Non-Gaussianities from Inflation

Leonardo Senatore, Kendrick Smith & MZ

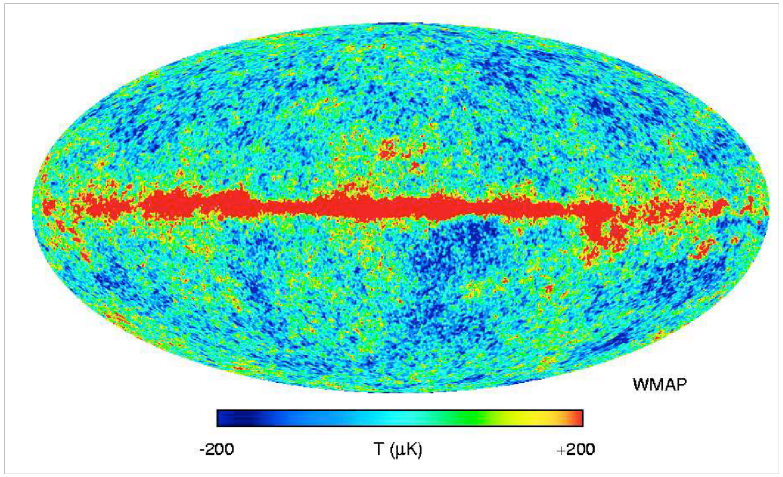
Lecture Plan:

- Lecture 1: Non-Gaussianities: Introduction and different take on inflation and inflation modeling.

- Lecture II: Non-Gaussianities: Some technical details

- Lecture III: CMB Polarization: physical origin. Information encoded in the CMB polarization.

The Era of Precision Cosmology



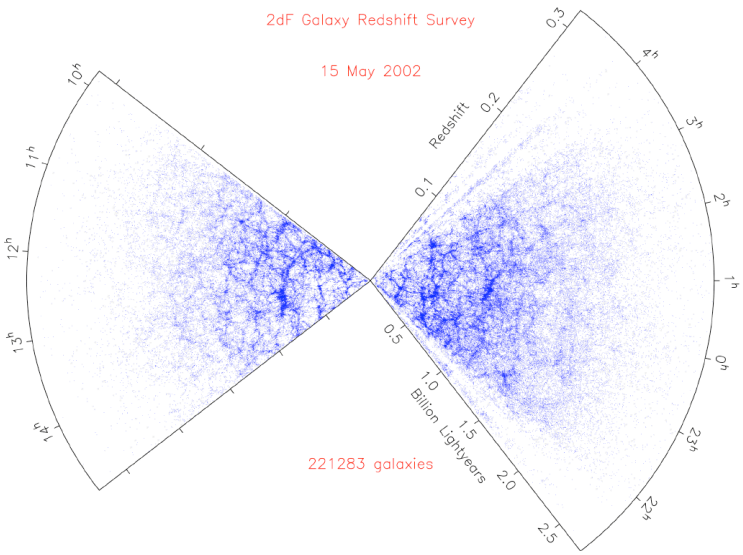


Table 6. Cosmological Parameter Summary

Description	Symbol	*WMAP*-only	*WMAP*+BAO+SN
Parameters for Standard ΛCDM Model [a]			
Age of universe	t_0	13.69 ± 0.13 Gyr	13.73 ± 0.12 Gyr
Hubble constant	H_0	$71.9^{+2.6}_{-2.7}$ km/s/Mpc	70.1 ± 1.3 km/s/Mpc
Baryon density	Ω_b	0.0441 ± 0.0030	0.0462 ± 0.0015
Physical baryon density	$\Omega_b h^2$	0.02273 ± 0.00062	0.02265 ± 0.00059
Dark matter density	Ω_c	0.214 ± 0.027	0.233 ± 0.013
Physical dark matter density	$\Omega_c h^2$	0.1099 ± 0.0062	0.1143 ± 0.0034
Dark energy density	Ω_Λ	0.742 ± 0.030	0.721 ± 0.015
Curvature fluctuation amplitude, $k_0 = 0.002$ Mpc^{-1} [b]	$\Delta^2_{\mathcal{R}}$	$(2.41 \pm 0.11) \times 10^{-9}$	$(2.457^{+0.092}_{-0.093}) \times 10^{-9}$
Fluctuation amplitude at $8h^{-1}$ Mpc	σ_8	0.796 ± 0.036	0.817 ± 0.026
$l(l+1)C_{220}^{TT}/2\pi$	C_{220}	$5756 \pm 42\ \mu K^2$	$5748 \pm 41\ \mu K^2$
Scalar spectral index	n_s	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$
Redshift of matter-radiation equality	z_{eq}	3176^{+151}_{-150}	3280^{+88}_{-89}
Angular diameter distance to matter-radiation eq.[c]	$d_A(z_{eq})$	14279^{+186}_{-189} Mpc	14172^{+141}_{-139} Mpc
Redshift of decoupling	z_*	1090.51 ± 0.95	$1091.00^{+0.72}_{-0.73}$
Age at decoupling	t_*	380081^{+5843}_{-5841} yr	375938^{+3148}_{-3115} yr
Angular diameter distance to decoupling [c,d]	$d_A(z_*)$	14115^{+188}_{-191} Mpc	14006^{+142}_{-141} Mpc
Sound horizon at decoupling [d]	$r_s(z_*)$	146.8 ± 1.8 Mpc	145.6 ± 1.2 Mpc
Acoustic scale at decoupling [d]	$l_A(z_*)$	$302.08^{+0.83}_{-0.84}$	$302.11^{+0.84}_{-0.82}$
Reionization optical depth	τ	0.087 ± 0.017	0.084 ± 0.016
Redshift of reionization	z_{reion}	11.0 ± 1.4	10.8 ± 1.4
Age at reionization	t_{reion}	427^{+88}_{-65} Myr	432^{+90}_{-67} Myr
Parameters for Extended Models [e]			
Total density [f]	Ω_{tot}	$1.099^{+0.100}_{-0.085}$	1.0052 ± 0.0064
Equation of state [g]	w	$-1.06^{+0.41}_{-0.42}$	$-0.972^{+0.061}_{-0.060}$
Tensor to scalar ratio, $k_0 = 0.002$ Mpc^{-1} [b,h]	r	< 0.43 (95% CL)	< 0.20 (95% CL)
Running of spectral index, $k_0 = 0.002$ Mpc^{-1} [b,i]	$dn_s/d\ln k$	-0.037 ± 0.028	$-0.032^{+0.021}_{-0.020}$
Neutrino density [j]	$\Omega_\nu h^2$	< 0.014 (95% CL)	< 0.0065 (95% CL)
Neutrino mass [j]	$\sum m_\nu$	< 1.3 eV (95% CL)	< 0.61 eV (95% CL)
Number of light neutrino families [k]	N_{eff}	> 2.3 (95% CL)	4.4 ± 1.5

[a] The parameters reported in the first section assume the 6 parameter ΛCDM model, first using *WMAP* data only, then using *WMAP*+BAO+SN data.

[b] $k = 0.002$ Mpc^{-1} $\longleftrightarrow$ $l_{eff} \approx 30$.

[c] Comoving angular diameter distance.

[d] $l_A(z_*) \equiv \pi d_A(z_*) r_s(z_*)^{-1}$.

[e] The parameters reported in the second section place limits on deviations from the ΛCDM model, first using *WMAP* data only, then using *WMAP*+BAO+SN data. A complete listing of all parameter values and uncertainties for each of the extended models studied is available on LAMBDA.

[f] Allows non-zero curvature, $\Omega_k \neq 0$.

[g] Allows $w \neq -1$, but assumes w is constant.

[h] Allows tensors modes but no running in scalar spectral index.

[i] Allows running in scalar spectral index but no tensor modes.

[j] Allows a massive neutrino component, $\Omega_\nu \neq 0$.

[k] Allows N_{eff} number of relativistic species. The last column adds the HST prior to the other data sets.

The initial conditions for the “standard” FRW phase

Could inflation be a part of our standard history of the Universe with the same level of confidence as Big Bang Nucleosynthesis?

The Horizon Problem

Why is the Universe so homogeneous?
Why is the temperature of the CMB in directions separated by many degrees?

$\dot{a}$ decreases during the standard FRW phase.

$$v = Hr = \dot{a}x$$

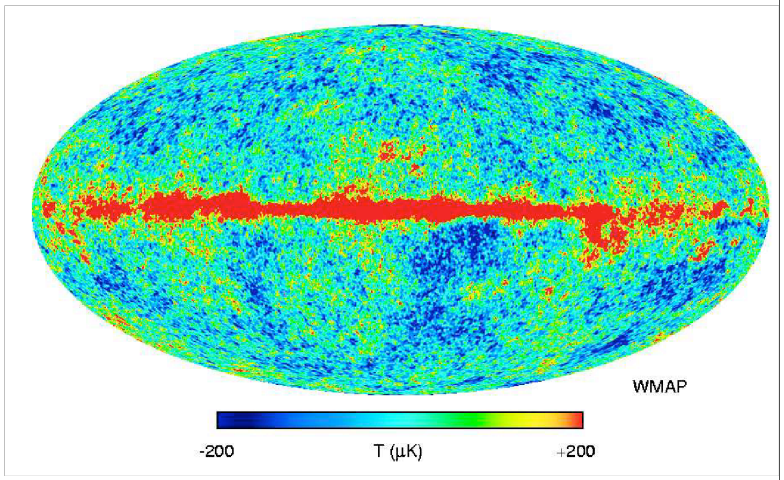
Ratio between the size of our observable Universe and the horizon at BBN is 10^{11} .

