our pipeline, using either the optimal or suboptimal estimator. We apply the suboptimal estimator to 3-year raw maps for consistency with [11]. *Bottom panel:* Comparison between 5-year results (optimal estimator, raw maps) reported in [10] and results obtained from our pipeline using the optimal or suboptimal estimator. We apply the suboptimal estimator to 5-year clean maps for consistency with [10].

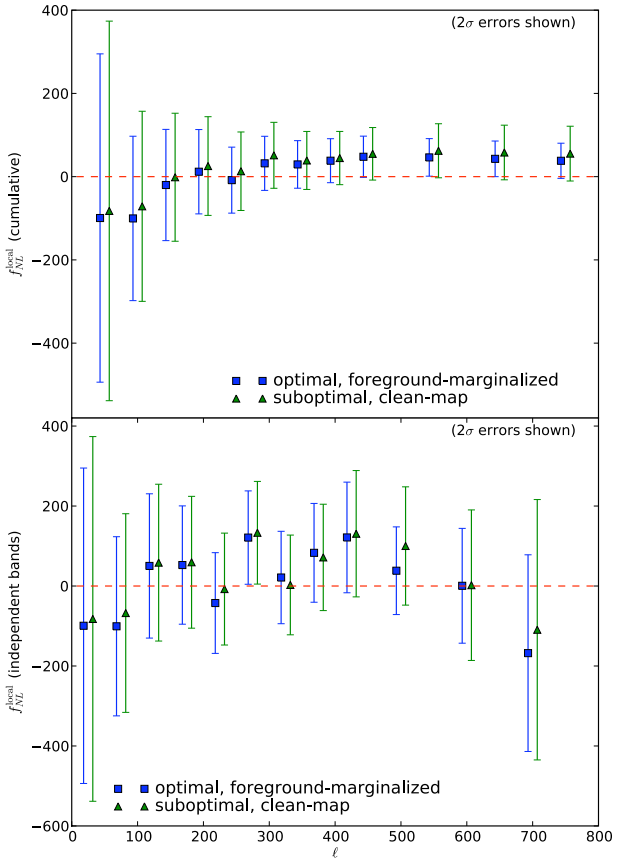


Figure 2: Constraints on $f_{NL}^{\rm local}$ using 5-year data and KQ75 mask, using both the optimal estimator (squares) and the old estimator applied to clean maps (triangles). The top panel shows cumulative results (constraints using all the information up to a given ℓ) while the bottom one shows contributions from separate ℓ bins. Our overall $f_{NL}^{\rm local}$ estimate, taking $\ell_{\rm max} = 750$, is (38 ± 21) for the optimal estimator and (55 ± 33) for suboptimal.

