Cluster weak lensing

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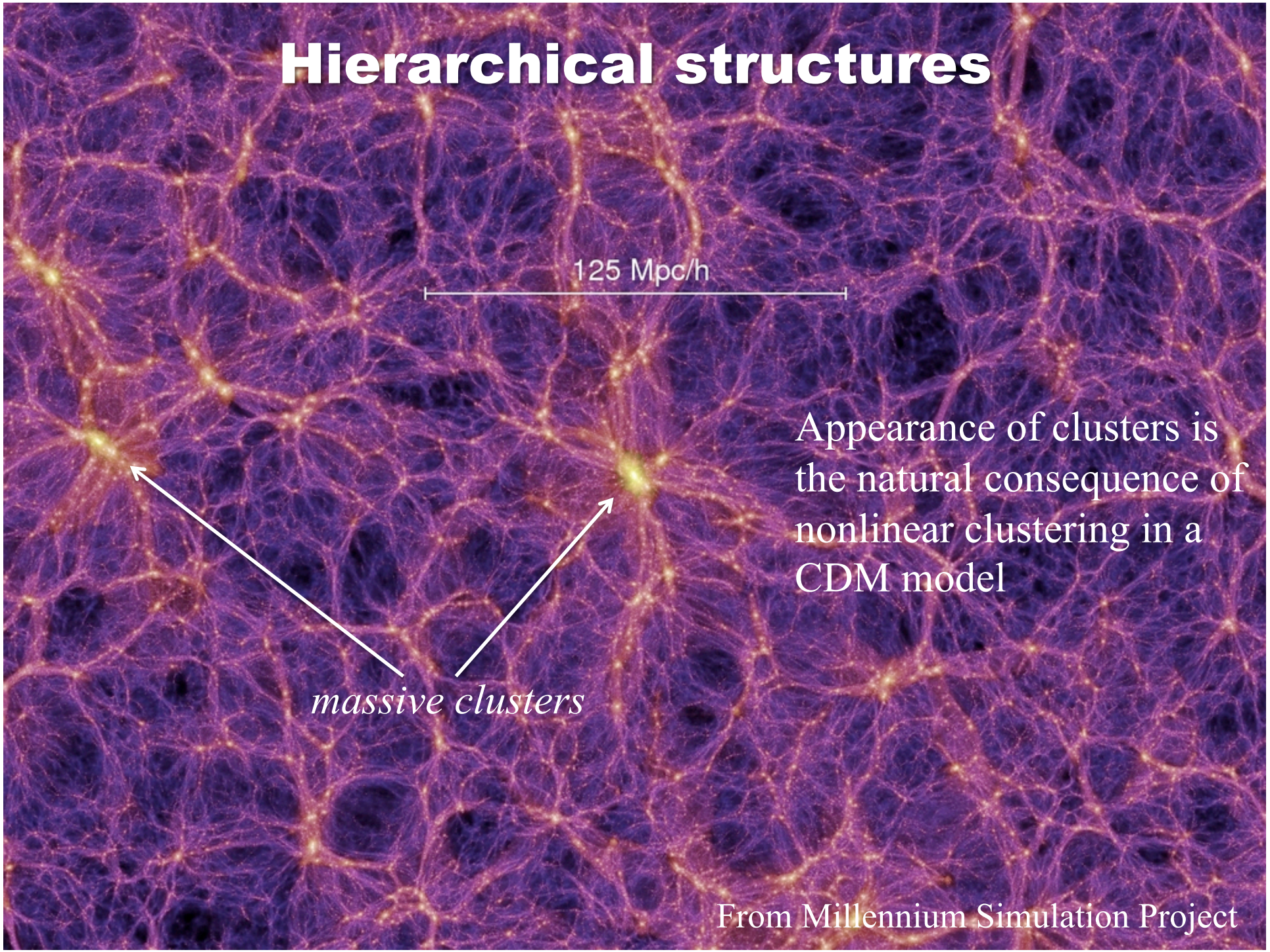
@ Cosmology at the Beach, Jan 14, 2010

Hierarchical structures

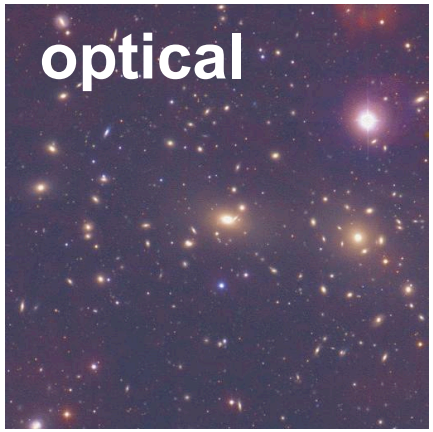


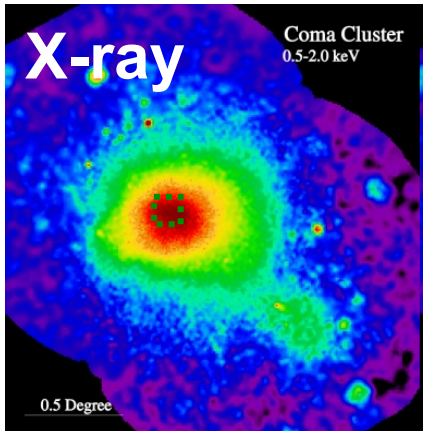
Appearance of clusters is the natural consequence of nonlinear clustering in a CDM model

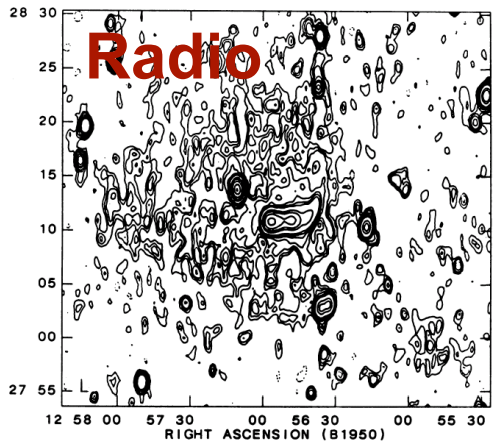
From Millennium Simulation Project

Galaxy Clusters

- Most massive gravitationally bound objects
  - 10^14 ~ 10^15 M_sun (100 – 1000 galaxies)
  - Strongest S/N of the lensing signals
  - DM plays a dominant role to the formation ⇔ for a galaxy, baryonic effect is important
  - Suitable for testing the CDM scenarios on small scales <1Mpc
- Astronomically very interesting objects to study
  - Seen with various wavelengths (radio, optical, X-ray)
  - Connection between DM (gravity), hot gas (baryonic matter) and galaxies (a tiny part of baryons); **100:10:1**

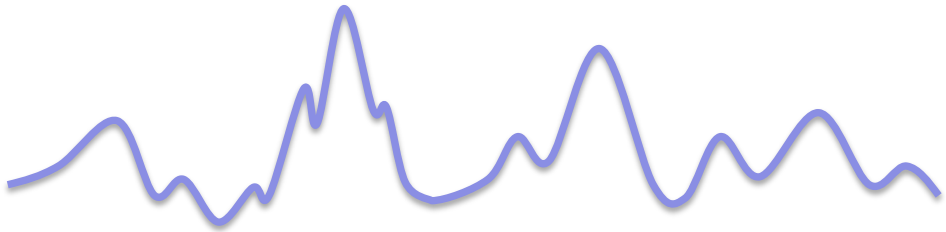






Cosmological Use of Clusters: Halo Mass Function

Tiny density fluctuations at z~1000: δ_m~10^-3

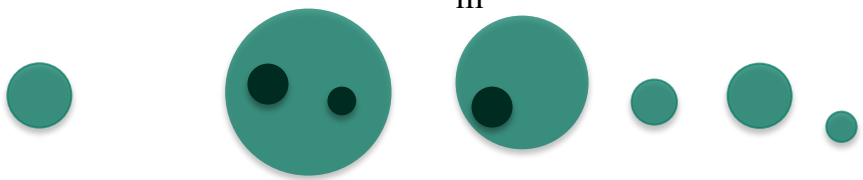


Gravitational instability

(gravity ⇔ cosmic expansion)

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\bar{\rho}_m\delta_m = 0$$

Halo formation at z~0: δ_m>>1



Gaussian seed density fluctuations

+

Spherical collapse model (or N-body simulation)

Mass function:

$$\frac{dn}{dM} \propto \exp\left(-\frac{\delta_c^2}{2\sigma^2(M)}\right)$$

@cluster mass scales

The mass function can be a powerful probe of cosmology (e.g. DE)

Halo mass function (contd.)

Angular number counts of clusters

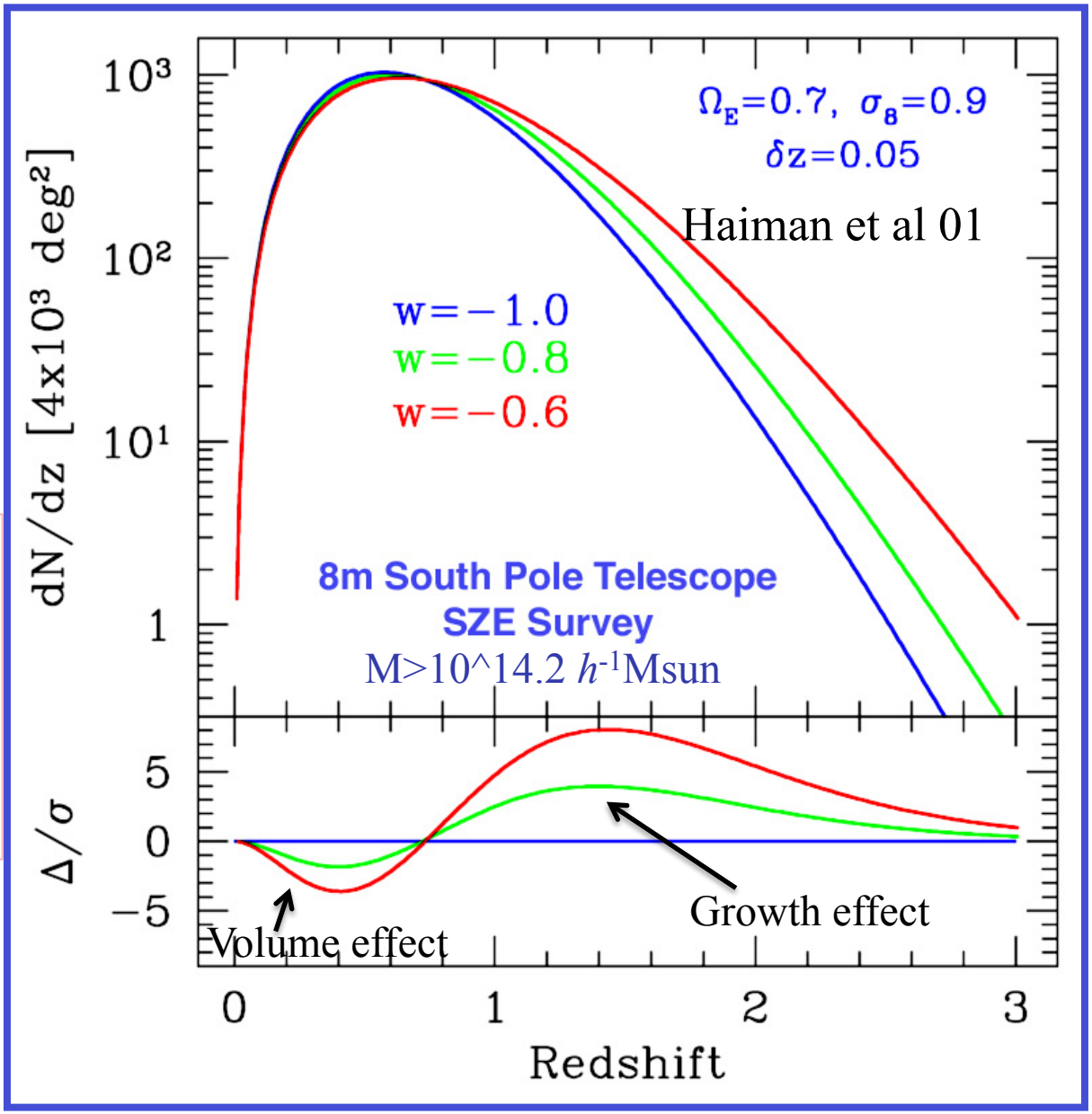
$$\frac{d^2N}{dzdM} = \Omega_S \frac{d^2V}{dzd\Omega}\frac{dn}{dM}$$

DE equation of state: w

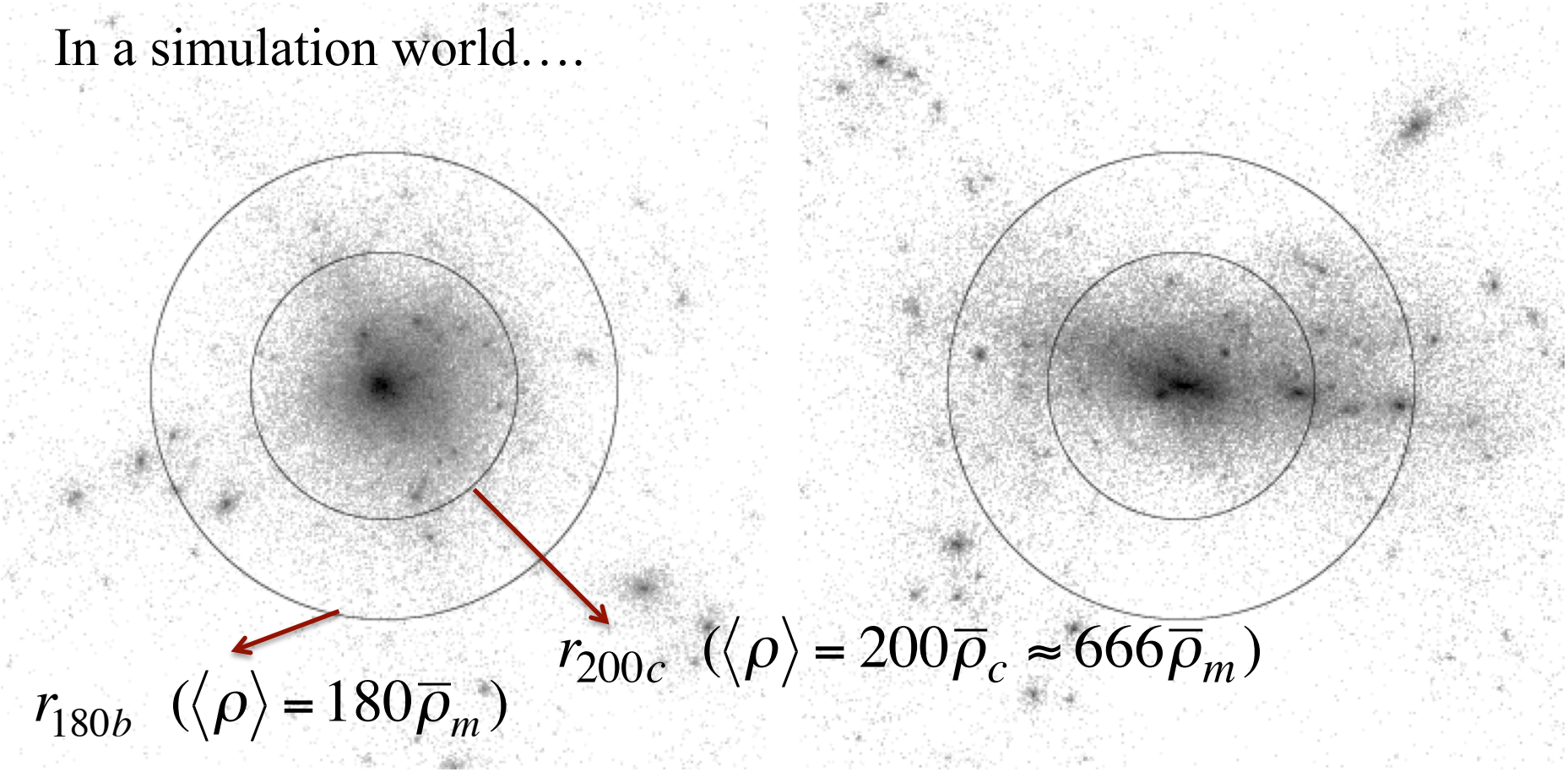
$$w \equiv \frac{p_{\rm de}}{\rho_{\rm de}}$$

$$\rightarrow \rho_{\rm de} \propto (1+z)^{3(1+w)} \quad \text{for} \quad w = \text{const.}$$