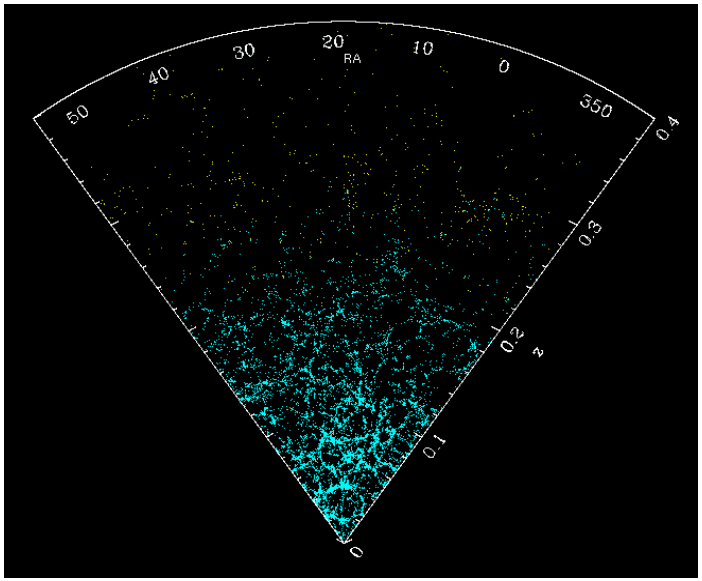
The standard model seems incomplete

Structure formation

We have a standard model that is very successful observations but it requires some initial seeds.

We have evidence that those seeds were in place very early on, perhaps as early as 10^{-36} sec after the BB (1/H for T ~ 10^{15} GeV).





The initial conditions are not trivial. How did they come about?

What caused the Bang in the Big Bang?

The fact that we now know that perturbations were produced during the Bang phase opens the opportunity to measure more numbers and increases the chances that we will be able to figure out what the Bang was.

The horizon "problem" means that there are many independent samples, many independent Universes.

Inflation is our standard model but, could we believe it with the same level of confidence as Big Bang Nucleosynthesis?

Inflation

I need a definition. For this talk I will take:

A period of accelerated expansion with

$$\dot{H} << H^2 \qquad\qquad a(t) \approx e^{Ht}$$

$$v = Hr = \dot{a}x \approx e^{Ht}Hx$$

Scales leave the horizon during inflation.

The triumph of Inflation: the origin of perturbations

What about the initial fluctuations?

Inflation can naturally create the seeds for structure, which is probably the most remarkable the feature of the theory.

Inflation

Why do we know that inflation will also lead to fluctuations?

The period of accelerated expansion eventually comes to an end.

The physical system responsible for the acceleration, the substance filling the Universe must be able to keep track of time. It is in some sense a very small clock.

Small things are governed by the laws of quantum mechanics.

There is a limit to how well one can keep track of time with a very small clock. As a result inflation will end at slightly different times in different regions. *This is the source of the initial seeds in inflationary models*.

Inflation

A period of accelerated expansion with:

$$\dot{H} << H^2$$

Quantum fluctuations in the clock that determines when inflation ends are the source of the initial seeds.

Can this paradigm one day become incorporated in our understanding of the Universe at the same level as BBN?

Standard approach:

Write down a particular inflationary models. This usually involves writing down some specific Lagrangian, usually for one or many scalar fields and computing various observables. Argue that your model is the most "natural".

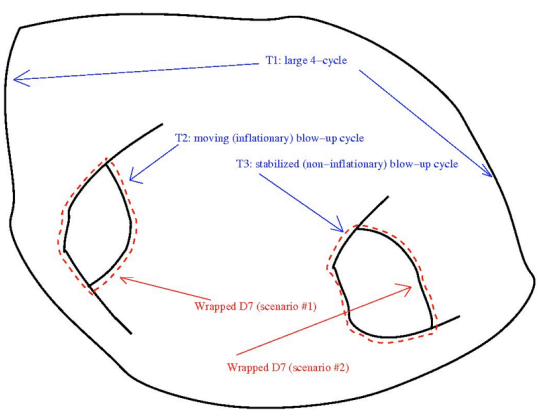
Unfortunately we do not currently have a direct connection between the "inflaton" and laboratory physics. So there is a wide range of inflationary models motivated by different criteria.

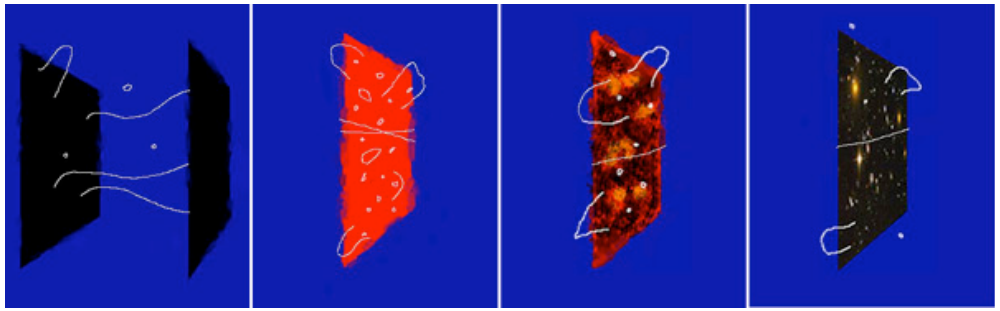
It is difficult to see in this "standard" approach which of the predictions are model independent and which are not. It is also difficult to see how one would conclude that inflation indeed happened (maybe Gravity Waves?).

In technical terms:

Very quickly it all looks like a fairy tail.

Could inflation one day be a part of our standard story of how the Universe came to be in the same way as BBN is? Could this happen just on the basis of astronomical observations?





Technical interlude

Effective theory of inflation:

Chung, Creminelli, Fitzpatrick, Kaplan & Senatore. 0709.0293

Quantum fluctuations in the clock that determines when inflation ends are the source of the initial seeds.

What can the dynamics of this clock be? It is characterized by just a handful of numbers.

I will try to summarize how that is done so that you see that you never need to draw a potential, etc just need to specify a few numbers. Of course in some sense this is old news as predictions in the standard approach typically depend only on the slow roll parameter.

Forget the fairy tale.

Of course the fairly tale may be true, but the point is that *we do not need to talk about it*.

Will need the action of the perturbations to calculate the how large the quantum fluctuations are.

$$\frac{S}{\hbar} = \frac{1}{\hbar}\int dt \frac{M}{2}(\dot{x}^2 - \omega^2 x^2)$$

$$\delta x^2 \sim \frac{\hbar}{M\omega}$$

Effective theory of inflation: Chung, Creminelli, Fitzpatrick, Kaplan & Senatore. 0709.0293

Use the measured time in the clock as the time coordinate.
The clock disappears from the action, everything is in the metric.
Can still make time dependent transformations of the spatial coordinates but time has been fixed. Terms must respect the residual symmetry.

$$S_{\text{E.H. + S.F.}} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}M_{\text{Pl}}^2 R + M_{\text{Pl}}^2 \dot{H} g^{00} - M_{\text{Pl}}^2(3H^2 + \dot{H}) + \\ + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 + \ \ldots. \\ - \frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^{\mu}{}_{\mu} - \frac{\bar{M}_2(t)^2}{2}\delta K^{\mu}{}_{\mu}{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^{\mu}{}_{\nu}\delta K^{\nu}{}_{\mu} + \ldots\Big]$$

$\delta K_{\mu\nu}$ the variation of the extrinsic curvature of constant time surfaces
Has one more derivative.

Expansion in fluctuations and in derivatives. Coefficients in the first line are such that the action starts quadratic.

Can reintroduce the clock. Call its fluctuations π

$$t \longrightarrow t - \pi$$

$$g^{00} \quad \longrightarrow \quad \frac{\partial(t+\pi)}{\partial x^\mu}\frac{\partial(t+\pi)}{\partial x^\nu} g^{\mu\nu}$$

For the models discussed here the terms involving π are the dominant.

$$g^{00} \quad \longrightarrow \quad -1 - 2\dot{\pi} - \dot{\pi}^2 + \frac{1}{a^2}(\partial_i \pi)^2$$

$$S_{\text{E.H.}+\text{S.F.}} = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}M_{\text{Pl}}^2 R + M_{\text{Pl}}^2 \dot{H} g^{00} - M_{\text{Pl}}^2(3H^2+\dot{H}) +$$

$$+\frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 +$$

$$-\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^{\mu}{}_{\mu} - \frac{\bar{M}_2(t)^2}{2}\delta K^{\mu}{}_{\mu}{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^{\mu}{}_{\nu}\delta K^{\nu}{}_{\mu} + \ldots\Big]$$

$$g^{00} \quad \rightarrow \quad -1 - 2\dot{\pi} - \dot{\pi}^2 + \frac{1}{a^2}(\partial_i\pi)^2$$

$$S_\pi = \int d^4x\sqrt{-g}\left[-M_{\text{Pl}}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3\right]$$

Parameters: $H, \quad \dot{H}, \quad \ddot{H}, \quad M_2, \quad M_3$

Structure is set by the symmetries, the requirement that everything can be incorporated into the metric by a suitable choice of coordinates. Specific signs and coefficients in front of various terms, requirement of certain interactions, difference between time and space derivatives.