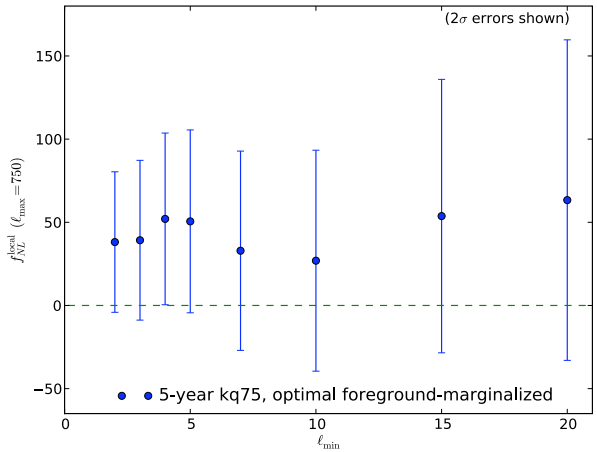
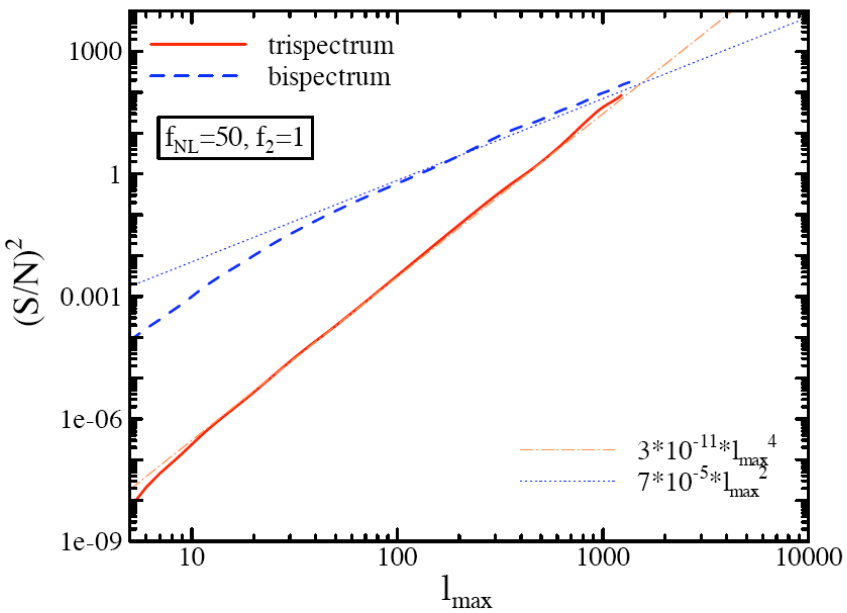


Figure 7: $f_{NL}^{\rm local}$ estimates with varying minimum multipole $\ell_{\rm min}$, using the optimal foreground-marginalized estimator, five-year data, KQ75 mask, and taking $\ell_{\rm max} = 750$ throughout.

Could one extract more information using other statistics?

4 Point function



Other statistics have been suggested such as Minkowski functionals, various wavelets, etc. It has also been suggested that higher order terms in the expansion can be detected.

FIG. 2: Predicted signal-to-noise ratio squared, $(S/N)^2$, of the angular trispectrum and bispectrum with the full radiation transfer function for $f_{\rm NL} = 50$ and $f_2 = 1$. Power-law fits are also shown. Note that the power-law fits break down at $l_{\rm max} \gtrsim 3000$, where the gravitational lensing effects become important: $(S/N)^2$ ceases to grow at $l_{\rm max} \sim 3000$ [2, 6].

Kogo & Komatsu, Hu & Okamoto

One can compute the full likelihood for fnl and calculate the best constraints that can be obtained by any statistic, the Cramer-Rao bound. Basically all the information is contained in the three point function.

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Construction of the Action

$$S = \int d^4x \sqrt{-g}\Big[\frac{1}{2}M_{\rm Pl}^2 R - c(t)g^{00} - \Lambda(t) + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 + \\ -\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + ...\Big] \ ,$$

Exercises:

1. Show that the various terms are invariant under time dependent spatial changes in coordinates
2. Vary the action with respect to the metric to obtain background equations. Plug solution back into the action for the fluctuations. Convince yourself that the action starts quadratic.

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\big[c(t)+\Lambda(t)\big]$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{3M_{\rm Pl}^2}\big[2c(t)-\Lambda(t)\big] \ .$$

$$S = \int d^4x \sqrt{-g}\Big[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2(3H^2+\dot{H}) + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 + \\ -\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + ...\Big] \ . \qquad (10)$$

Connection to scalar field examples

Simple Examples:

$$\int d^4x\,\sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 - V(\phi)\right] \to \int d^4x\,\sqrt{-g}\left[-\frac{\dot{\phi}_0(t)^2}{2}g^{00} - V(\phi_0(t))\right]$$

$$\dot{\phi}_0(t)^2 = -2M_P^2\dot{H} \text{ and } V(\phi(t)) = M_{\rm Pl}^2(3H^2 + \dot{H})$$