Note that the right plot uses σ_8 normalization: the same number density of clusters at present for all models



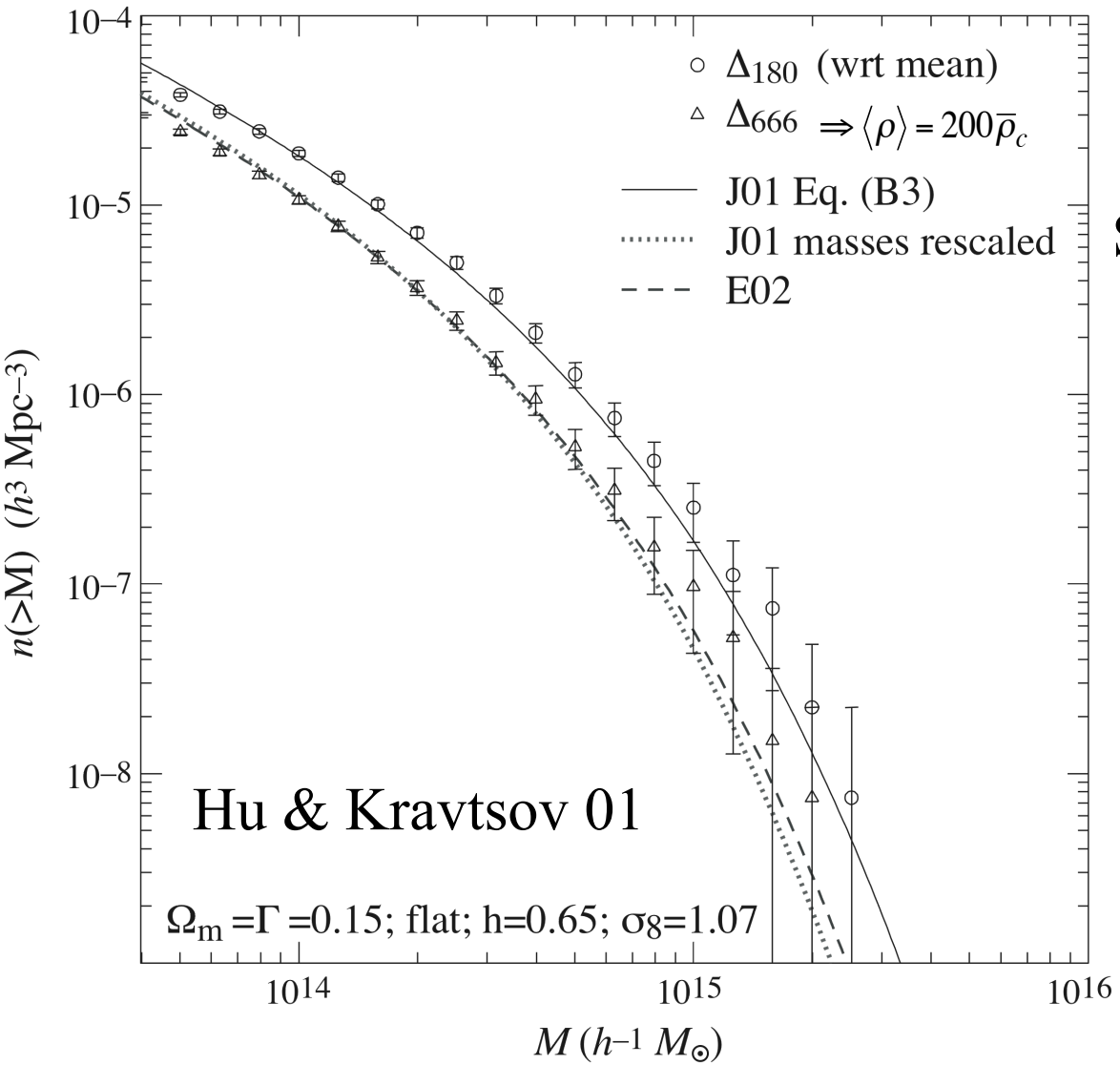
Issue: cluster mass



$$M_\Delta(< r_\Delta) = \int_{r<r_\Delta} d^3\boldsymbol{x}\, \rho(\boldsymbol{x}) \;\Rightarrow\; n(M_\Delta)$$

White 02

Cosmological Use of Clusters:
Halo Mass Function



Gaussian seed density fluctuations

+

Spherical collapse model (or N-body simulation)

Mass function: $n(>M)$

$$\frac{dn}{dM} \propto \exp\left(-\frac{\delta_c^2}{2\sigma^2(M)}\right)$$

@cluster mass scales

The mass function can be a powerful probe of cosmology (e.g. DE)

Cluster mass (contd.)

- In a real world, there is no unique definition of cluster mass; no clear boundary with the surrounding structures
  - Need to estimate the mass such that the definition is closer to the way used in simulations; e.g. spherical overdensity mass
- Have to infer cluster masses (including DM) from the observables (optical, X-ray, *lensing*)
- Cluster counting experiment requires the well-calibrated mass-observable relation for cosmology
  - For a SPT-like survey (4000 deg^2), the mass proxy relation needs to be known to a few % accuracy σ_{lnM}~0.01

Vikhlinin et al. 2009: Chandra

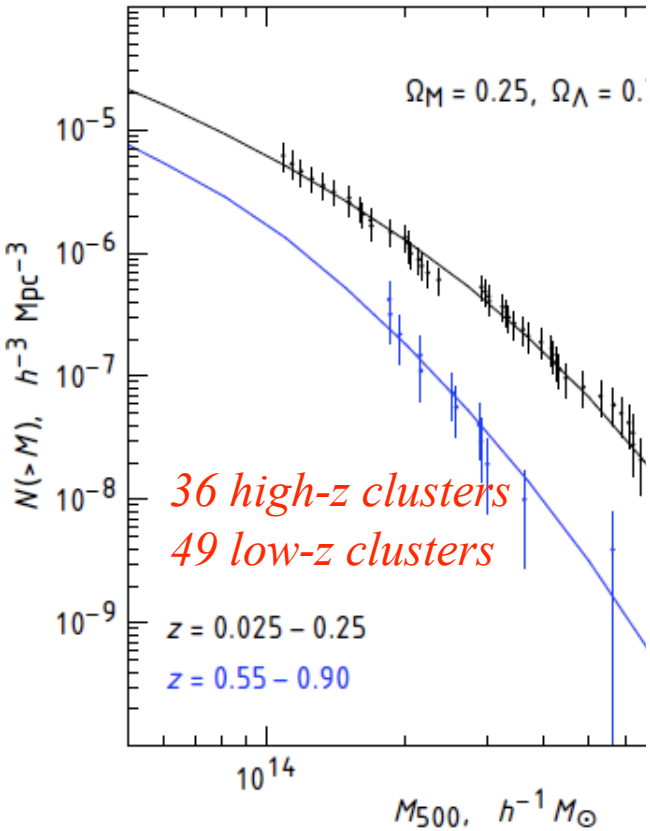
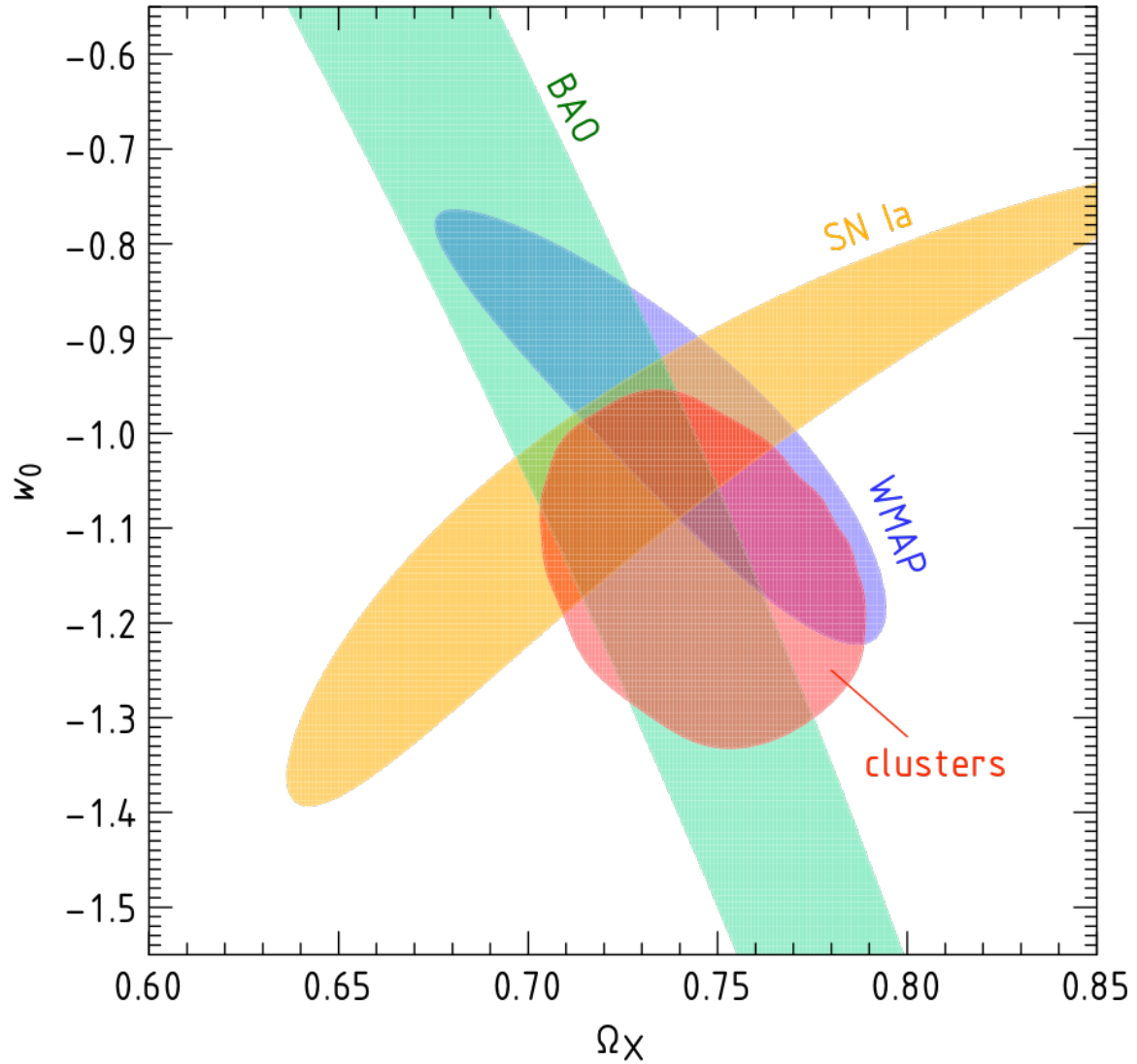
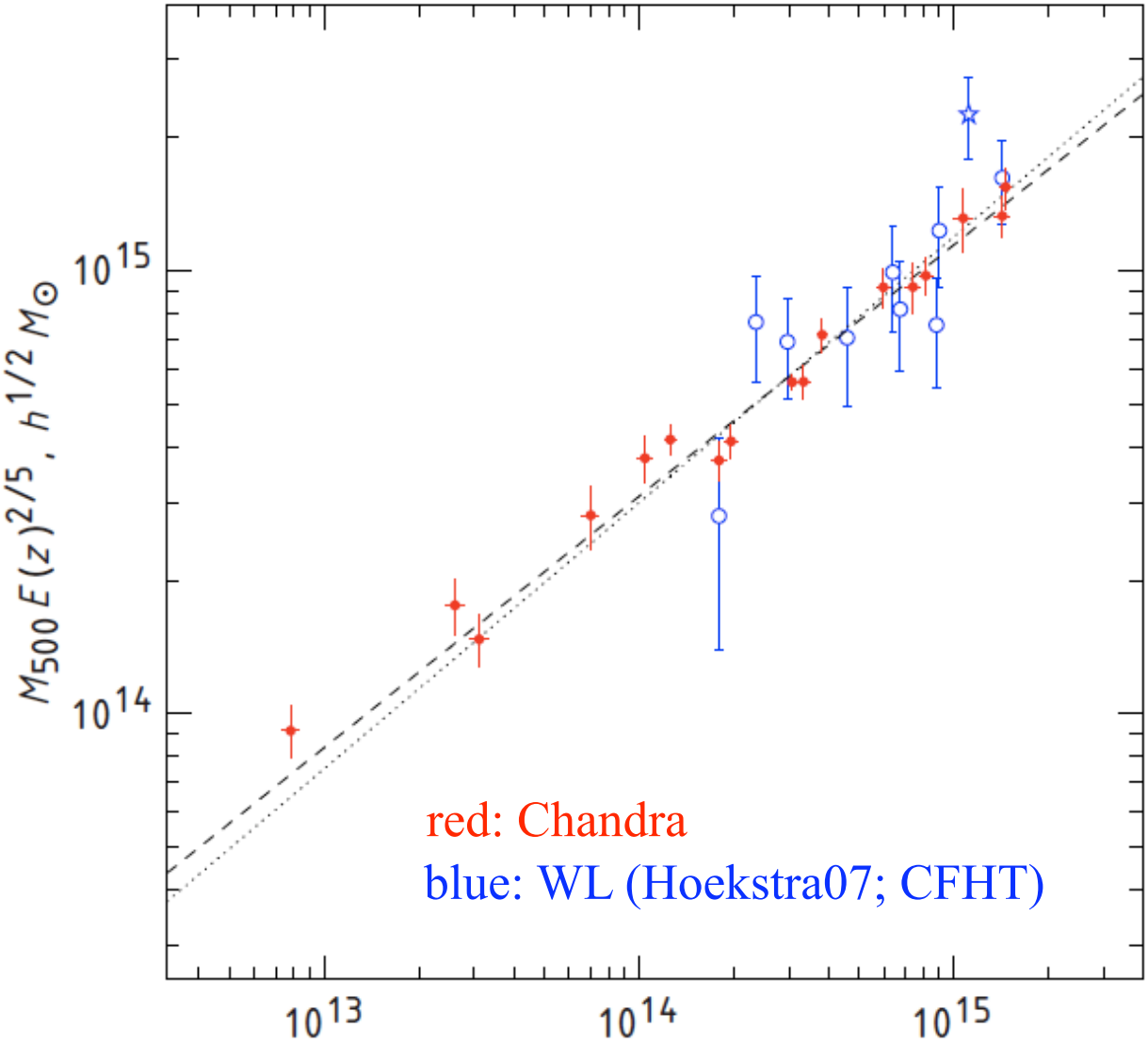