$$S_\pi = \int d^4x\sqrt{-g}\left[-M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3\right]$$

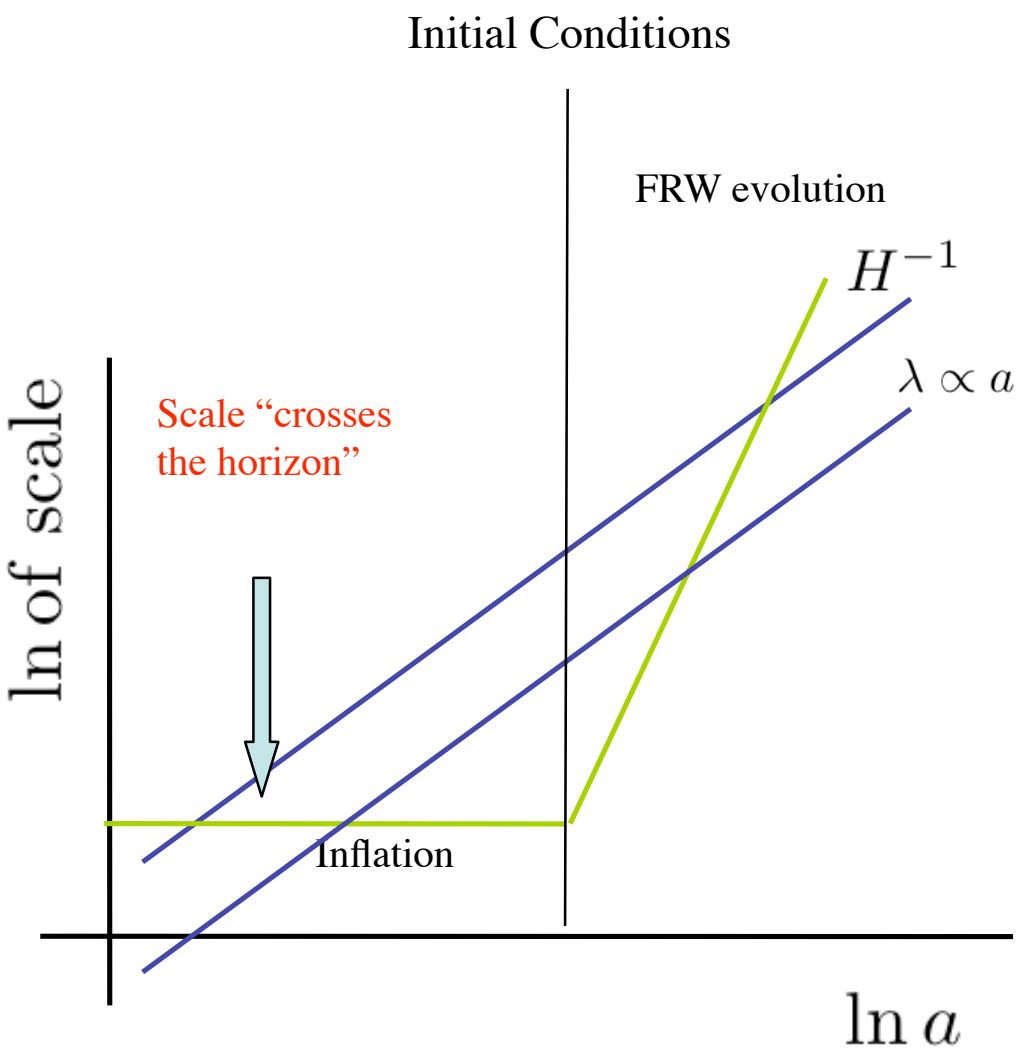
M_2 changes the dispersion relation of modes, introducing a "sound speed". Same term that changes the propagation speed generates interactions.

$$M_2^4 = -\frac{1-c_s^2}{c_s^2}\frac{M_{\rm Pl}^2\dot{H}}{2}$$

$$\omega^2 = c_s^2 k^2$$

$$S_\pi = \int d^4x\,\sqrt{-g}\left[-\frac{M_{\rm Pl}^2\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{M_{\rm Pl}^2\dot{H}}{c_s^2}\dot{\pi}\frac{(\partial_i\pi)^2}{a^2} - \frac{2}{3}\frac{\tilde{c}_3}{c_s^4}M_{\rm Pl}^2\dot{H}\dot{\pi}^3\right]$$

Connecting to observations



The clock fluctuations are "frozen" at horizon crossing (frequency of order H). *We are probing the theory at an energy H which is roughly constant in time*. Note that the range of scales we are talking about could be huge, as much as e^{60}.

What we need to calculate is the additional expansion of one region relative to the other due to the clock fluctuations:

$$\zeta = H\pi = \frac{\dot{a}}{a}\pi = \frac{\Delta a}{a}$$

Curvature fluctuations

Observational Consequences

The Effective theory of Multifield inflation (in preparation)
Leonardo Senatore, Matias Zaldarriaga

Non-Gaussianities in Single Field Inflation and their Optimal Limits from the WMAP 5-year Data.
Leonardo Senatore, Kendrick M. Smith, Matias Zaldarriaga, . May 2009. 46pp. Brief entry
e-Print: **arXiv:0905.3746**

Optimal limits on f_{NL}^{local} from WMAP 5-year data.
Kendrick M. Smith, Leonardo Senatore, Matias Zaldarriaga, . Jan 2009. 21pp. Brief entry
Published in **JCAP 0909:006,2009**.
e-Print: **arXiv:0901.2572** [astro-ph]

Limits on f_NL parameters from WMAP 3yr data.
Paolo Creminelli, (ICTP, Trieste) , Leonardo Senatore, (Harvard U.) , Matias Zaldarriaga, (Harvard U. & Harvard-Smithsonian Ctr. Astrophys.) , Max Tegmark, (MIT) . IC-2006-108, HUTP-06-A0041, Oct 2006. 10pp.
Published in **JCAP 0703:005,2007**.
e-Print: **astro-ph/0610600**

Estimators for local non-Gaussianities.
Paolo Creminelli, (ICTP, Trieste) , Leonardo Senatore, (MIT) , Matias Zaldarriaga, (Harvard U., Phys. Dept. & Harvard-Smithsonian Ctr. Astrophys.) . HUTP-06-A0016, MIT-CTP-3737, IC-2006-028, Jun 2006. 25pp.
Published in **JCAP 0703:019,2007**.
e-Print: **astro-ph/0606001**

Limits on non-gaussianities from wmap data.
Paolo Creminelli, Alberto Nicolis, (Harvard U., Phys. Dept.) , Leonardo Senatore, (MIT, LNS & MIT) , Max Tegmark, (MIT) , Matias Zaldarriaga, (Harvard U., Phys. Dept. & Harvard-Smithsonian Ctr. Astrophys.) . HUTP-05-A0038, MIT-CTP-3670, Sep 2005. 20pp.
Published in **JCAP 0605:004,2006**.
e-Print: **astro-ph/0509029**

Observational Consequences: 2-pt function

$$S_\pi = \int d^4x\sqrt{-g}\left[-M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3\right]$$

When interactions are small I just have a collection of harmonic oscillators. Wave function in the vacuum state is a Gaussian resulting in Gaussian initial fluctuations. All it left to measure in that limit is the two point correlation function.
Translational invariance demands only a function of distance.
Simple arguments demand it to be close to scale invariant, ie power law with specific slope.
Not that much to measure.

$$k^3|\zeta_k|^2 = A(\frac{k}{k_*})^{(n-1)}$$

Scale invariance a consequence of time translation invariance:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} << 1$$

Scale invariance is a natural consequence of having a fluctuating clock in a near de-Sitter background. Probably difficult to get in another way, especially if there are 60- efolds of scale invariant fluctuations, but still does not leave much to measure.

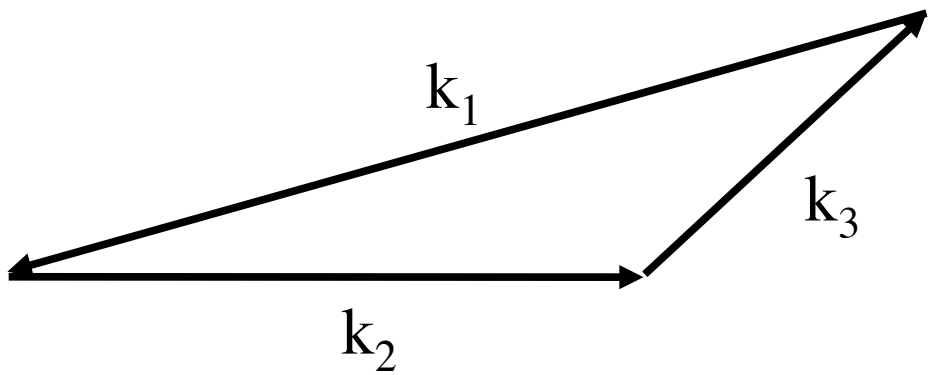
This is especially true because specific values of the two numbers that can be measured are not really predictable because they are not a the direct consequence of the symmetries but depend on the details of the background.

For example they do just follow from

$$\epsilon \equiv -\frac{\dot{H}}{H^2} << 1$$

but depend on the value of the ratio.