K-Inflation

$$L = P(X,\phi), \text{ with } X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.$$

$$S = \int d^4x\,\sqrt{-g}\,P(\dot{\phi}_0(t)^2 g^{00}, \phi(t))$$

$$M_n^4(t) = \dot{\phi}_0(t)^{2n}\partial^n P/\partial X^n$$

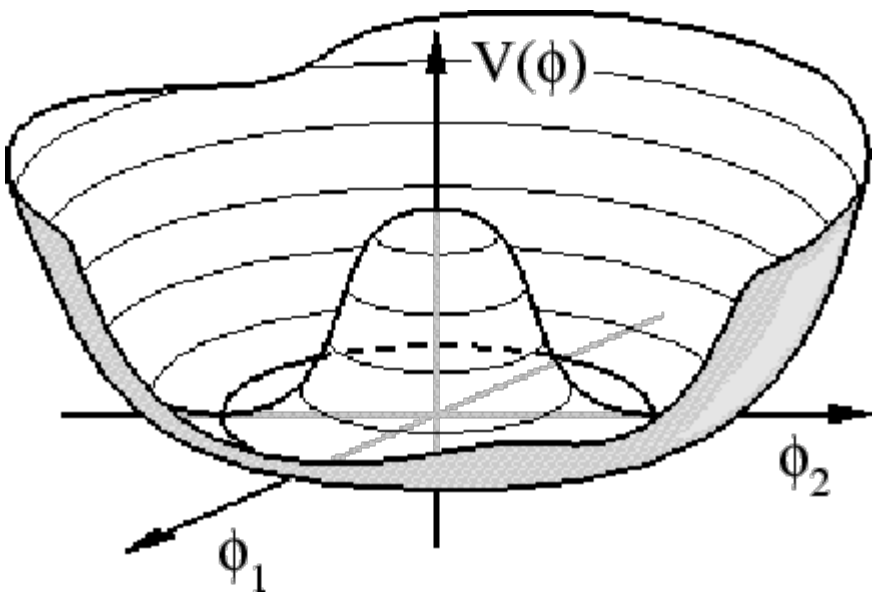
Introducing the π: An analogy

$$\mathcal{L} = |D_\mu \phi|^2 - V(|\phi|) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\phi \to e^{i\theta(x)} \phi$$

$$D_\mu \phi = \partial_\mu \phi + i q A_\mu \phi$$

$$A_\mu \to A_\mu - \frac{1}{q} \partial_\mu \theta$$



a radius of the minimum

reparametrize ϕ

$$\phi = \rho e^{i\alpha/a}$$

What is the Lagrangian for the fluctuations?

Call M the curvature in the radial direction.

$$\mathcal{L} = \frac{1}{2}(\partial\rho)^2 - \frac{1}{2}M^2\rho^2 - \frac{1}{4}F^2 - q^2a^2A_\mu A^\mu + \frac{1}{2}(\partial\alpha)^2 + \frac{1}{2}aq\partial_\mu\alpha A^\mu$$