FIG. 2.— Illustration of sensitivity of the cluster mass fun
models (with only the overall normalization at $z = 0$ adjuste
from Fig. 1, which for the high-z cluster we show only the r
models are computed for a cosmology with $\Omega_\Lambda = 0$. Both t
function is changed because it is derived for a different dist
thresholds corresponding to $\Delta_{crit} = 500$ are different. Whe
of $z > 0.55$ clusters is in strong disagreement with the data



State-of-the-art mass proxy

Vikhlinin et al. 07

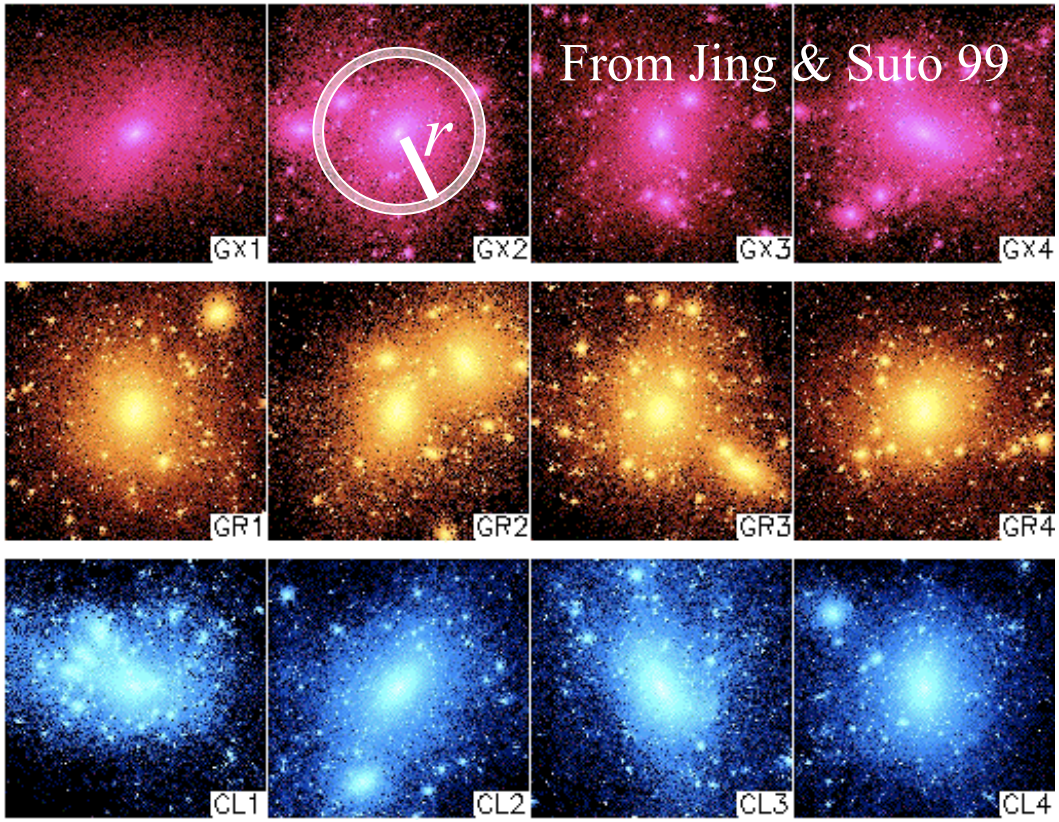


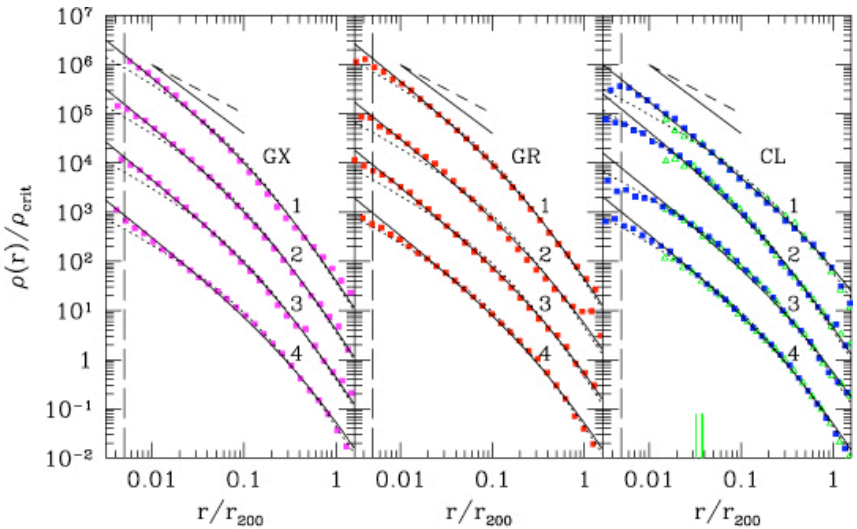
$$Y_X = M_X \times T_{gas,X}$$

$\sigma_{\ln M} \sim 10\%$

Internal structure of halo

- Simulation-based predictions: the appearance of a characteristic, universal density profile (Navarro, Frenk & White 96, 97; **NFW profile**)





NFW profile

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$$

Outer: ρ∝r^-3

Inner: ρ∝r^-1

In addition, halo shape is by nature triaxial (Jing & Suto 01)

NFW profile

- An NFW profile is specified by 2 parameters
- Useful to express the NFW profile in terms of the cluster mass and the halo concentration parameter

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$$

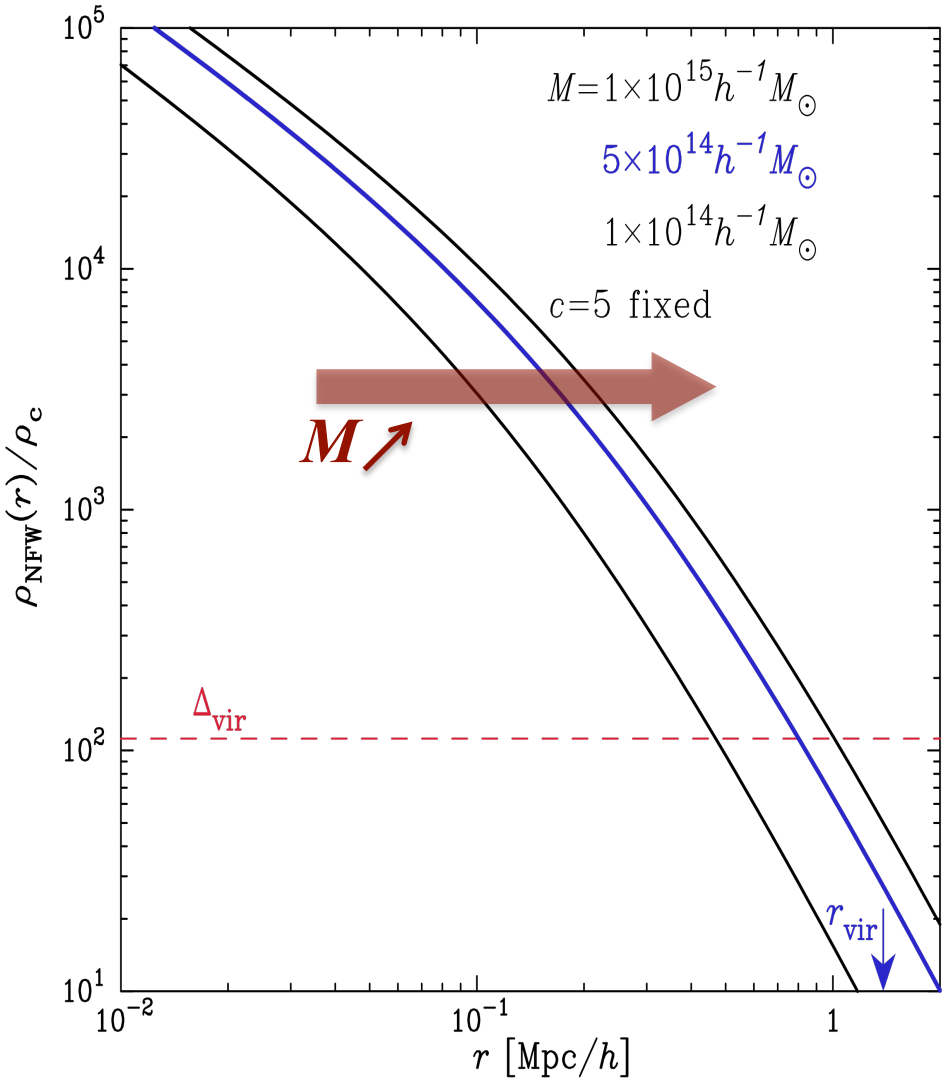
$$+\begin{cases} M_\Delta = \dfrac{4\pi}{3} r_\Delta^3 \bar{\rho}_m \Delta & : \text{defines the halo boundary for a given } \Delta \\ M_\Delta = \displaystyle\int_{r<r_\Delta} 4\pi r^2 dr\, \rho_{NFW}(r) & : \text{sets the interior mass of } \rho_{\mathrm{NFW}} \text{ to } M_\Delta \end{cases}$$

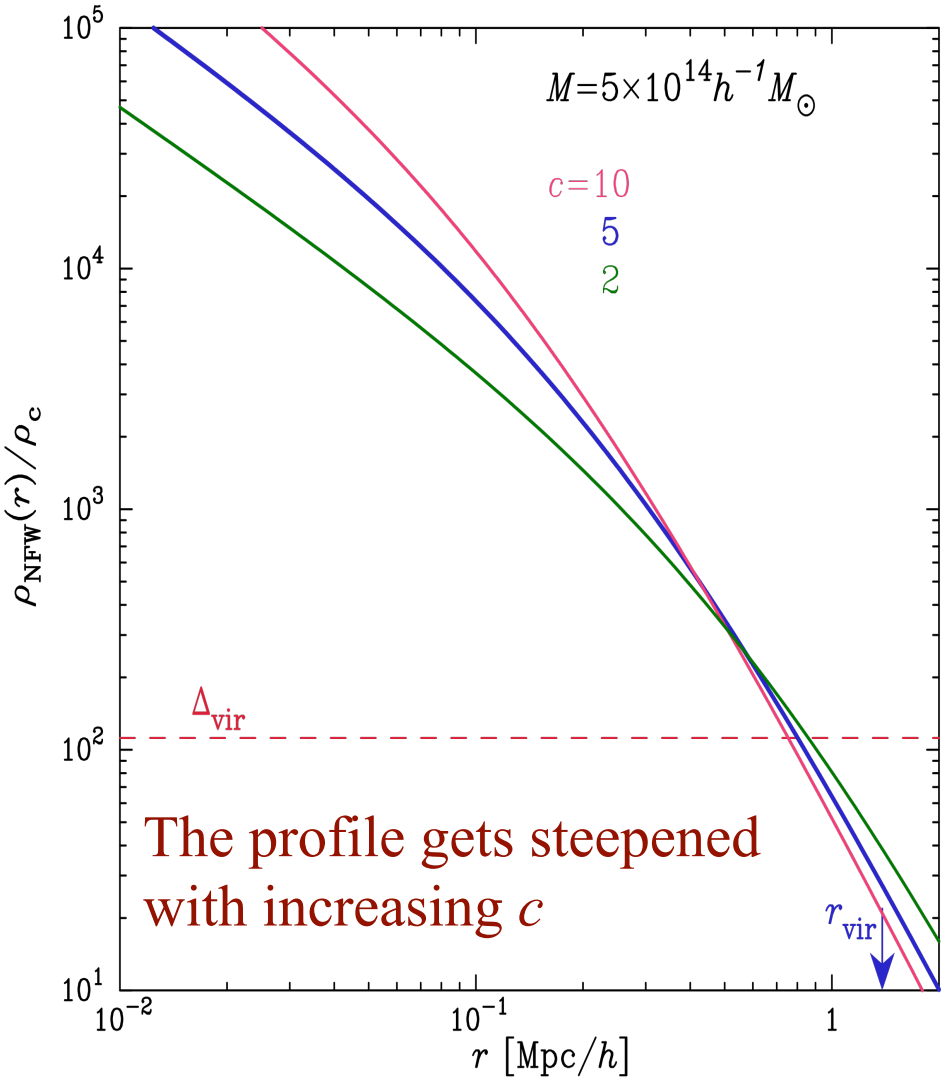
$$\boxed{\rho_{\mathrm{NFW}}(r; M_\Delta, c_\Delta) \quad (\text{note}: c_\Delta \equiv r_\Delta / r_s)}$$

- Can infer the halo mass from the measured halo profile

NFW profile (contd.)