Observational Consequences: 3-pt function



$$k_1^6 \langle \zeta_{k1} \zeta_{k2} \zeta_{k3} \rangle = F(\frac{k_2}{k_1}, \frac{k_3}{k1})$$

Higher order moments, departure from Gaussianity are sensitive to the interactions.
Even after requiring scale invariance and translation invariance the three point function is still an arbitrary function of two continuous variable.

$$S_\pi = \int d^4x\sqrt{-g}\left[-M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3\right]$$

Just two specific shapes for this two shapes are predicted from each of the two possible interactions:

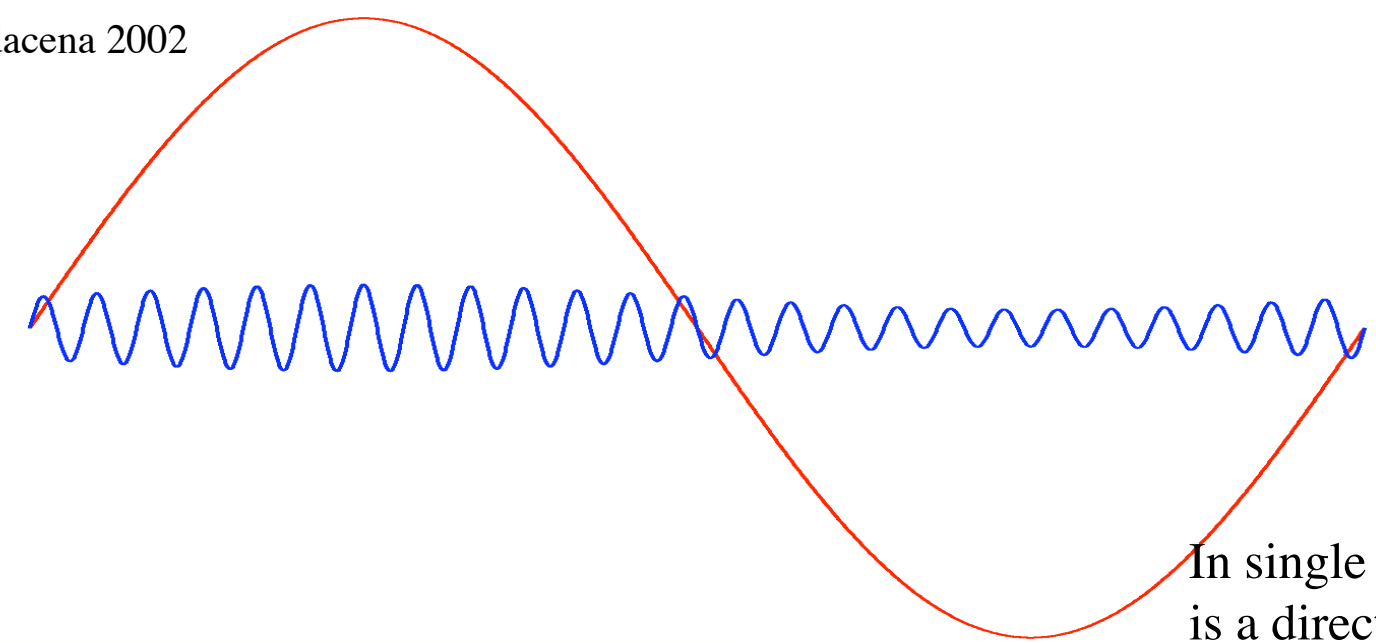
$$\dot{\pi}^3 \quad \dot{\pi}(\partial_i\pi)^2$$

The Squeezed limit

$k_3 << k_1 \sim k_2$

$$\zeta(x) = \zeta_g(x) + f_{NL}\zeta_g^2(x) + \cdots$$

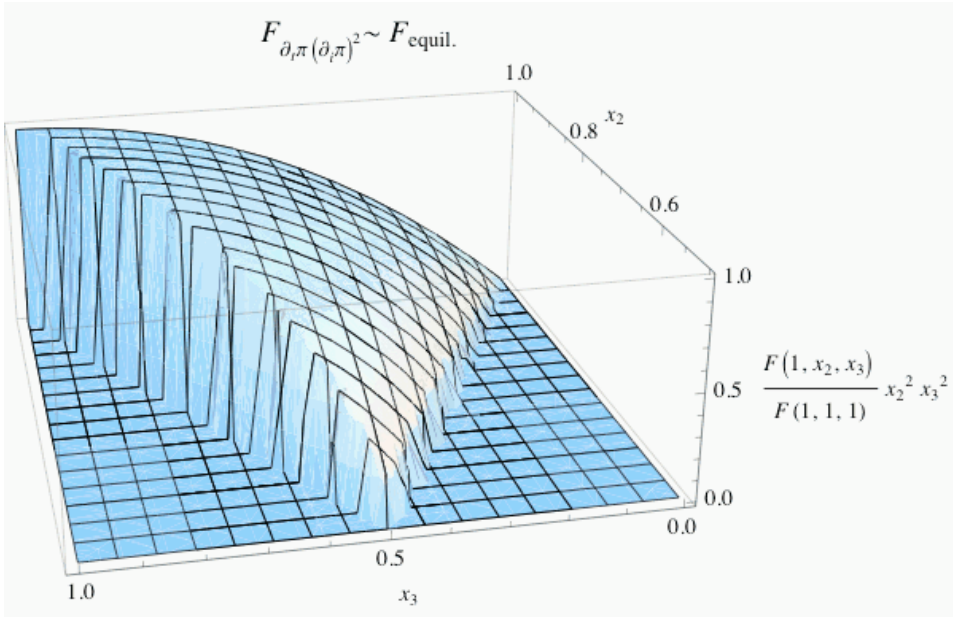
Maldacena 2002

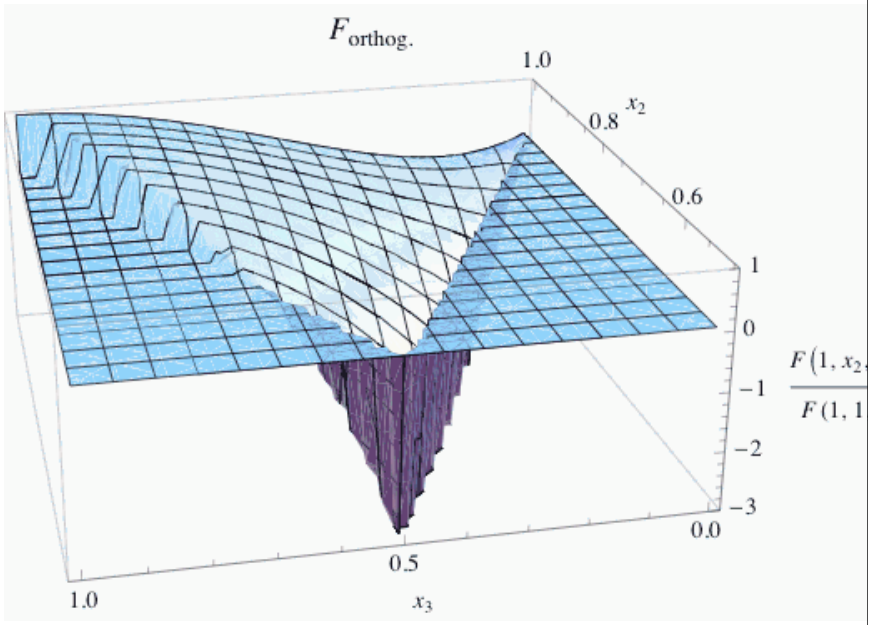


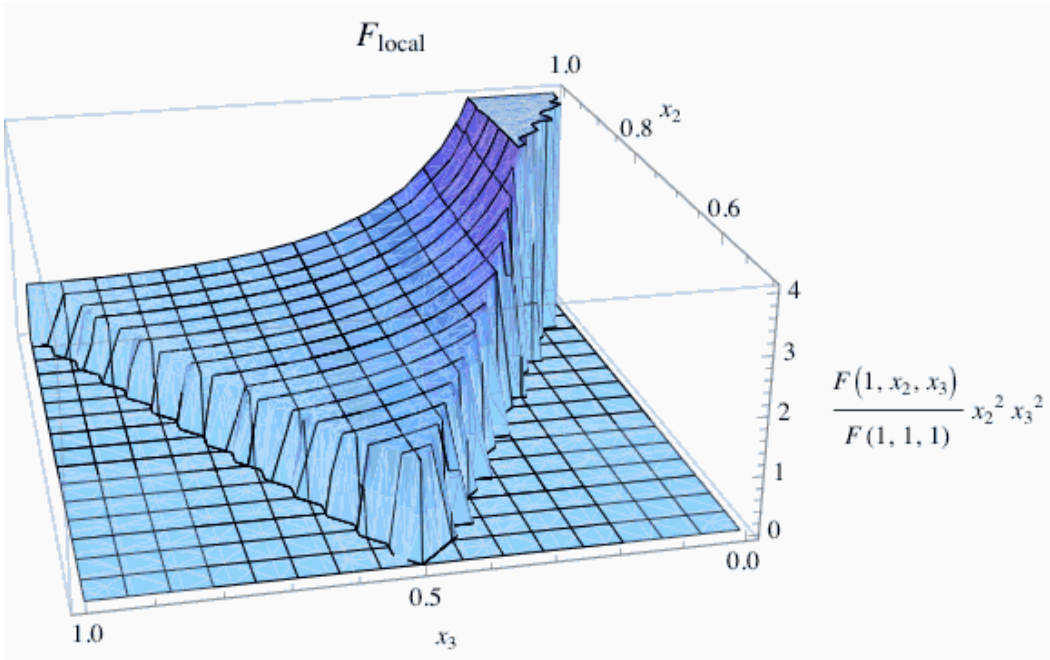
In single clock models there is a direct connection between the departures from scale invariance and the three point function.