Massive radial excitations

Vector modes

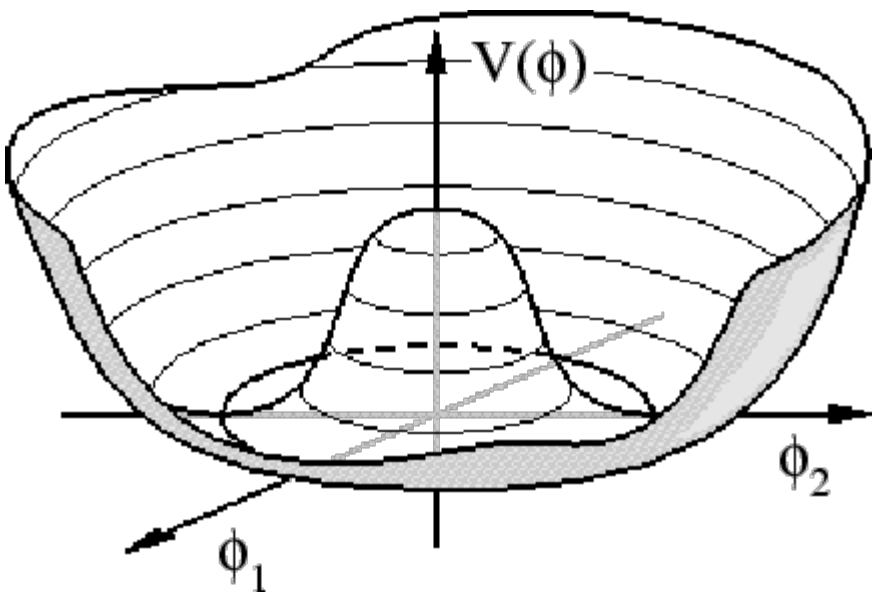
Goldstone Boson

$$\phi \to e^{i\theta(x)}\phi$$

$$\phi = \rho e^{i\alpha/a}$$

Can always choose gauge where α is 0. At low energies massive vector that has three polarization. $m_A^2 = q^2a^2$

However at energies above the mass of the vector the mixing terms are unimportant so talking about the Goldstone is useful.



The gauge symmetry of the example is equivalent to the freedom in GR to change the time coordinate. The gauge where there is no Goldstone (the Unitary gauge) is equivalent to the gauge where there is no π

$$\alpha \to \pi$$

$$E_{\rm mix} = m_A \to \sqrt{\epsilon}H$$

$$||\vec{\phi}|| = 1$$

$$\phi = (\pi_1, \pi_2, \cdots, \sigma) \qquad \sigma = \sqrt{1 - \pi_1^2 - \pi_2^2 - \cdots}$$

$$||\partial\vec{\phi}||^2 \to (\partial\pi_1)^2 + (\partial\pi_2)^2 + \cdots + \frac{(\vec{\pi} \cdot \partial\vec{\pi})^2}{1 - \pi_1^2 - \pi_2^2 - \cdots}$$

$$m_A < E < \frac{m_A}{q}$$

The Goldstones interact but there is a range of energies for which they are both weakly coupled and are decoupled from the rest.
In Inflation because H falls exactly in this range, the description with the Goldstones is quite simple.

Gravitational waves

$$S_{\text{E.H.}+\text{S.F.}} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}M_{\text{Pl}}^2 R + M_{\text{Pl}}^2 \dot{H} g^{00} - M_{\text{Pl}}^2(3H^2+\dot{H}) + \\ + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 + \\ - \frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots\Big]$$

The action for gravitational wave perturbations in unchanged by the additional terms. Each polarization of the gravity waves, once properly normalized, has the same quadratic action as a massless scalar field so it will have fluctuations.

$$M_{\text{PL}} h_{\mu\nu} \to \phi \qquad\qquad h_{\mu\nu} \sim \frac{H}{M_{\text{PL}}}$$

The energy scale of inflation: gravitational waves

Inflation predicts the presence of a stochastic Background of Gravitational Waves