- Dependences of $\rho_{\rm NFW}(r)$ on M and c





Estimating cluster mass with WL

- Model-dependent method: the measured lensing signals are compared with the model predictions (e.g. NFW model) to obtain the best-fit mass parameter
  - Individual clusters basis vs. statistical basis (cross-correlations of clusters – background galaxy images)
  - Systematics inherent in this method
- (Less) model-dependent method
  - Aperture mass-type methods
- Combining different estimates/Self-calibration
  - Combine with X-ray, optical, SZ based estimates
  - Add the clustering information (b(M))

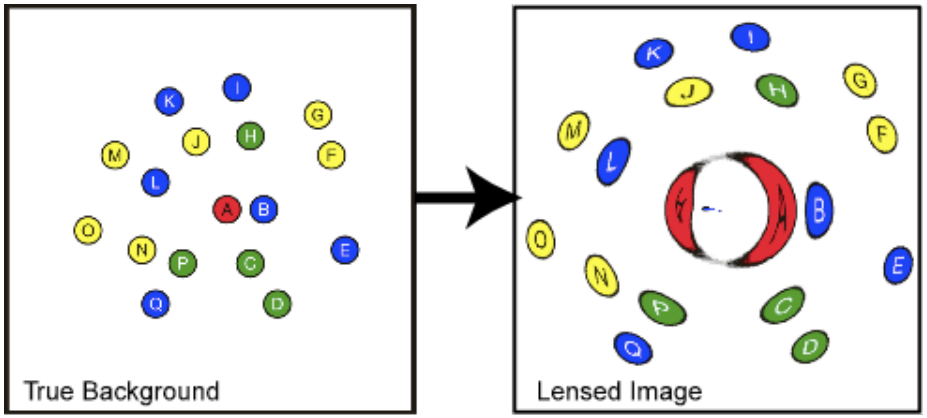
Note: strong and weak lensing

•Strong Lensing

– Multiple Images

– Large Arcs, Ring

– Obvious Distortion

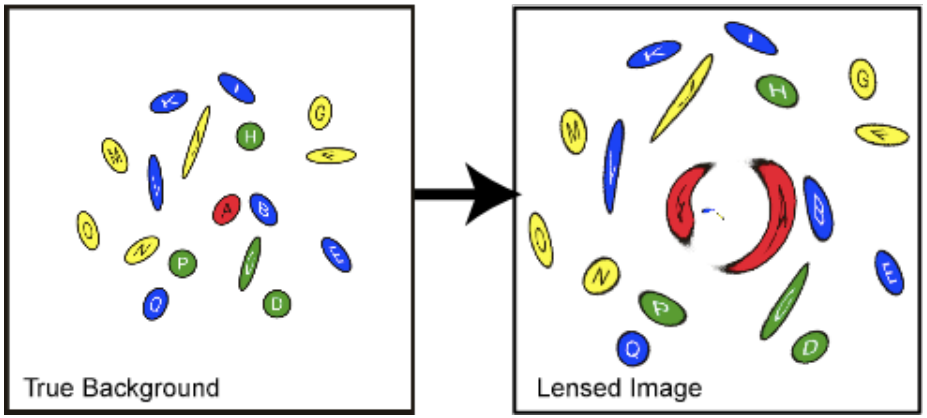


•Weak Lensing

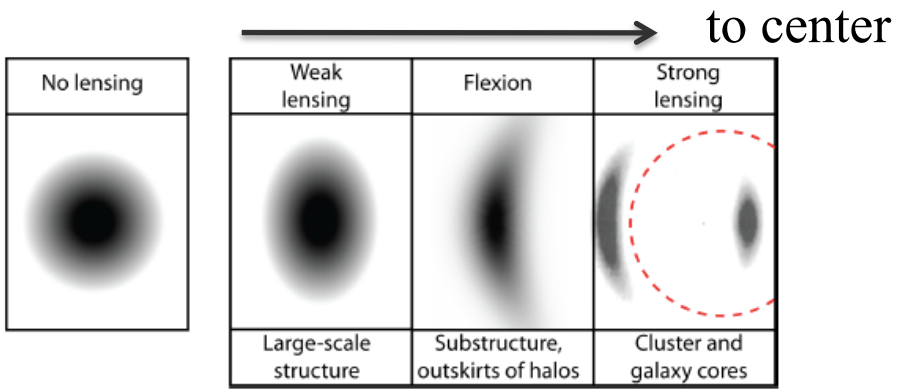
– Slight Stretching

– Distortion small compared to initial shape

– Statistical lensing

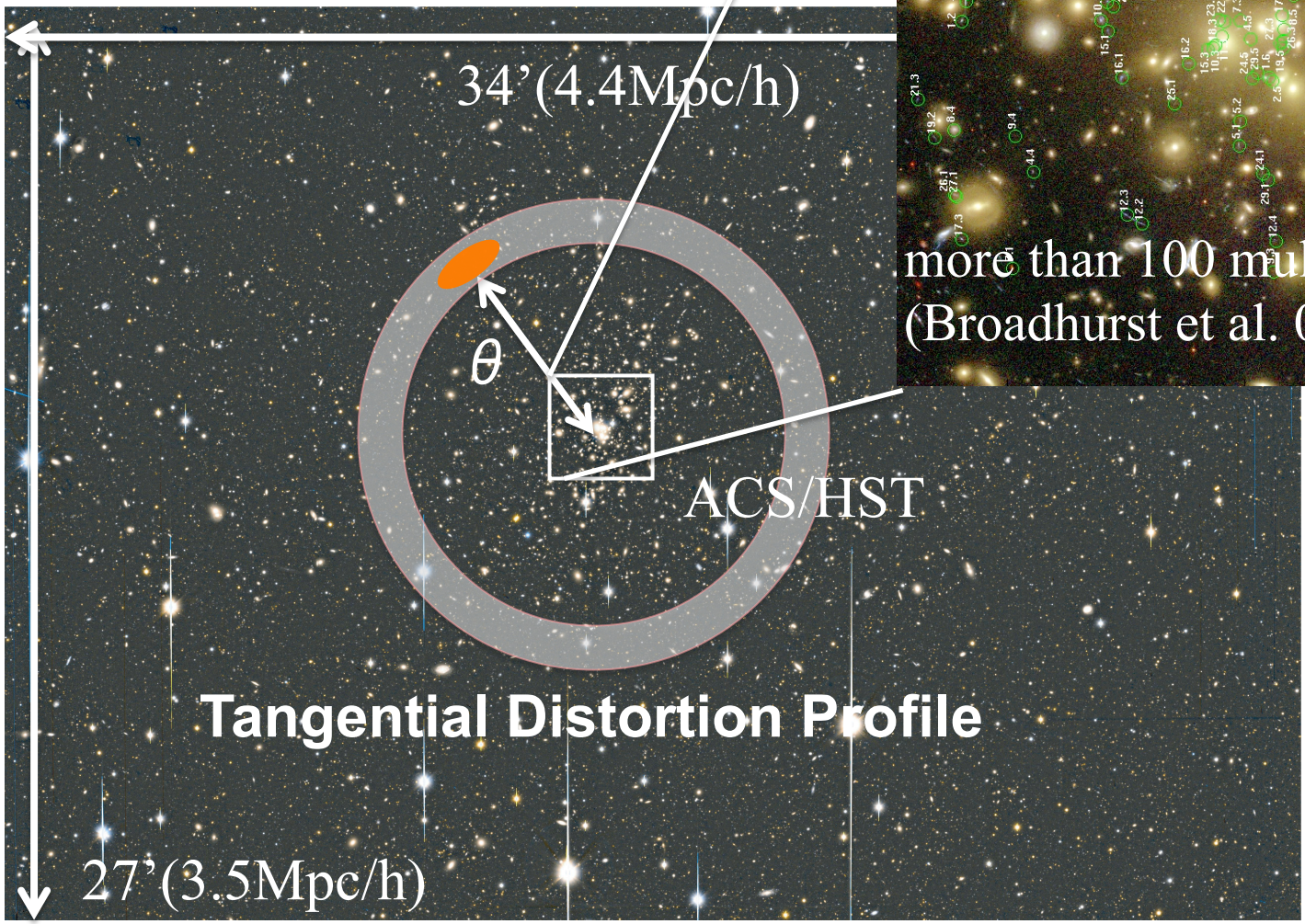


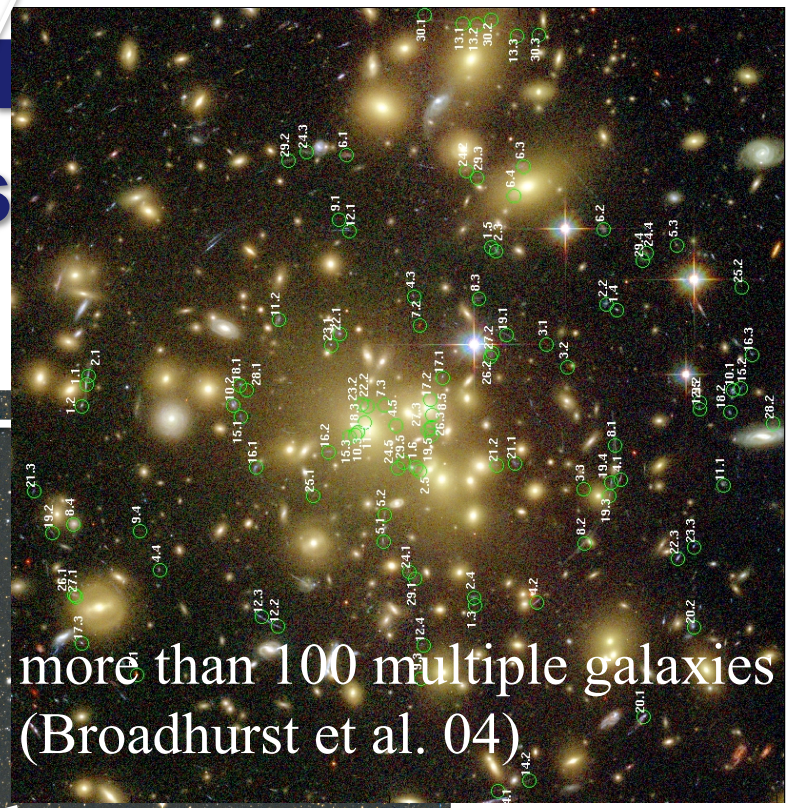
These two regime lensing are very complementary! The combination allows to probe the entire region of cluster (Broadhurst, MT+05)



Model-depend
tangential s

Example: A1689 (z=0.18)





- Virial radius of a massive cluster ~Mpc
- Subaru FoV covers the virial region of a cluster at z~0.2

Tangential shear fitting

- Measured shear profile

$$\gamma_+^{\rm obs}(\theta_i) = \gamma_+^{\rm cluster}(\theta_i) + \underline{\gamma_+^{\rm LSS}(\theta_i)} + \underline{\varepsilon_+(\theta_i)} + {\rm sys.}$$

intrinsic ellipticity

projection effect due to large-scale structures along the line of sight

- Model shear profile

$$\gamma_+^{\rm model}(\theta_i) = \gamma_+^{\rm model}(\theta_i; M_\Delta, c_\Delta)$$

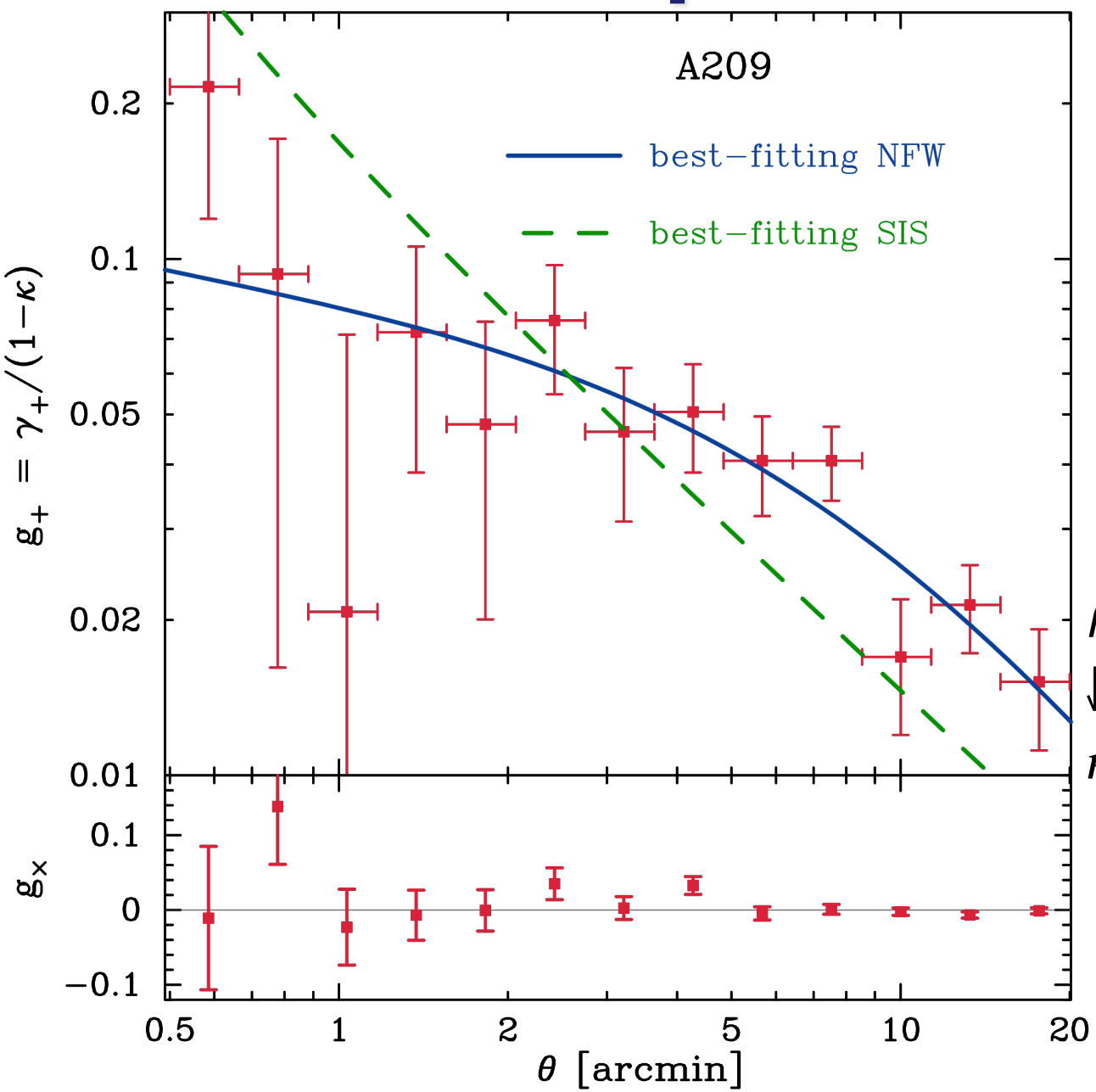
- A parameter estimation based on the χ^2 method

$$\chi^2 = \left[\gamma_+^{\rm obs}(\theta_i) - \gamma_+^{\rm model}(\theta_i; M, c, \ldots)\right] C_{ij}^{-1} \left[\gamma_+^{\rm obs}(\theta_j) - \gamma_+^{\rm model}(\theta_j; M, c, \ldots)\right]$$

Covariance is given as the sum of the intrinsic ellipticities and the cosmic shear contamination

$$C_{ij} = \frac{\sigma_\varepsilon^2}{N_{gi}} \delta_{ij}^K + \xi(\theta_i, \theta_j)$$