Non-Gaussianities $\sim (n-1)10^{-5}$

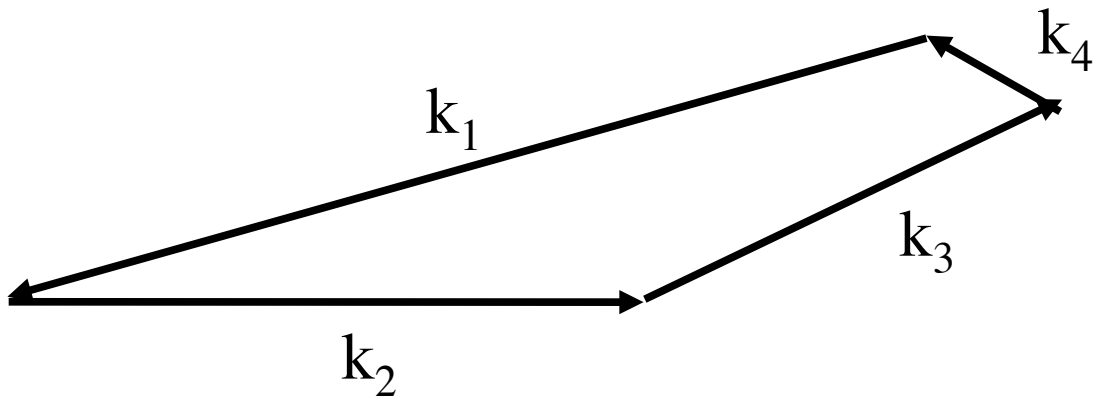
The shapes in pictures







Observational Consequences: 4-pt function



$$k_1^9 \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = F(\frac{k_2}{k_1}, \frac{k_3}{k_1}, \frac{k_4}{k_1}, \theta, \phi)$$

Higher order moments have even more freedom.
Even after requiring scale invariance and translation invariance the four point function is still an arbitrary function of five continuous variable.

Senatore & MZ in preparation.

Could one have an observable 4-pt function without a much larger 3-pt?

$$S_{\text{E.H.}+\text{S.F.}} = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}M_{\text{Pl}}^2 R + M_{\text{Pl}}^2\dot{H}g^{00} - M_{\text{Pl}}^2(3H^2+\dot{H}) +$$
$$+\frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 +$$
$$-\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^{\mu}{}_{\mu} - \frac{\bar{M}_2(t)^2}{2}\delta K^{\mu}{}_{\mu}{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^{\mu}{}_{\nu}\delta K^{\nu}{}_{\mu} + \ldots\Big]$$

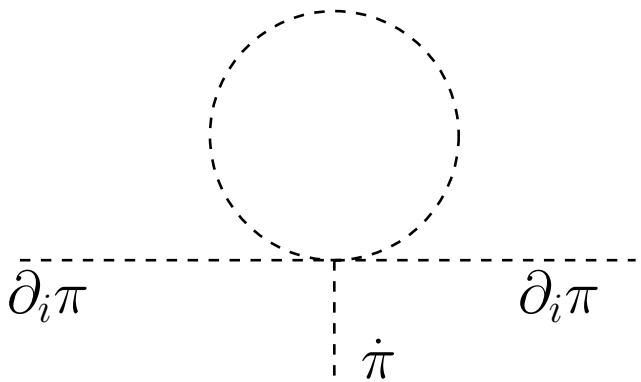
$$g^{00} \quad \rightarrow \quad -1 - 2\dot{\pi} - \dot{\pi}^2 + \frac{1}{a^2}(\partial_i\pi)^2$$

Although there are quartic interactions inside $(g_{00}+1)^2$ and $(g_{00}+1)^3$ quartic terms, those are very small given current constraints on the three point function.

$$\tau_{\text{NL}}\zeta^2 \sim \left.\frac{L_4}{L_2}\right|_{E\sim H} \sim \frac{1}{c_s^4}\zeta^2 \qquad \Rightarrow \qquad \tau_{\text{NL}} \sim \frac{1}{c_s^4}\ .$$

Could one have an observable 4-pt function without a much larger 3-pt?

Could one have a $(g_{00}+1)^4$ terms without $(g_{00}+1)^2$ and $(g_{00}+1)^3$ ones?



$$M_4^4(\delta g^{00})^4 \to M_4^4\left(16\dot{\pi}^4 - 32\dot{\pi}^3(\partial_\mu\pi)^2 + 24\dot{\pi}^2(\partial_\mu\pi)^4 - 8\dot{\pi}(\partial_\mu\pi)^6 + (\partial_\mu\pi)^8\right)$$

Although loop corrections generate 3-pt interactions they can be consistently small.

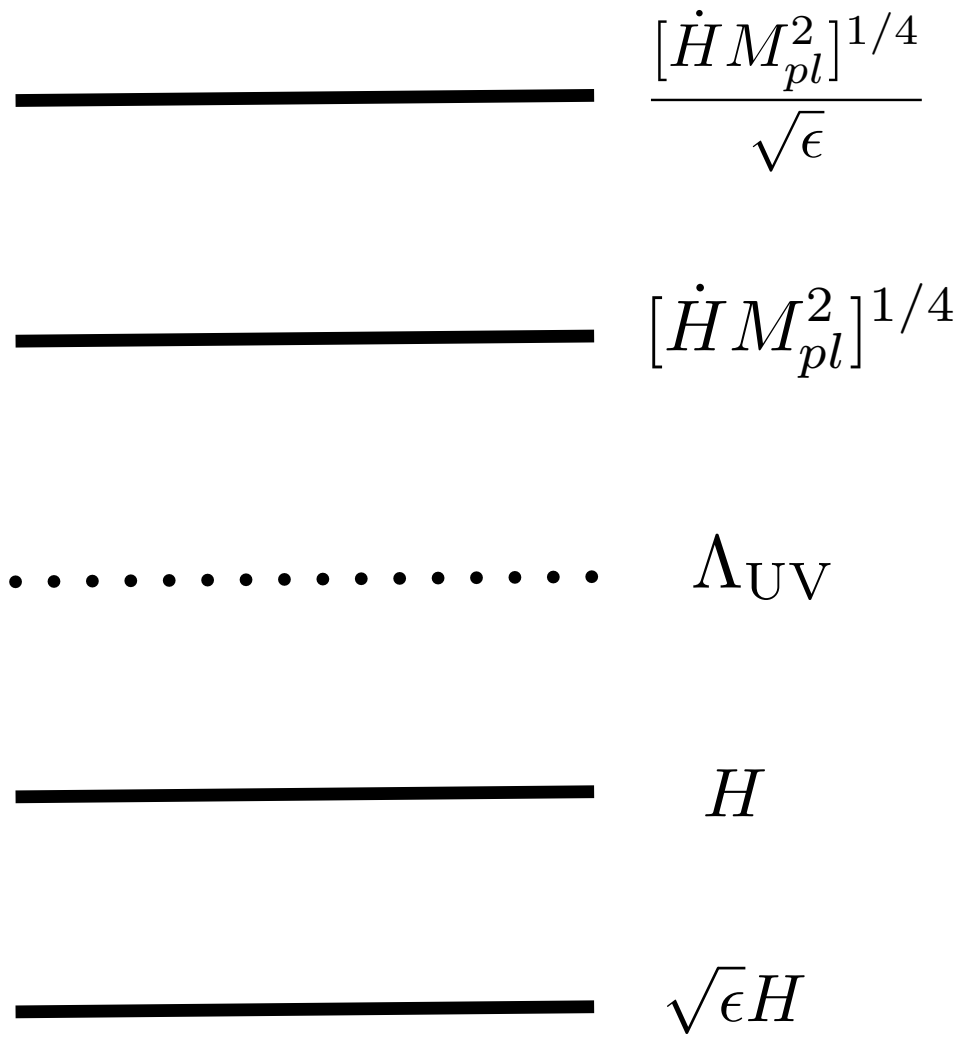
$$M_4^4(\delta g^{00})^4 \qquad \to \qquad \dot{H}M_{\rm Pl}^2(\delta g^{00})^2\ ,\dot{H}M_{\rm Pl}^2(\delta g^{00})^3 \qquad f_{NL} \sim 1$$

$$k_1^9 \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = F(\frac{k_2}{k_1}, \frac{k_3}{k_1}, \frac{k_4}{k_1}, \theta, \phi)$$

Although an arbitrary function of five continuous variable only one possibly large shape from single clock models, created by the interaction

$$\dot{\pi}^4$$

Energy Scales



The structure of the interactions is very constrained, leading to much fewer possibilities than allowed by translational invariance and scale invariance.

Non-Gaussianities provide an avenue to get convinced that inflation indeed happened.

Observational Status:

Data was first used to search for the local type of non-Gaussianity, the shape that cannot be created by single field models. This shape however is the "simplest" one to write down.

After it was realized that single field models could not produce the local type of non-Gaussianity it was soon realized that inflationary models could be constructed that had larger non-Gaussianities with another shape.