$$h_{ij} \sim \frac{H}{M_{PL}}$$

If $\frac{H}{M_{PL}} \sim 10^{-5}$ → Comparable to density perturbations

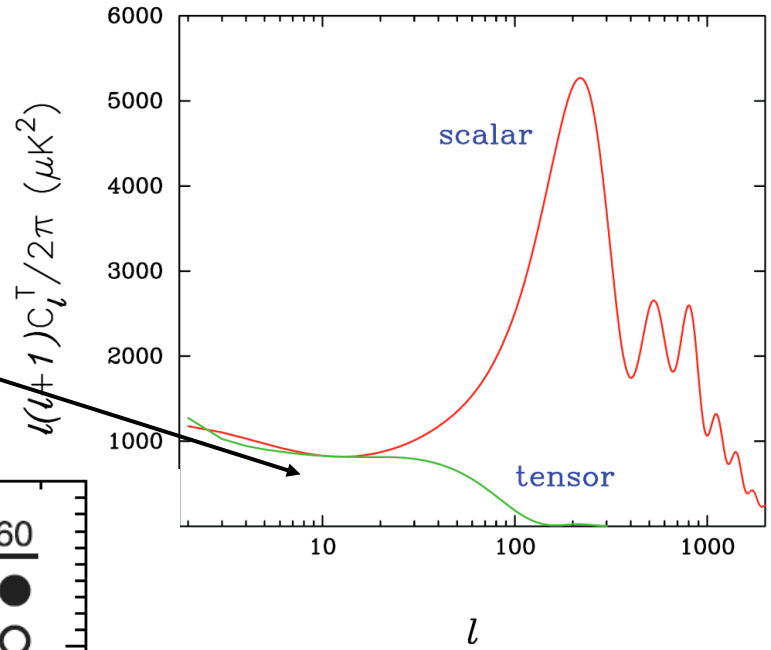
Directly measure the expansion rate during Inflation.

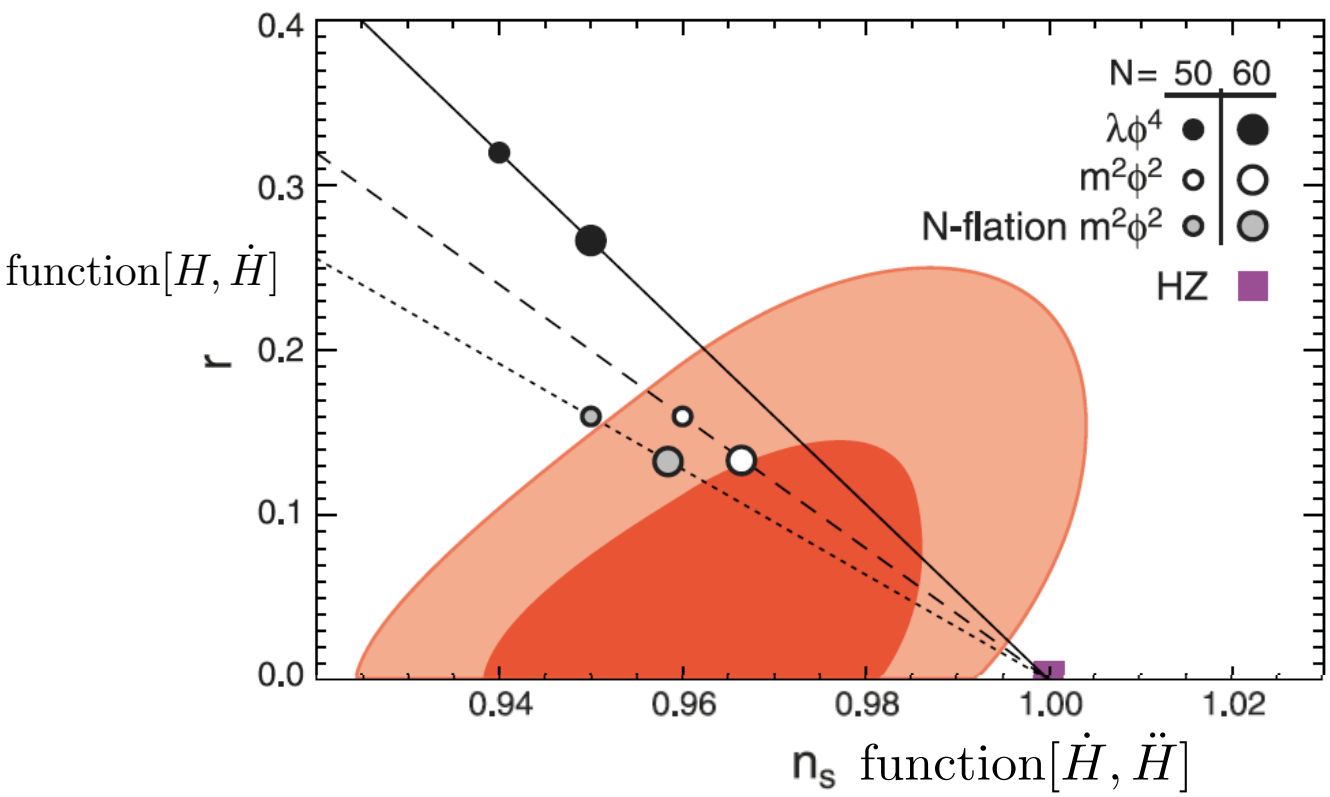
This measurement has taken a greater significance now that it appears that GW might not be observable in “string-inflation”.