Example 1: A209



$$\rho_{\rm NFW} \propto \frac{1}{(r/r_s)(1+r/r_s)^2}$$

$$\rho_{\rm SIS} \propto \frac{1}{r^2}$$

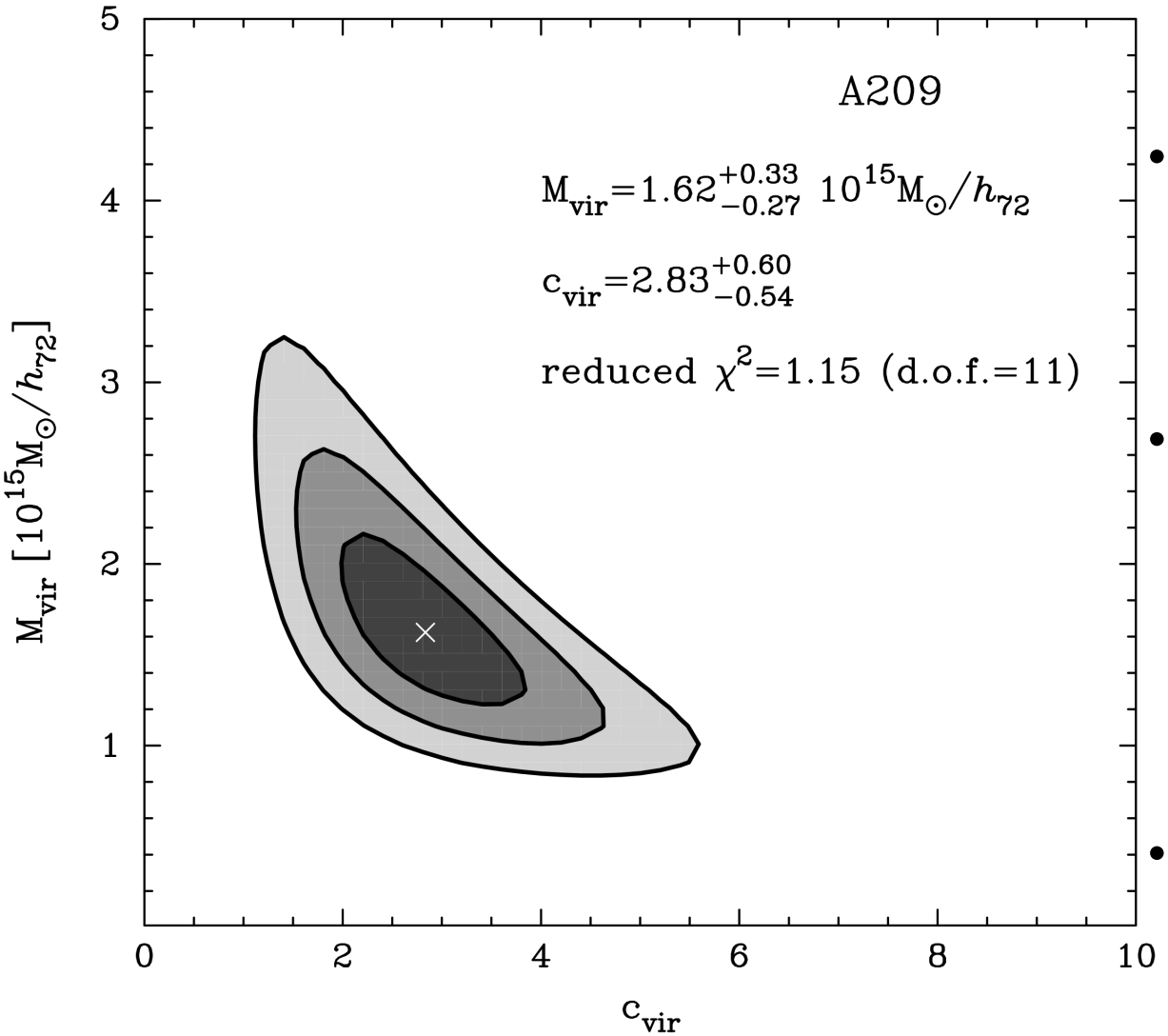
$\rho_{3D}(r; M_\Delta, c_\Delta)$

$\downarrow$ projection

$\kappa(\theta; M_\Delta, c_\Delta), \gamma(\theta; M_\Delta, c_\Delta)$

Okabe, MT, + 09

The degeneracy with halo concentration



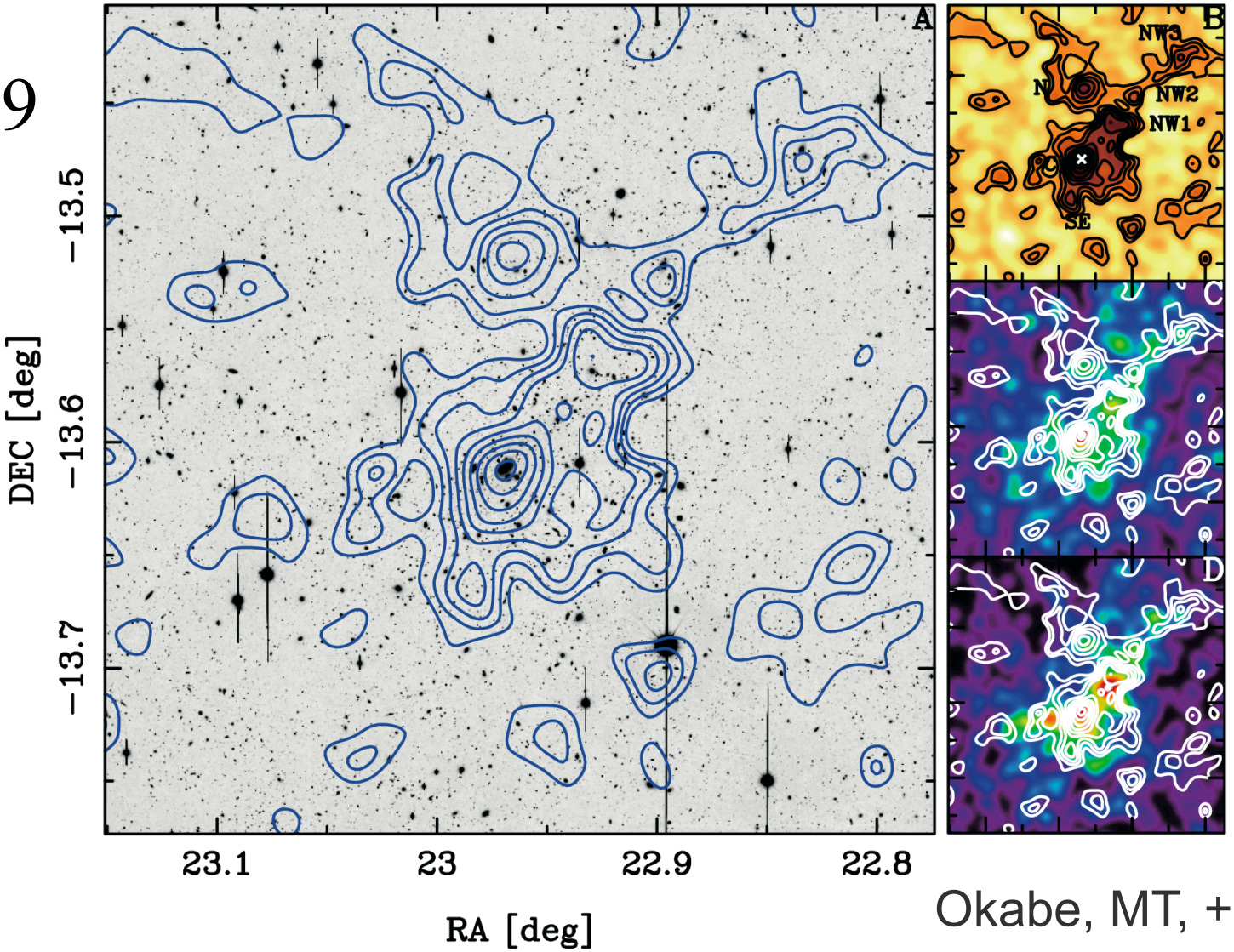
- The halo mass estimation is significantly degenerate with halo concentration
- Hence, imposing the theoretical prior (c~4) may cause a significant bias (~50%) in the mass estimate (Mandelbaum et al. 09)
- The spherical NFW model is a reasonable model?

Mass Maps

shear : $\gamma_\alpha \Rightarrow$ 2D mass density : κ

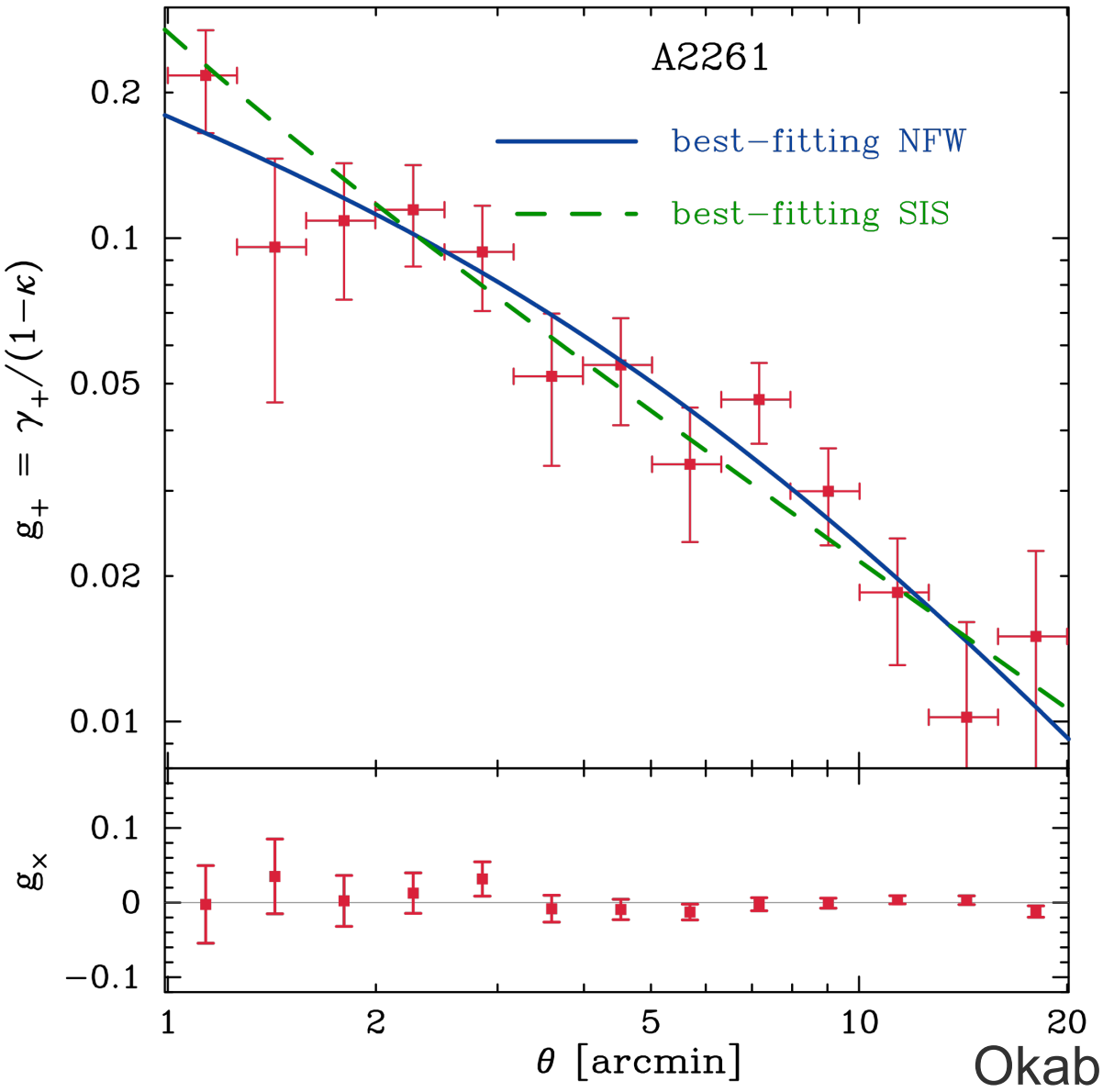
(the different pixels are correlated)