The development of the effective theory of inflation showed that there are really two shapes that need to be searched for. We now have upper limits for this two shapes that can be directly translated into constraints on the parameters of two parameters in the Lagrangian for the fluctuations.

$$S_\pi = \int d^4x \sqrt{-g} \left[-\frac{M_{\rm Pl}^2 \dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{M_{\rm Pl}^2 \dot{H}}{c_s^2} \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} - \frac{2\,\tilde{c}_3}{3\,c_s^4} M_{\rm Pl}^2 \dot{H} \dot{\pi}^3 \right]$$

In addition to the parameters that describe the expansion history there are two additional one, the sound speed and c3 (a dimensionless number of order one).
They can be directly constrained using WMAP.

Searching for the signal

The best data set is the one with the largest number of high signal to noise measurements (pixels, Fourier modes). Constraints go as

$$\frac{\langle \delta T^3 \rangle}{\langle \delta T^2 \rangle^{3/2}} \propto N_{\rm pix}^{-1/2}$$

WMAP and future all sky CMB experiments are the most promising but large scale structure is competitive for the local shape.

Surveys of hydrogen at high redshift using its 21 cm line could potentially do better for all shapes.

Current constraints on single clock models

From WMAP 5 yr data release

$$-125 \leq f_{NL}^{\rm equil} \leq 435 \quad 95\% \ \ {\rm CL}$$

$$-369 \leq f_{NL}^{\rm orthog} \leq 71 \quad 95\% \ \ {\rm CL}$$

$$\Longrightarrow \quad c_s \geq 0.011 \quad 95\% \ \ {\rm CL}$$

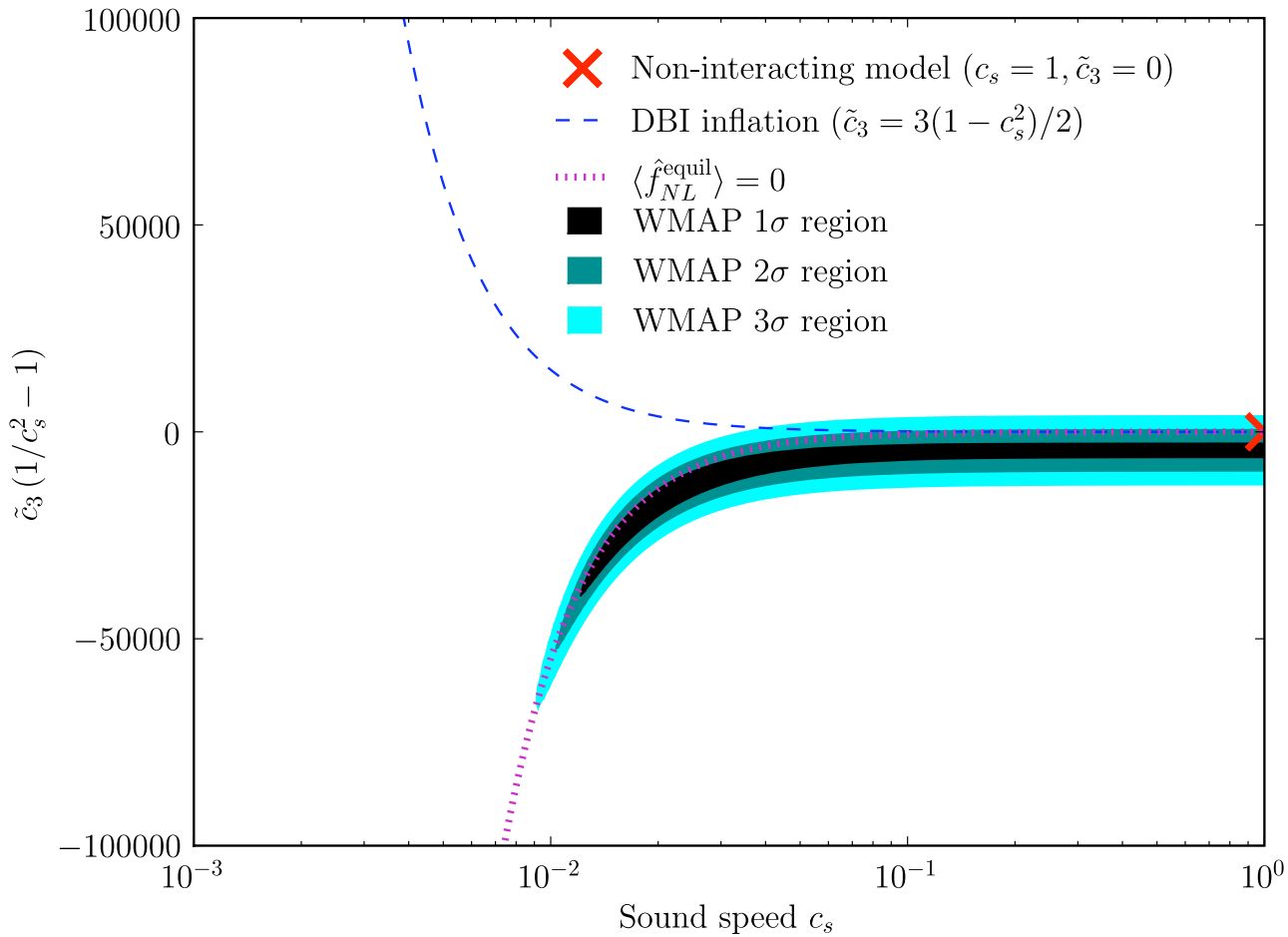
Power spectrum

$$\langle \Phi_{\vec{k}_1} \Phi_{\vec{k}_2} \Phi_{\vec{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\sum_i \vec{k}_i) F(k_1, k_2, k_3) \qquad F(k,k,k) = f_{NL} \cdot \frac{6\Delta_\Phi^2}{k^6} \ ,$$

$$f_{NL}^{\dot{\pi}(\partial_i \pi)^2} = \frac{85}{324}\left(1 - \frac{1}{c_s^2}\right) \ ,$$

$$f_{NL}^{\dot{\pi}^3} = \frac{10}{243}\left(1 - \frac{1}{c_s^2}\right)\left(\tilde{c}_3 + \frac{3}{2}c_s^2\right)$$

Smith, Senatore, MZ 0905.3746



$$c_s \geq 0.011 \qquad \text{at } 95\% \ \text{CL}$$

$$-125 \leq f_{NL}^{\rm equil.} \leq 435 \quad \text{at } 95\% \ \text{CL}$$

$$-369 \leq f_{NL}^{\rm orthog.} \leq 71 \quad \text{at } 95\% \ \text{CL}$$

One cannot put a bound on cs if only one shape is measured.

Current constraints on the local shape

$$-4 \leq f_{NL}^{\rm local} \leq 80 \quad 95\% \;\; {\rm CL}$$

Our error bars are roughly 40% smaller than previous analysis.

We outperform the previous analysis on both the large and small scales.

Our results are robust to doing different cuts on the data (3 yr vs 5yr, details of the mask, range of l used). All the differences we see are consistent with being statistical.

We see no evidence of foreground contamination and we are robust to the procedure used to subtract point sources.

Smith, Senatore & MZ 0901.2572

Inflation

A period of accelerated expansion with

$$\dot{H} << H^2$$

~~Quantum fluctuation in the clock that determines when inflation ends leads to the structure we see.~~

What if fluctuations had another source that dominated over the fluctuations of the clock? Can an analogous effective theory be constructed? What are the consequences? (Senatore and MZ in preparation)

Summary:

We now know how to talk about single field inflation just in terms of what is relevant to observations.

In the last few years our understanding of non-Gaussianities from single clock models has improved substantially. They tell us about the interactions between fluctuations in the clock and symmetries dictate the structure of these interactions. They can provide the "proof" that what we are seeing in the sky results from the quantum fluctuations in the clock that kept time during a quasi de-Sitter phase.

We expect:
There are only two interactions so the shape of the possible three point functions is fully determined by two parameters.

Only a single 4 point function shape could be large. Not yet been searched for in the data.

Currently the best upper limits come from WMAP. Planck should improve things by at least an order of magnitude.

• No non-Gaussianities in the squeezed limit (local type) for single field models (Maldacena's theorem).

• Only two operators provide interactions that can create a non-zero three point function. Thus there are two distinct shapes of non-Gaussianities that can be produced by any single clock inflationary model. *Only two coefficients encode the signatures of all possible single clock inflationary models*.

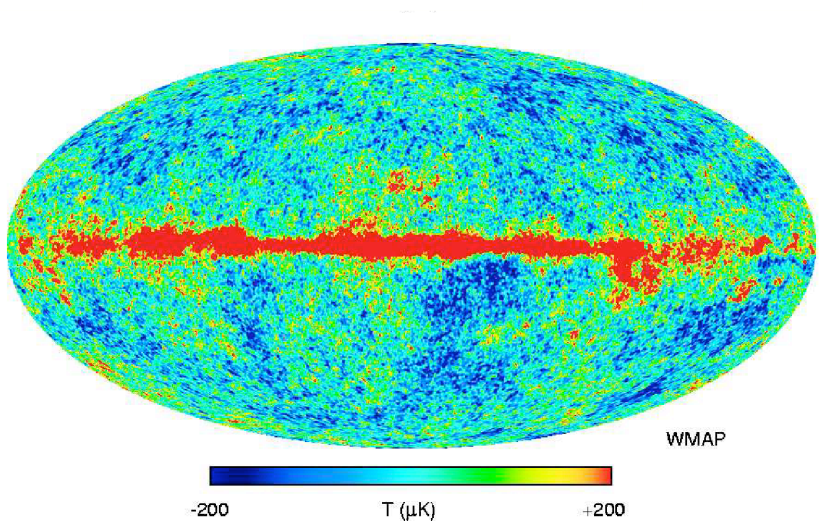
• The amplitude of non-Gaussianities are directly related to the "sound-speed" of the fluctuations, the lower the sound speed the larger the non-Gaussianities.