Current constraints come from fitting the shape of the temperature power spectrum.

Current constraints from WMAP 5 yrs

r is ratio of contributions on large scales





CMB probes a narrow range of energy scales

Effective field theory for Quintessence

$$S = \int d^4x\, a^3 \left[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 - \frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi \right.$$
$$\left. -\frac{\bar{M}^2}{2}\left(3H\dot{\pi} - 3\dot{H}\pi + \frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right)^2 \right]. \qquad (3.1)$$

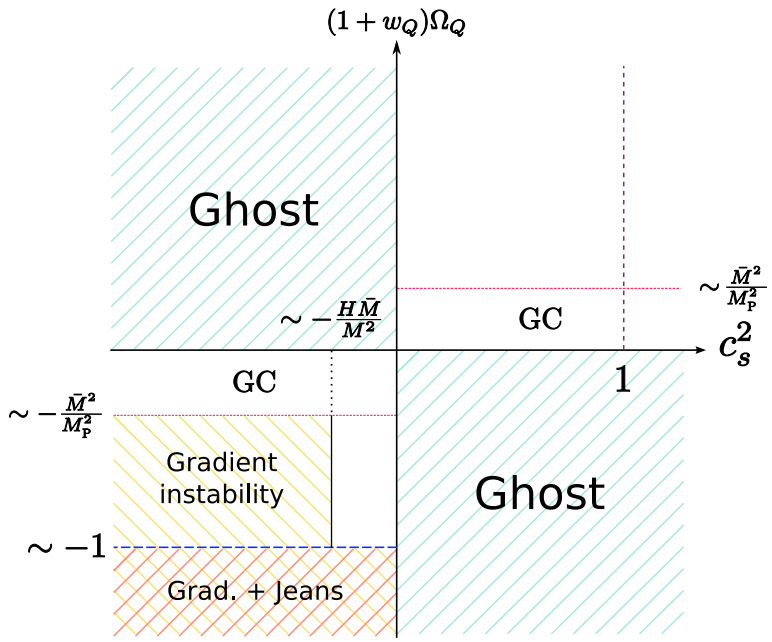


Figure 2: *On the quintessential plane, we show the theoretical constraints on the equation of state and speed of sound of quintessence, in the presence of the operator $\bar{M}$. Instability regions are dashed. Where $1 + w_Q$ and c_s^2 have opposite sign we have a ghost-like instability corresponding to negative kinetic energy. For $w_Q < -1$, the dashed regions in the left-lower panel is unstable by gradient ($c_s^2 \lesssim -H\bar{M}/M^2$) and Jeans ($(1 + w_Q)\Omega_Q \lesssim -1$) instabilities, while the strip close to the vertical axis corresponds to the stability window (2.29). Furthermore, the strip around the horizontal axis given in eq. (3.4) corresponds to the Ghost Condensate. Above this region, in the right-upper panel, we find standard k-essence.*

$$-c_s^2 \lesssim \left(\frac{H_0}{M_{\rm Pl}}\right)^{1/2} \sim 10^{-30}$$

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