A209

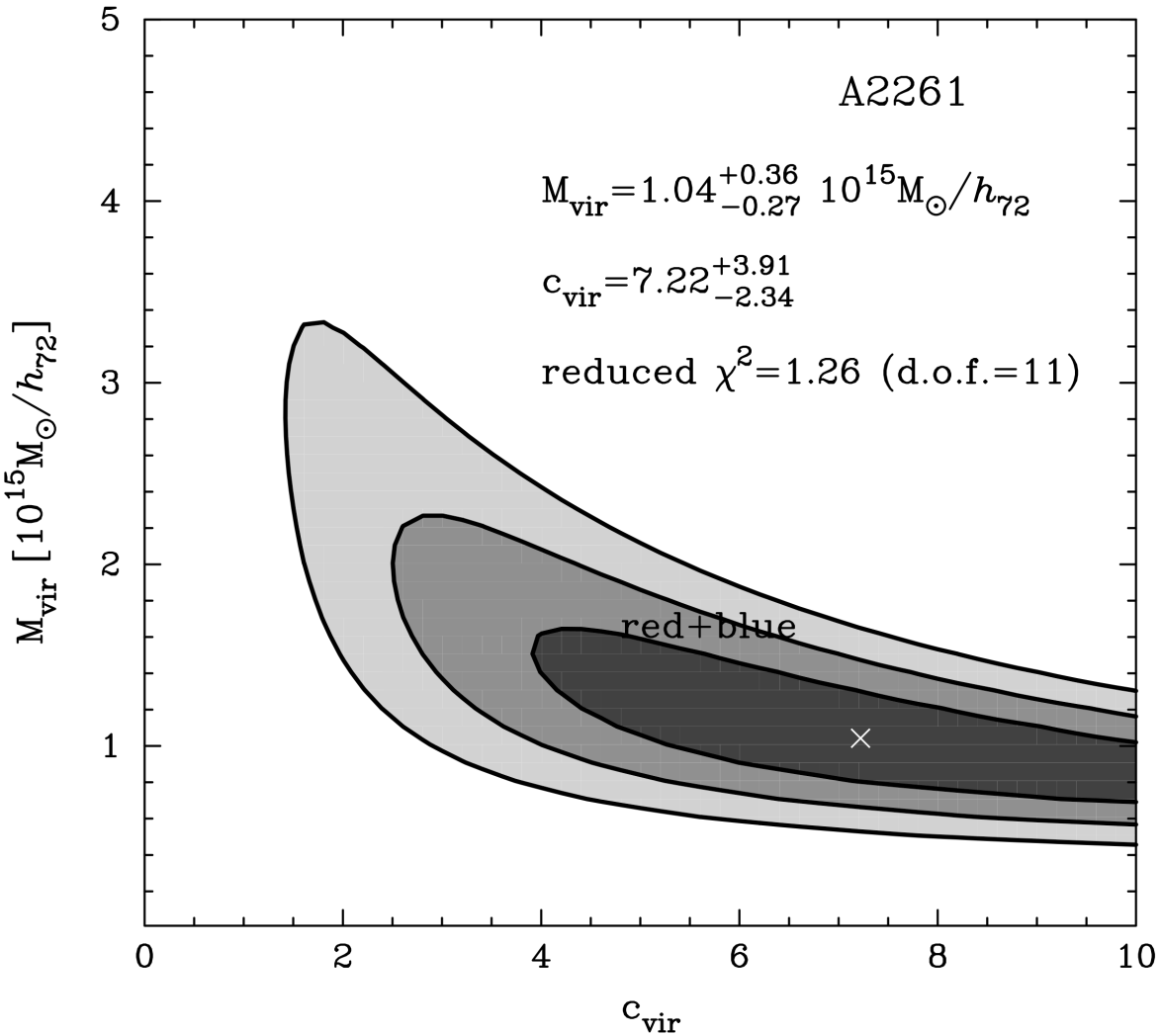


Okabe, MT, + 09

Example 2: A2261

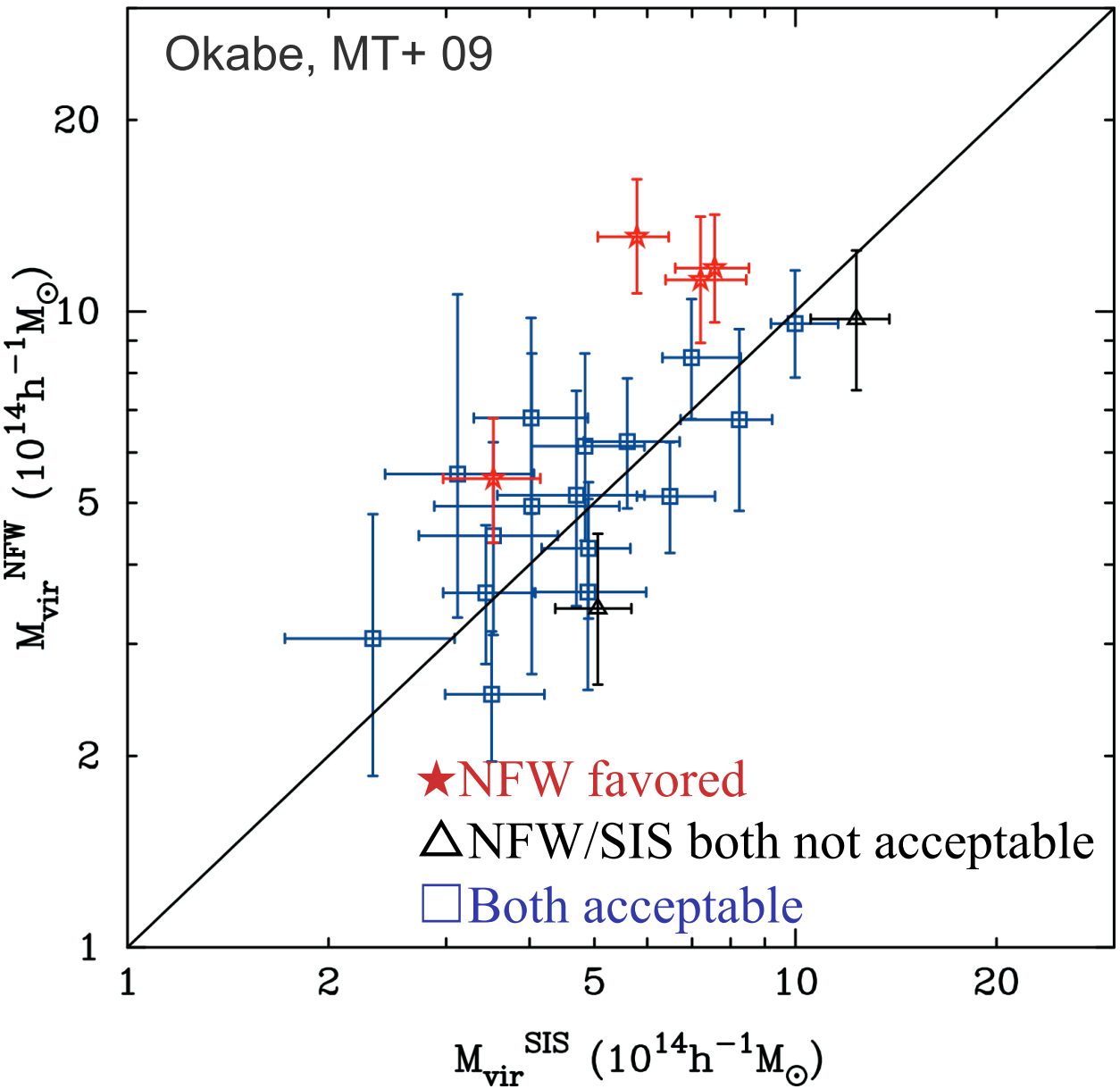


Okabe, MT, + 09



Even more stronger degeneracy. This level degeneracy is quite often….

Compiling ~20 clusters

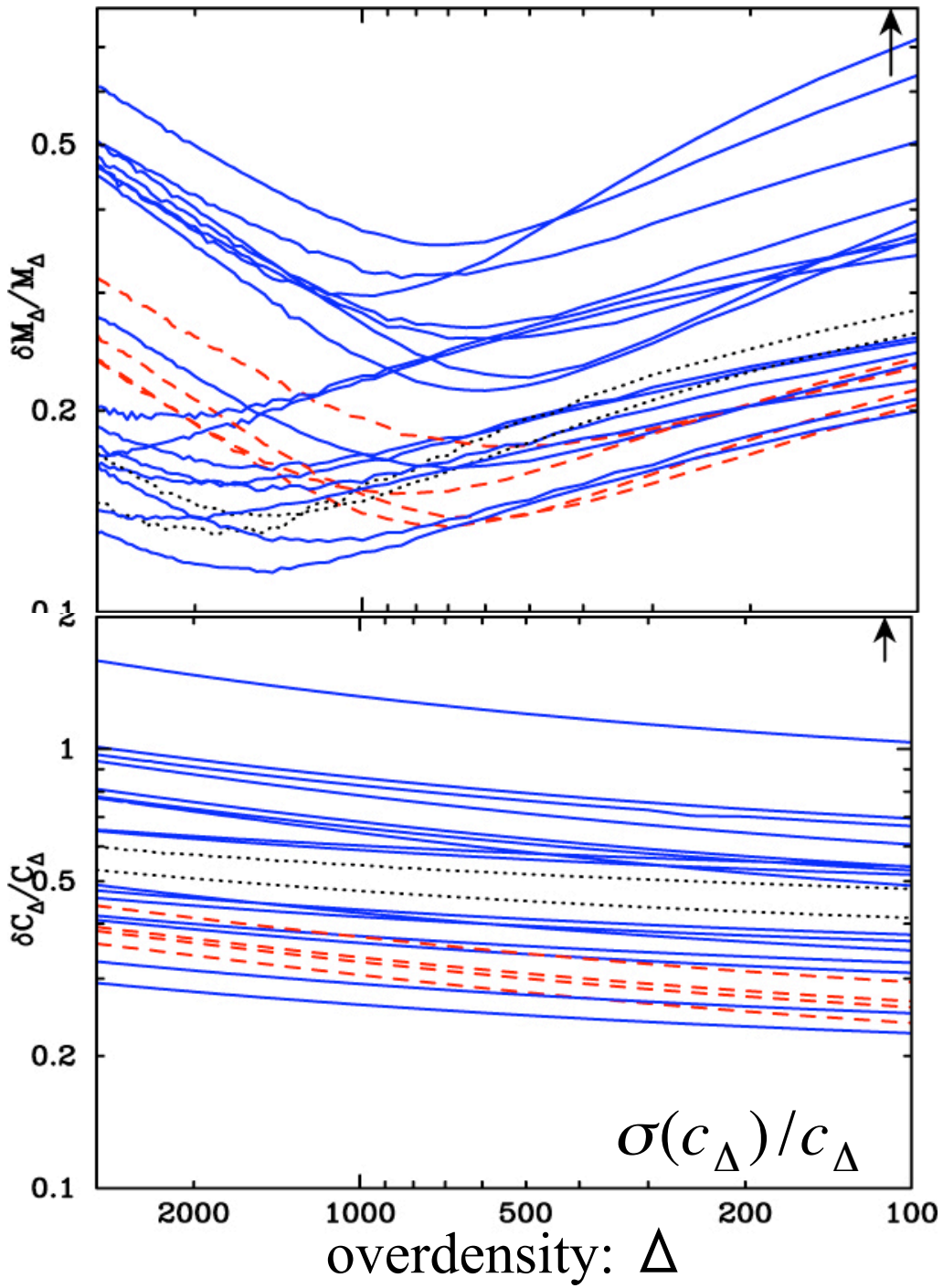


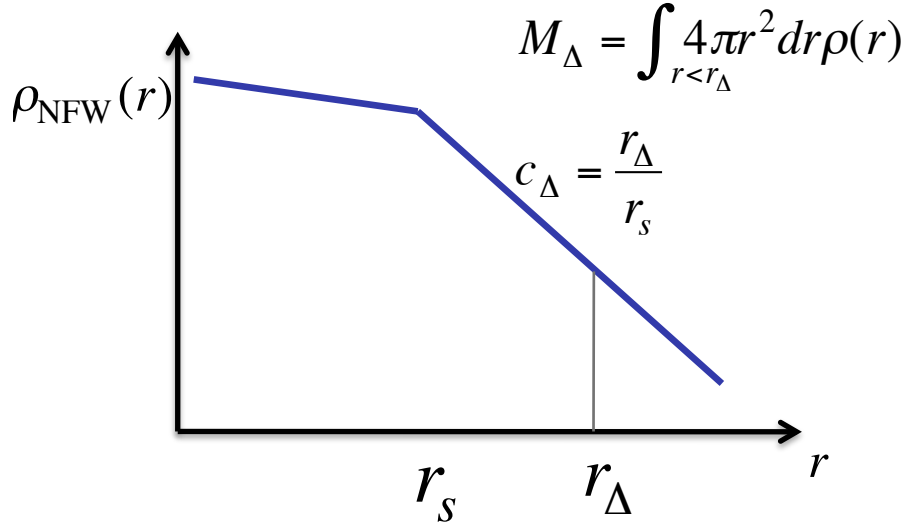
$$\rho_{NFW} \propto \frac{1}{(r/r_s)(1+r/r_s)^2}$$

$$\rho_{SIS} \propto \frac{1}{r^2}$$

- **All clusters: S/N>5 (typically S/N~10)**
- The mass estimates depend on the model assumed for the fitting
- The virial mass determination: accuracy 20-30%
- M_{NFW}/M_{SIS}~1.19

Mass determination (contd.)



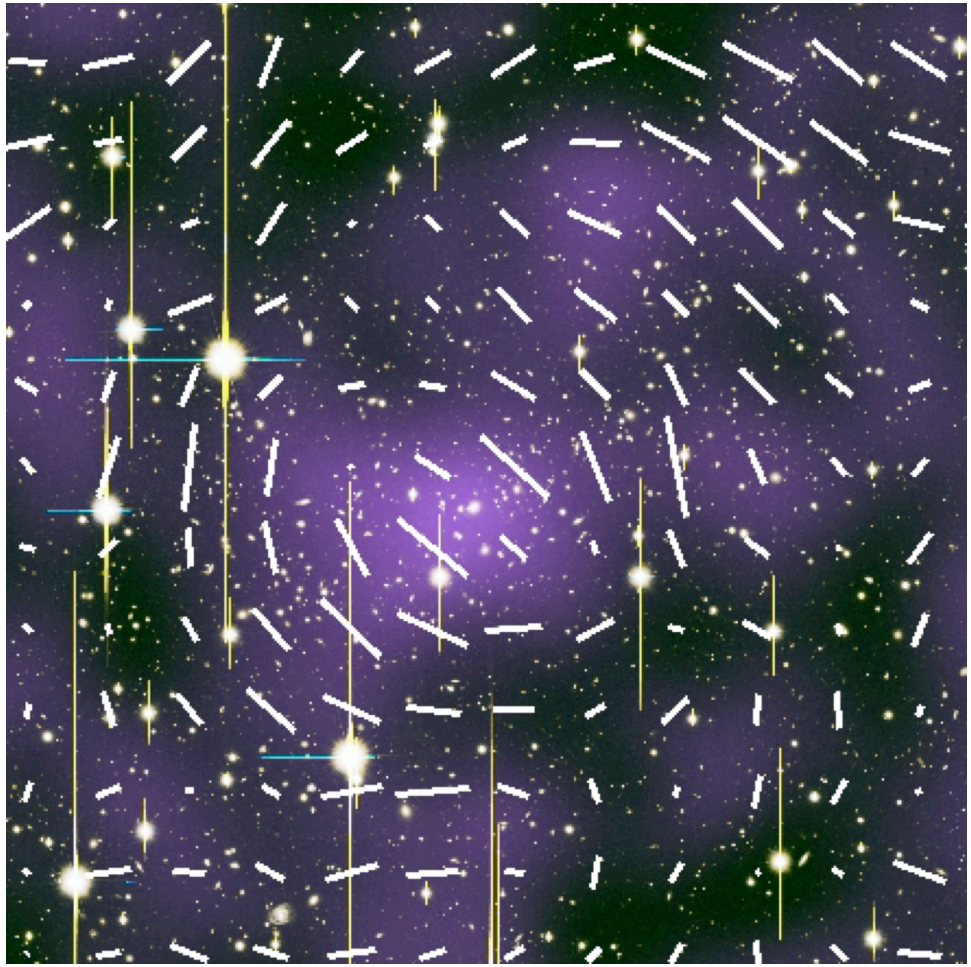


- A best accuracy in M is 10-20% when Δ=500-1000 is assumed
  - Over the radii the lensing signals have a largest S/N
- The concentration parameter is most accurately measured for the virial definition

Full use of 2D shear map

Oguri, MT, + to be submitted

A2390



- The cluster mass distribution is far from spherical symmetry, as predicted from the collisionless CDM model.
- Jing & Suto showed that simulated halos can be better described by a triaxial halo model than the spherical one
- Projecting the triaxial halo model along the l.o.s. gives the 2D mass density:

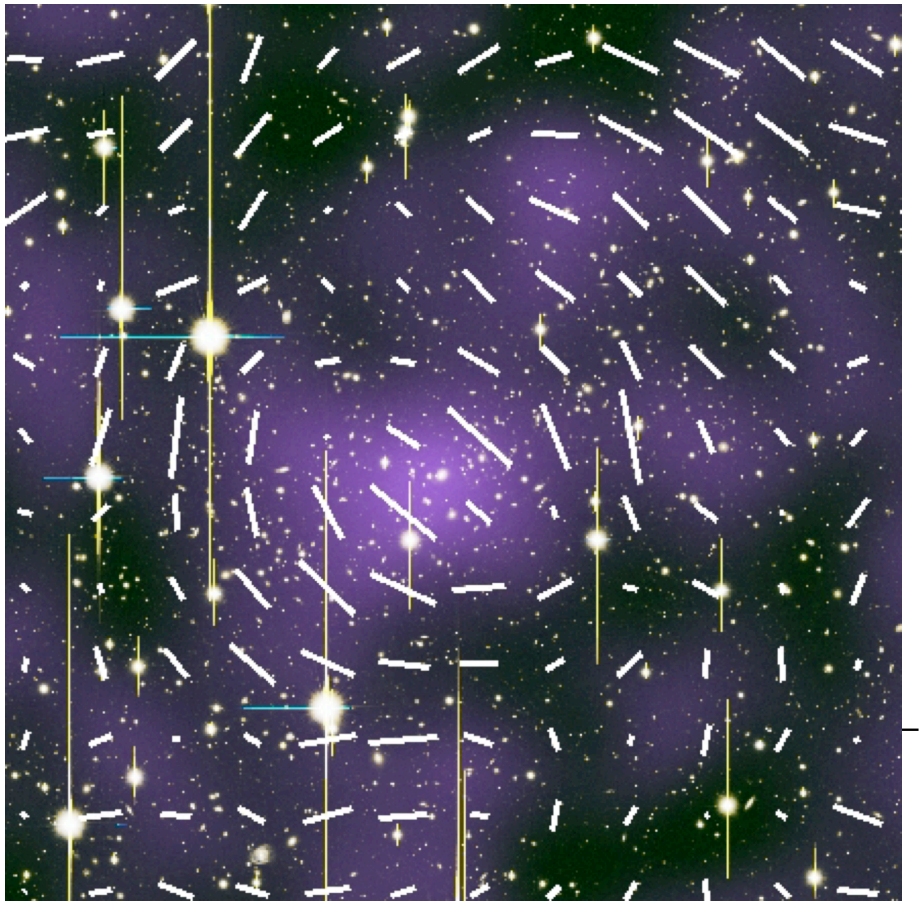
$$\kappa(x, y) = \kappa_{\rm sph}(\zeta),$$