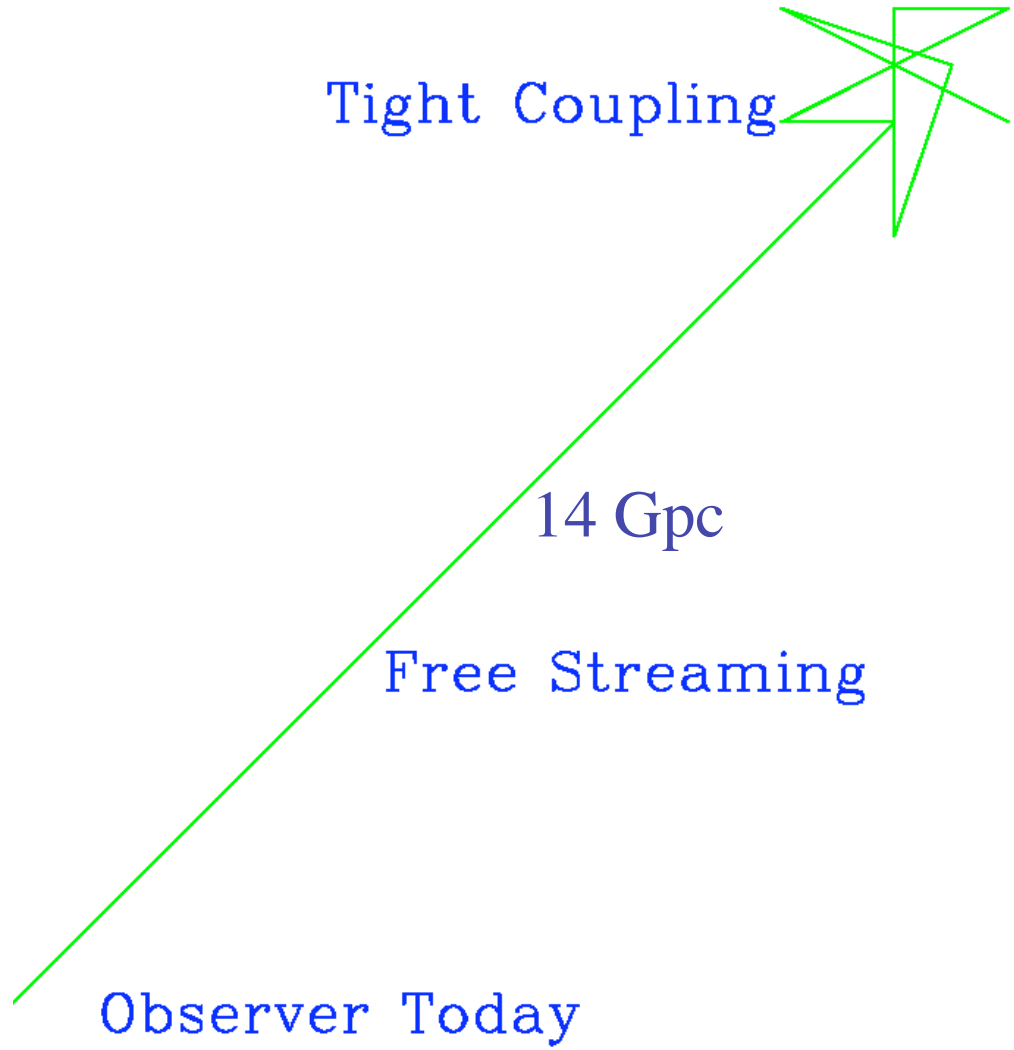
The basics of CMB Anisotropies

$$\frac{\delta T}{T} = \phi + \frac{\delta_\gamma}{4} + \frac{v_r}{c}$$

All 3 effects have the same origin

$$P \sim \lambda_T \nabla v$$





Analysis

Optimal weight

Three point function

$$\mathcal{E} = \frac{1}{N} \cdot \sum_{l_i m_i} \frac{\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle}{C_{l_1} C_{l_2} C_{l_3}} a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}$$

Amplitude of primordial fluctuations

$$a_{lm} = 4\pi i^l \int \frac{d^3k}{(2\pi)^3} \Delta_l^T(k) \Phi(k) Y_{lm}^*(\hat{k})$$

Primordial 3 point function

$$\langle \Phi(\mathbf{k}_1) \Phi(\mathbf{k}_2) \Phi(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) F(k_1, k_2, k_3)$$

Computationally very difficult unless F is factorizable:

$$F = g_1(k_1) g_2(k_2) g_3(k_3) + \cdots$$

Optimal weight is difficult to calculate when anisotropic noise and galactic cut are included.

Constraints on the local shape:

Smith, Senatore & MZ 0901.2572

Better analysis of WMAP 5 data that properly takes into account the effects of the galactic mask and the anisotropies in the noise.

Results in 40% smaller error bars even after foreground marginalization.

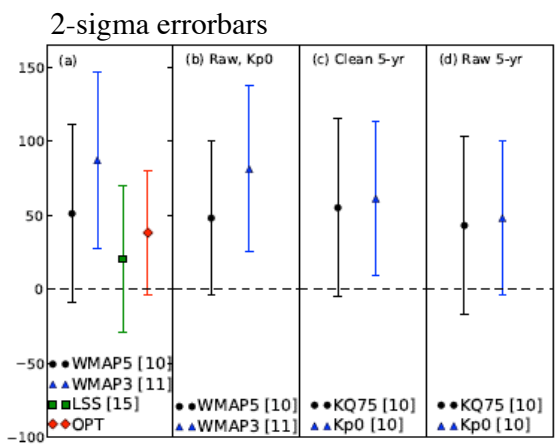


Figure 1: Current constraints on $f_{NL}^{\rm local}$. Errors in this figure and throughout the paper are 2-σ. Panel (a) best results from WMAP 5 years from [10] and WMAP 3 years from [11] together with the large scale structure results from [12] and the results from this paper using our optimal method (OPT). Panel (b) comparison of [10] and [11] for the same choice of analysis parameters ($l_{max} = 500$, raw maps and the Kp0 mask). Panels (c) and (d) show the effect of the mask for cleaned and raw maps respectively (from [10]).

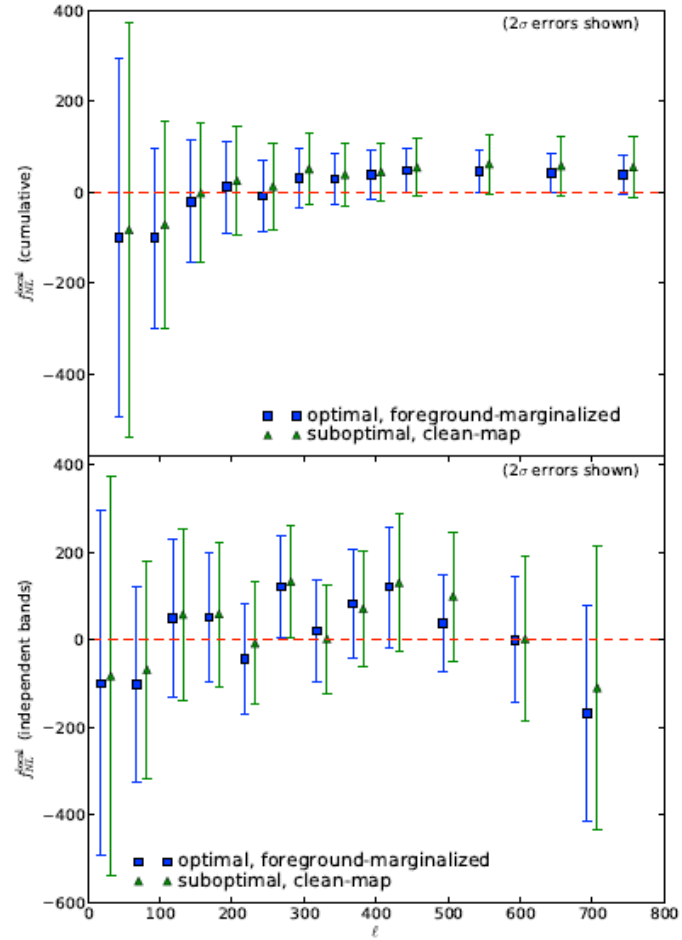
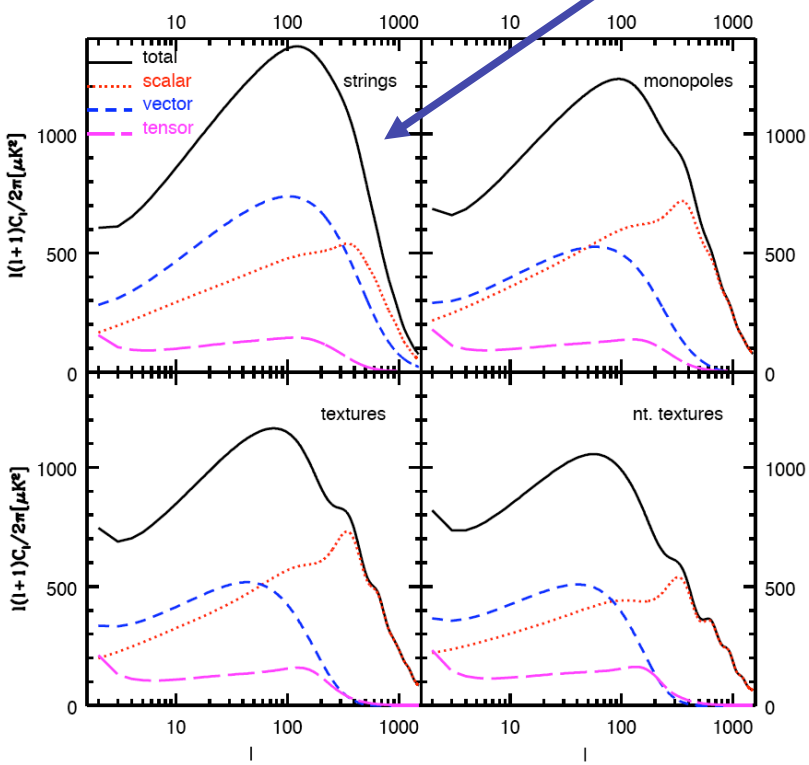