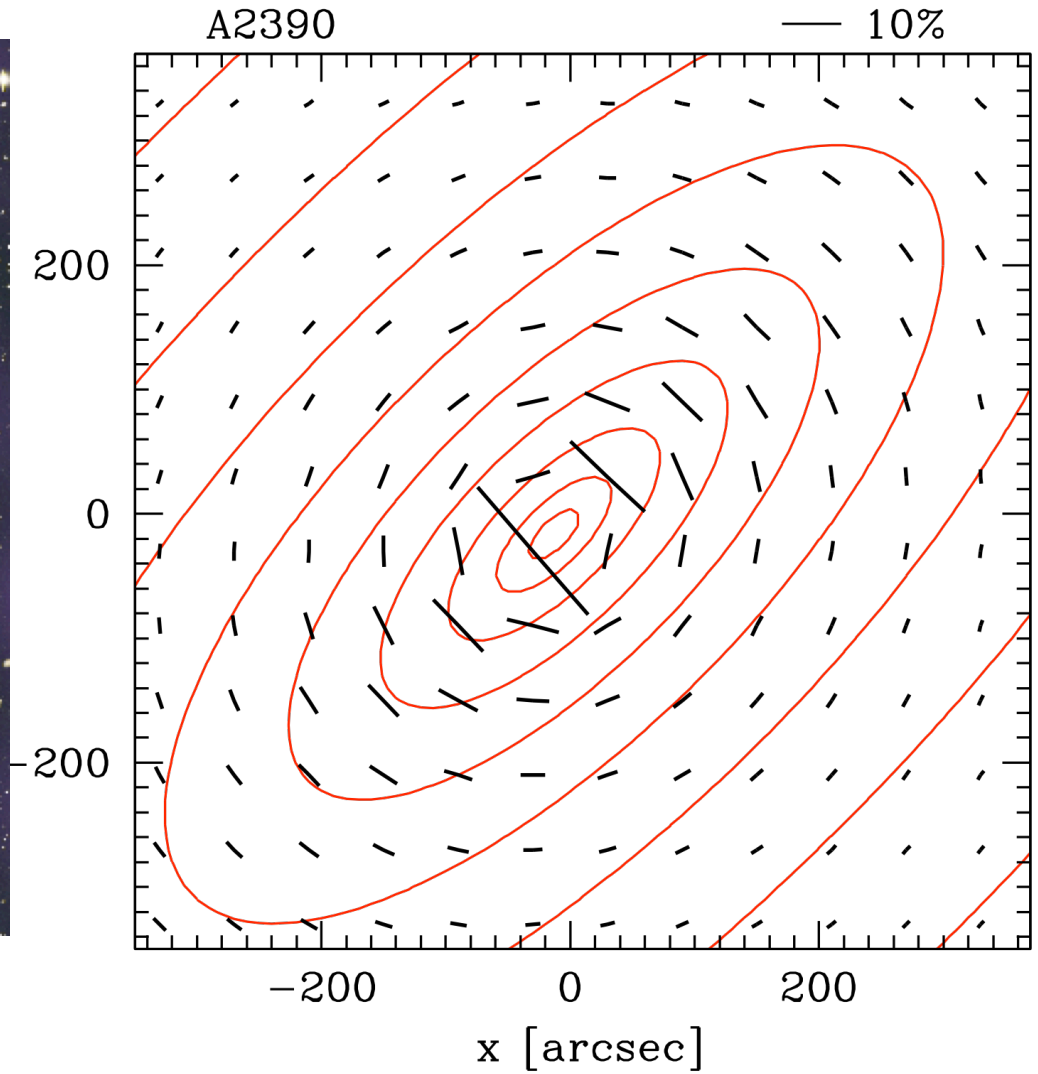
$$\zeta^2 = \frac{x'^2}{1-e} + (1-e)y'^2,$$

$$x' = x\cos\theta_e + y\sin\theta_e,$$

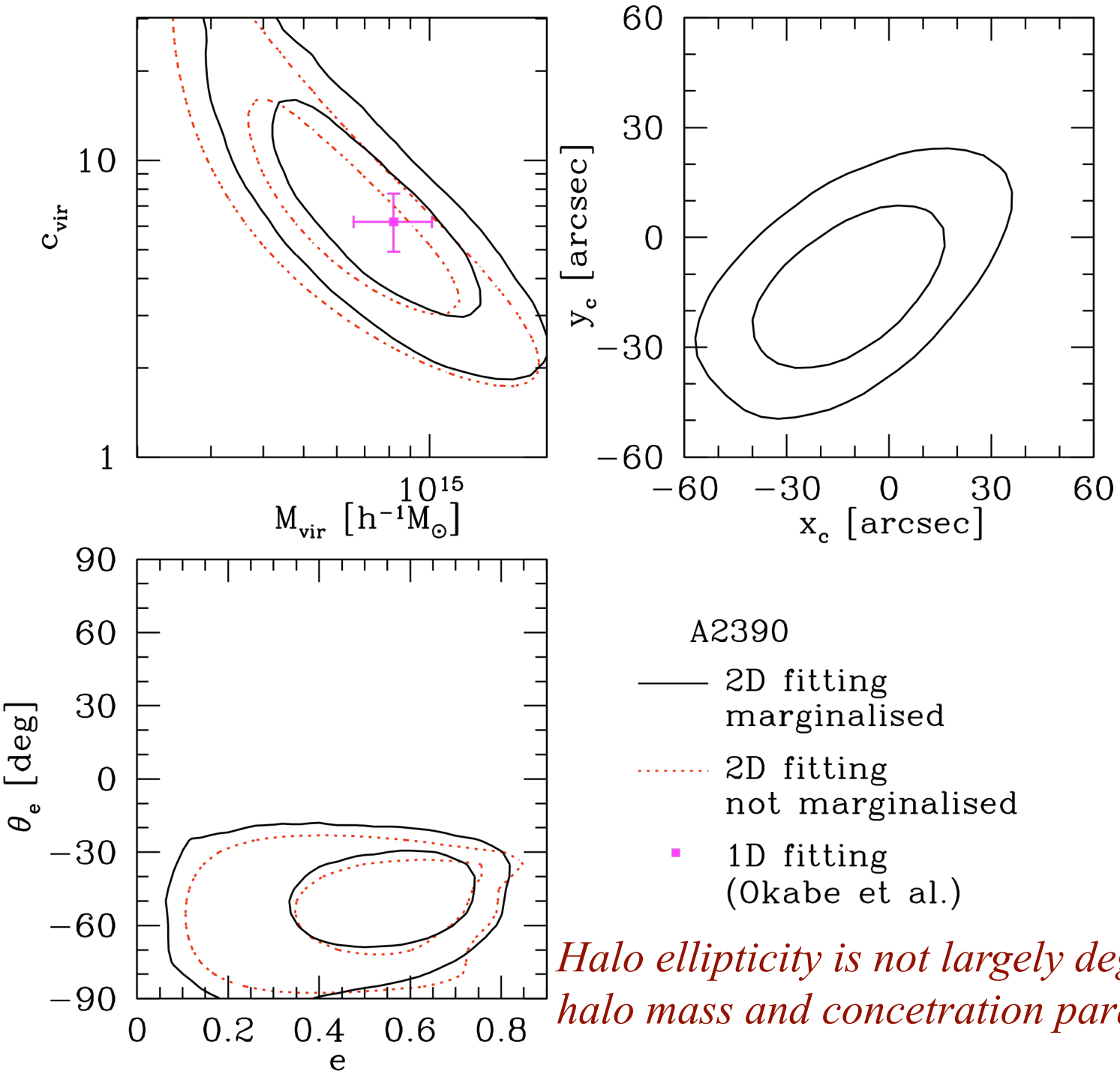
$$y' = -x\sin\theta_e + y\cos\theta_e,$$

2D shear fitting

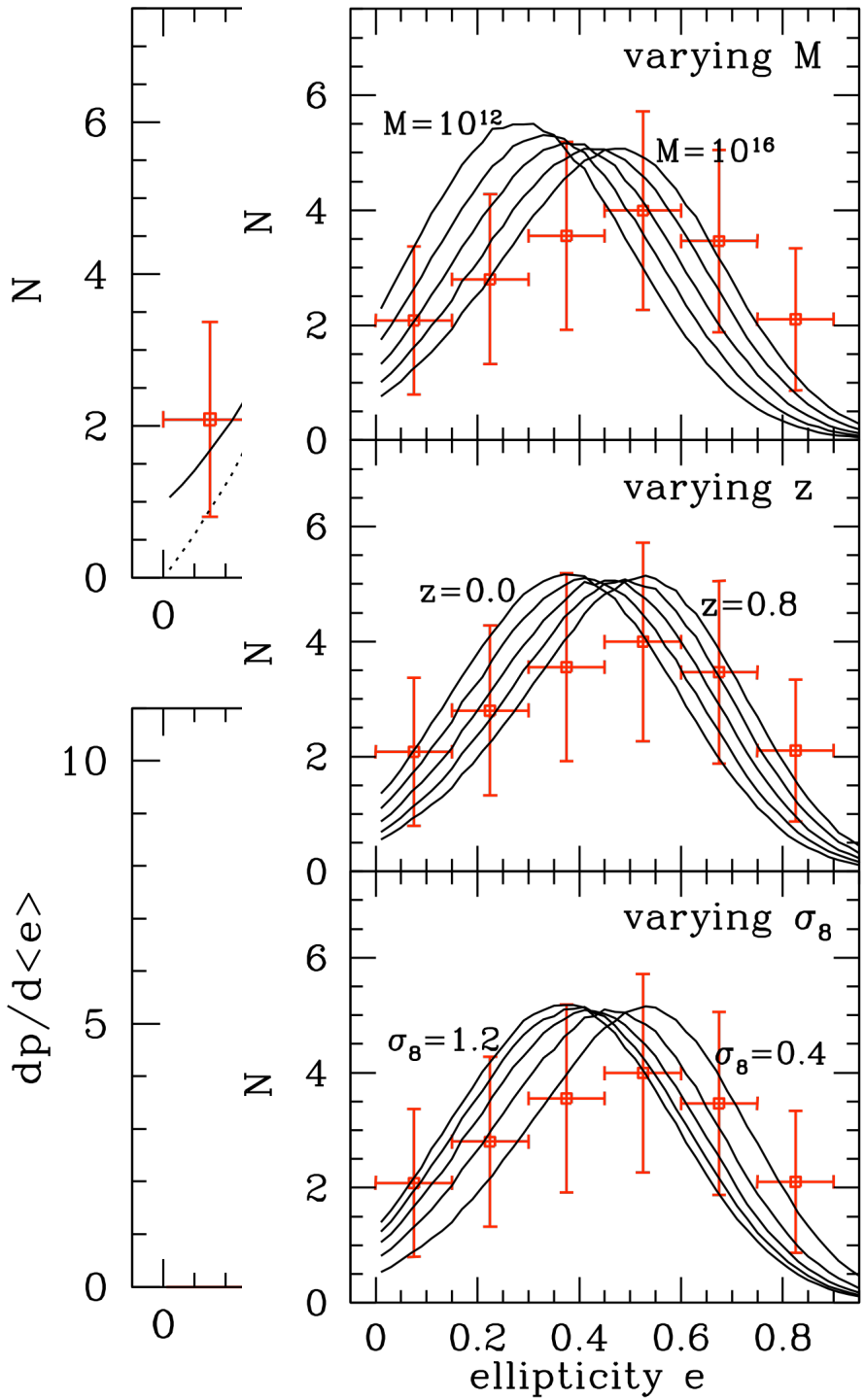




- In this particular case, e_2D=1-b/a=0.59
- Note that the iso-contours of shear amplitudes are not elliptical, needs to solve the 2D Poisson equation.



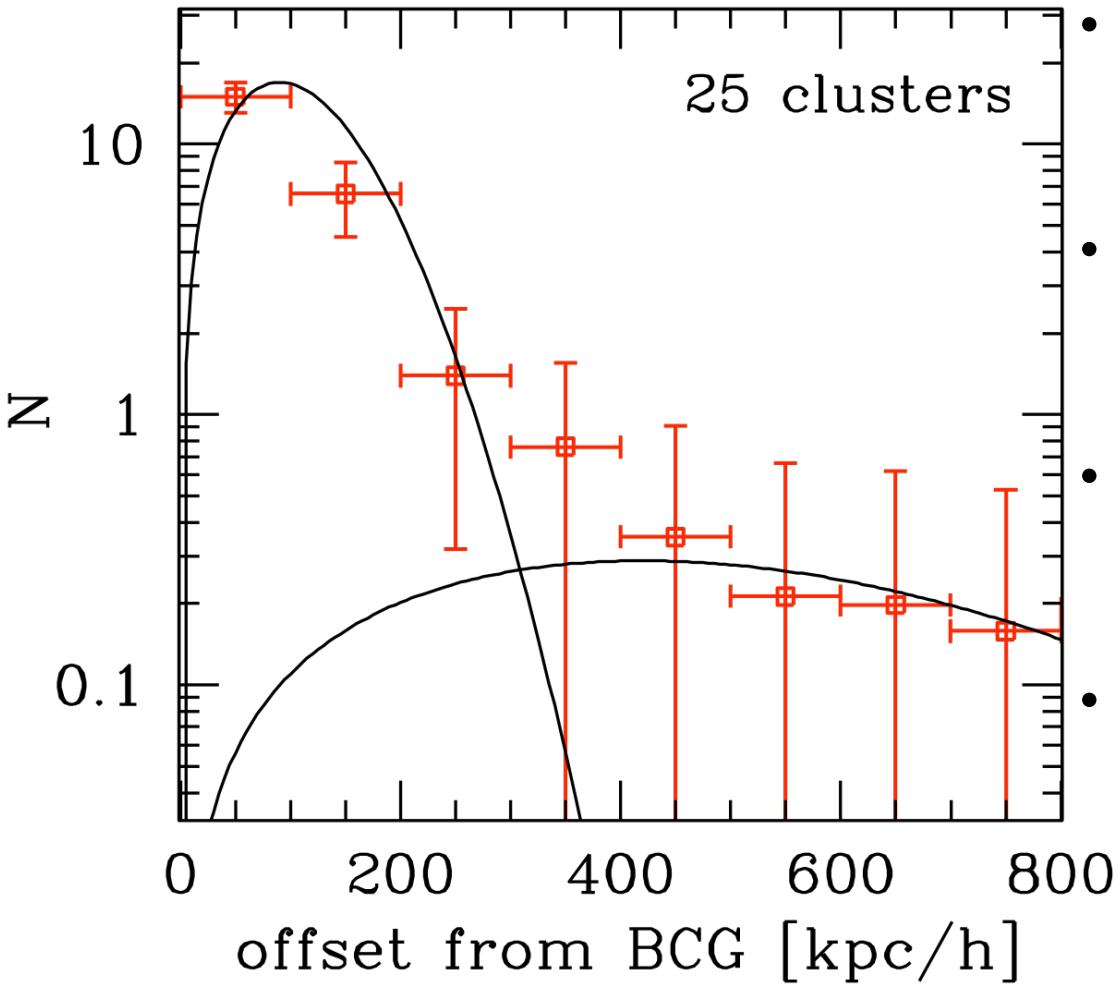
Halo ellipticity is not largely degenerate with halo mass and concetration parameters