Figure 2: Constraints on $f_{NL}^{\rm local}$ using 5-year data and KQ75 mask, using both the optimal estimator (squares) and the old estimator applied to clean maps (triangles). The top panel shows cumulative results (constraints using all the information up to a given ℓ) while the bottom one shows contributions from separate ℓ bins. Our overall $f_{NL}^{\rm local}$ estimate, taking $\ell_{\rm max} = 750$, is (38 ± 21) for the optimal estimator and (55 ± 33) for suboptimal.

When were perturbations created?

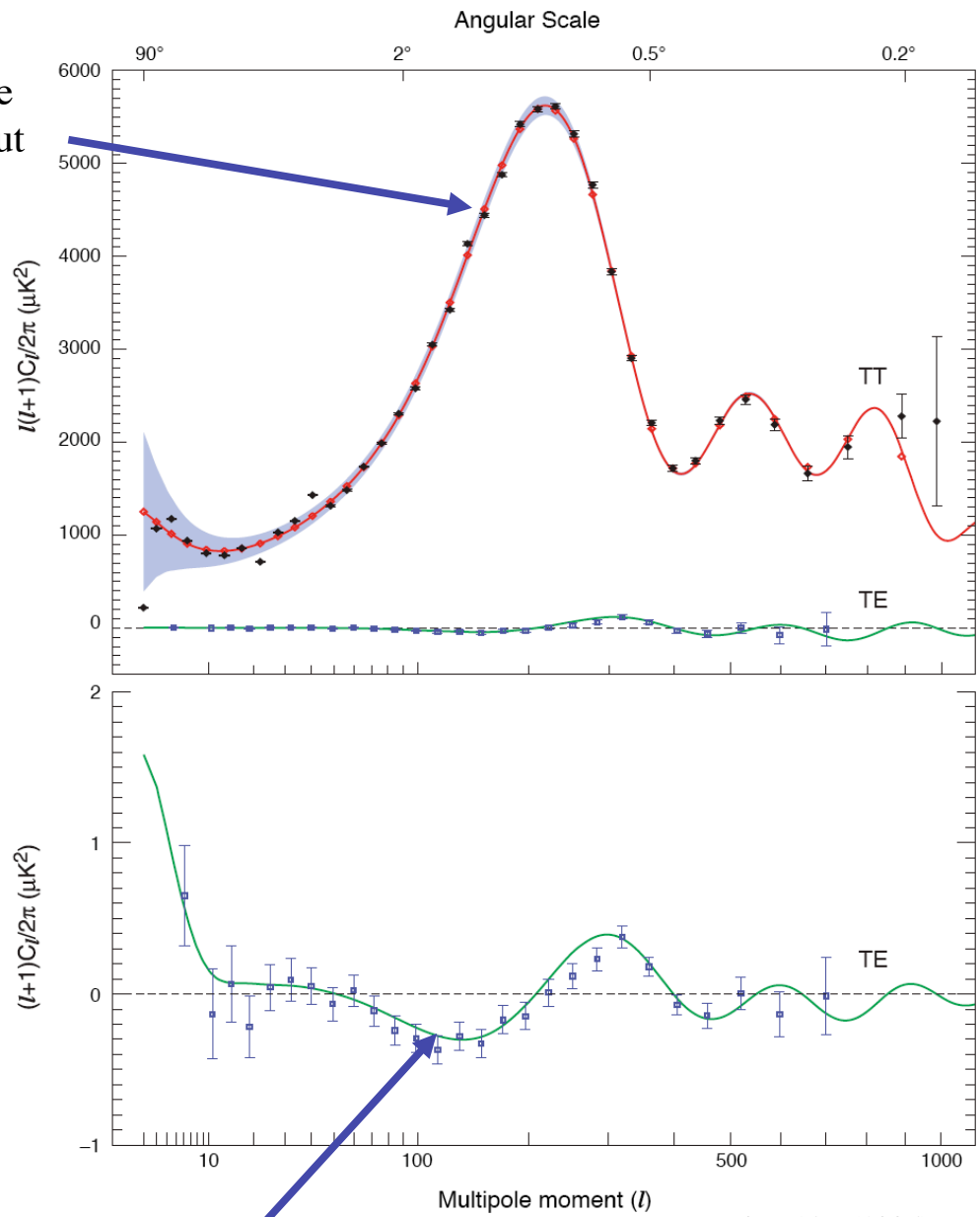
Sharp acoustic peaks are difficult to create without inflation

Causal Seeds



Pen, Seljak & Turok (1997)

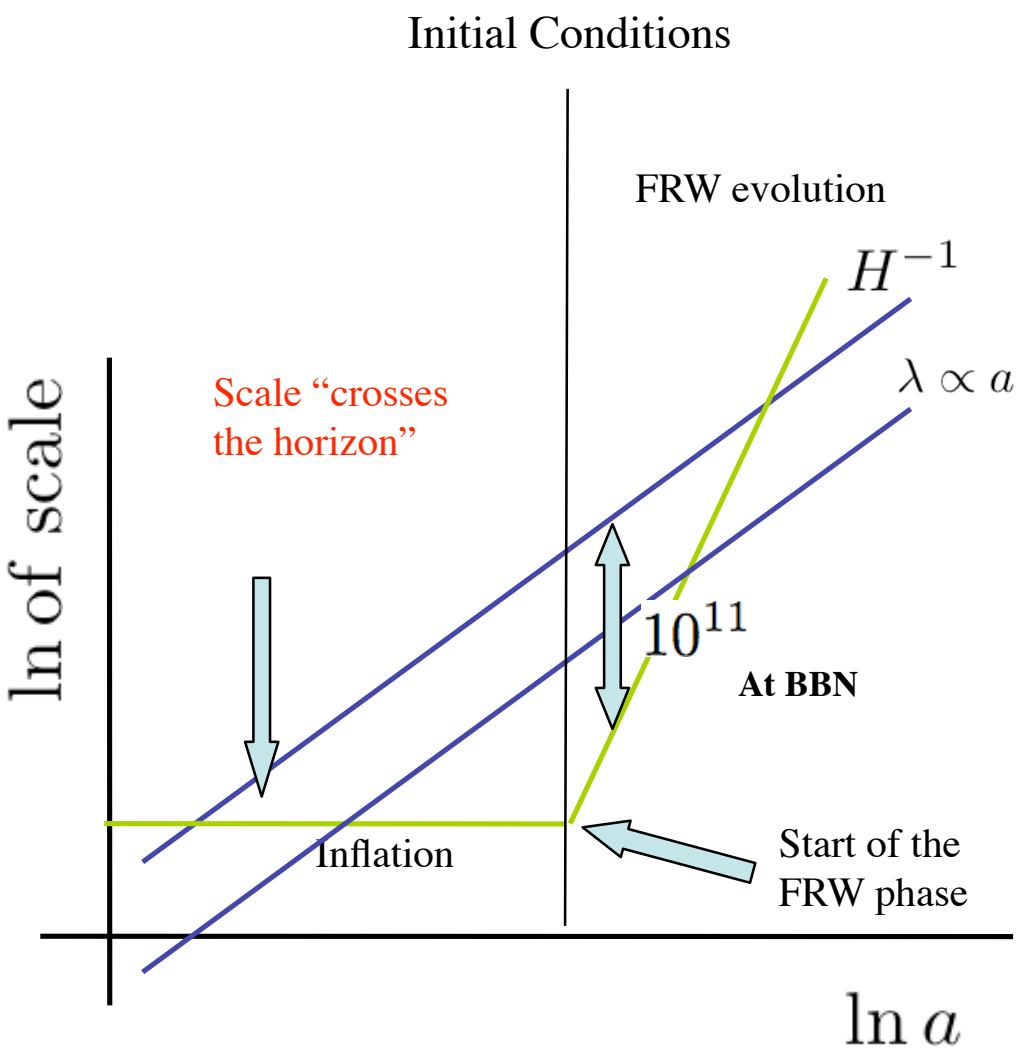
WMAP 3yrs



Negative peak imply fluctuations come from outside horizon

Hu & White (1996)
Spergel & MZ (1997)
Pieris et al WMAP (2003)

Inflation and the Horizon Problem



$$H = \frac{\dot{a}}{a}$$

$$R_H \sim H^{-1}$$

$$\lambda = \lambda_0 a$$

$$\frac{\lambda}{R_H} = \lambda_0 a H \propto \dot{a}$$

$$v = Hr = \dot{a}x$$

A period of accelerated expansion puts things outside the Horizon.