A detection of halo ellipticity

- A significant detection of halo ellipticity for 18 clusters, at 7σ level compared to the spherical model
- The ellipticity ~0.5 on average
  - X-ray images show e~0.2
  - Can exclude MOND?
- Remarkable agreement with the CDM predictions
- Not enough to discriminate the model differences

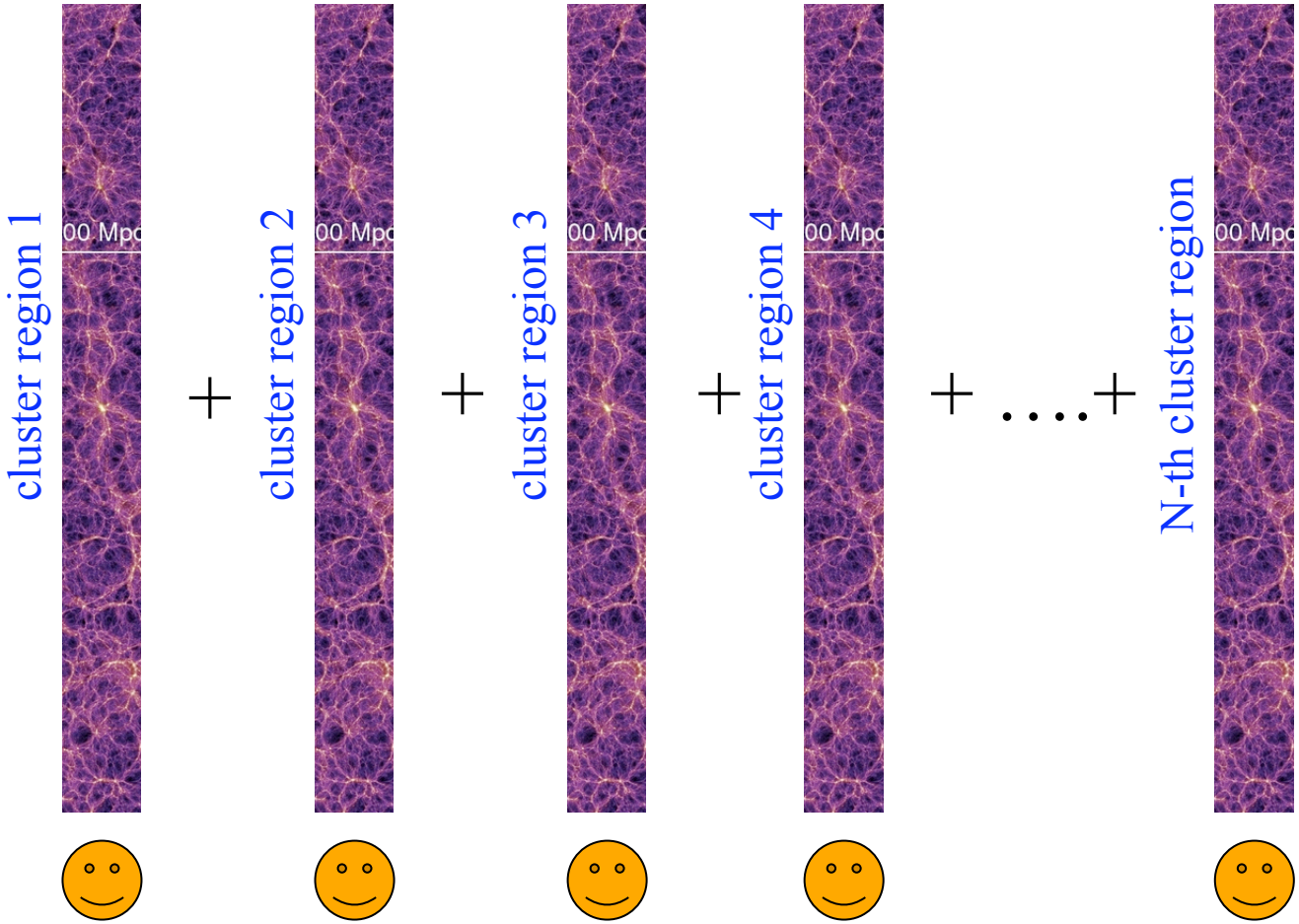
Halo center



- Halo center, constrained from lensing, is close to the position of brightest central galaxy
- However, some clusters (about 10% fraction) show large offsets
- Imply that the BCG is oscillating around the potential well for some clusters
- Quantify the impact of systematic errors in the stacked cluster lensing analysis

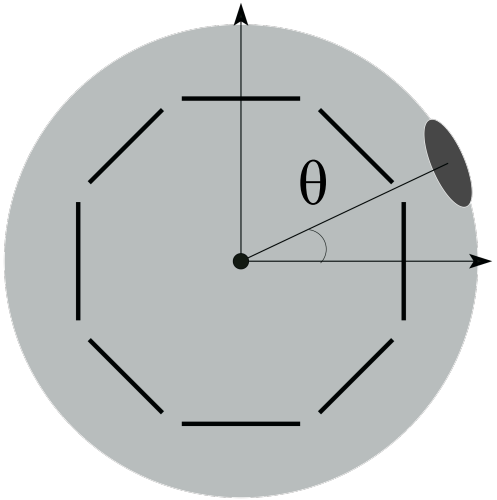
Stacked cluster lensing

$$\gamma_+^{\rm obs}(\theta_i) = \gamma_+^{\rm cluster}(\theta_i) + \gamma_+^{\rm LSS}(\theta_i) + \varepsilon_+(\theta_i)$$



$$\langle \gamma_+ \rangle(\theta) = \frac{1}{N_{cl}} \sum_{a=1}^{N_{cl}} \sum_{|\vec{\theta}'| \subset \theta} \gamma_{+(a\text{-th cluster})}(\vec{\theta}') \approx \left\langle \gamma_+^{\rm cluster} \right\rangle(\theta)$$

Stacked cluster lensing (contd.)

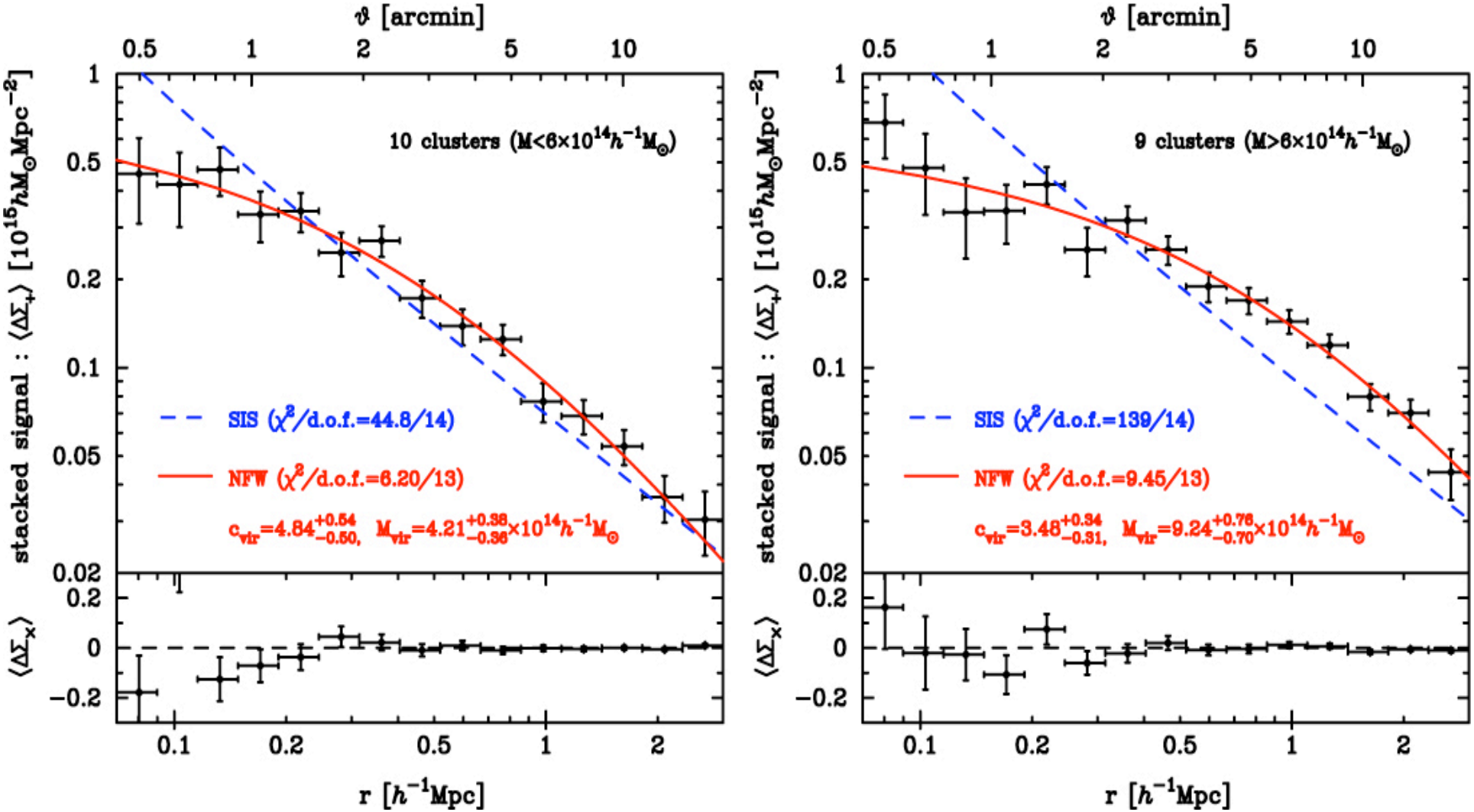


- The signal is *linear* (⇔cosmic shear)
  - Contaminating effects can be significantly reduced after stacking (if well behaved): projection effect and systematic errors, intrinsic alignment
- WL angular resolution improved due to the gain in background galaxies
- The stacking picks up *spherical* mass distribution in cluster regions, due to the statistical isotropy of the Universe
  - Easy to compare with models in order to easy cluster mass
- The stacking method probes average mass in each richness bins
  - Clusters need to be binned in each richness bin, based on the pre-observations (optical, X-ray, SZ, individual lensing itself)
- The centroid position of each cluster needs to be *a priori* known
- Lensing studies of individual clusters are still needed

Results: stacked lensing

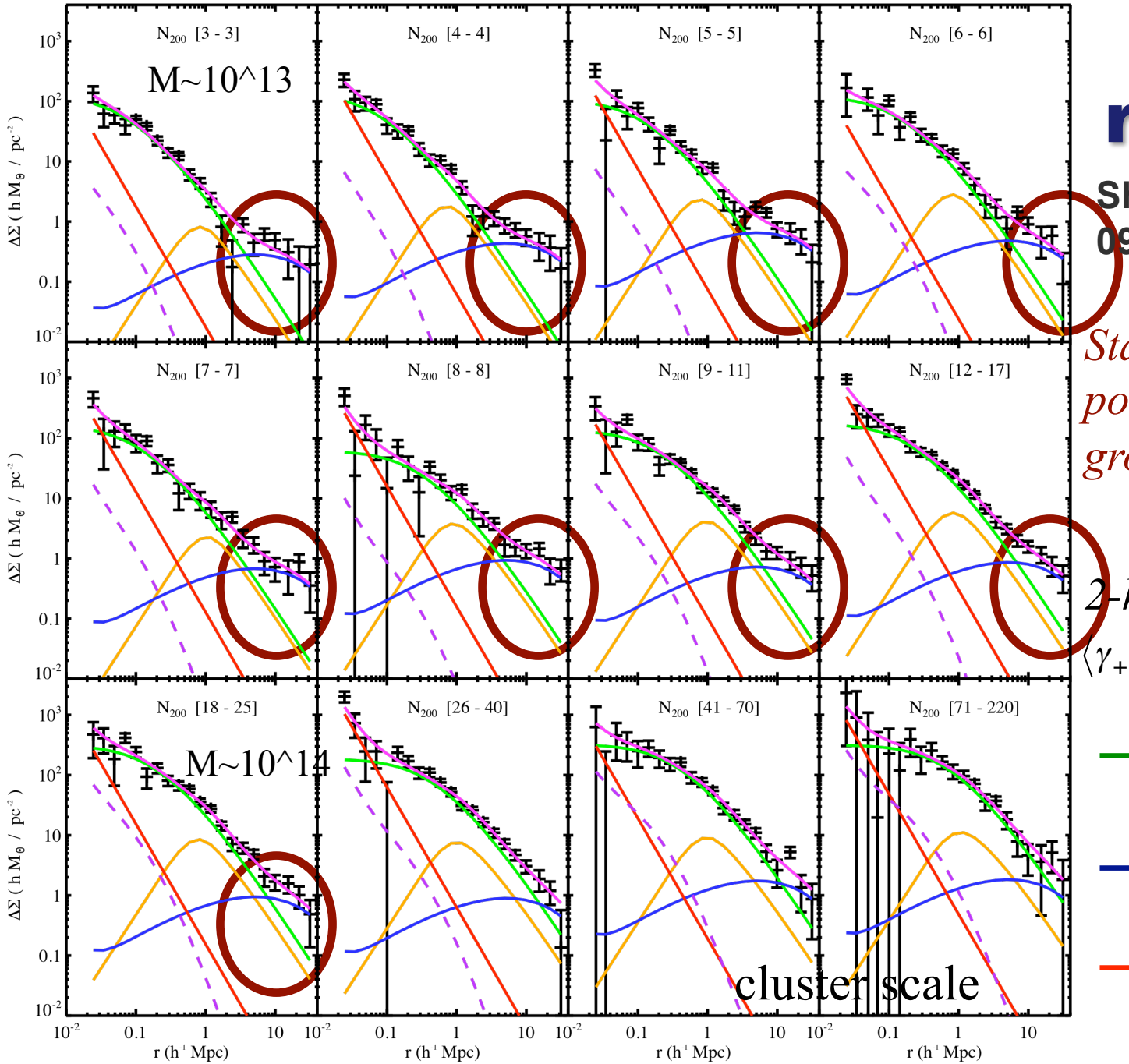
Okabe, MT+09

- For Subaru data, only ~10 clusters are enough to obtain the high S/N signals



$$\Delta\chi^2 = \chi^2_{\rm SIS} - \chi^2_{\rm NFW} = 39 \text{ and } 129 \text{ for low- and high-mass samples, respectively}$$

SDSS results



Sheldon et al. 09;Johnston et al. 07

Statistics is so powerful! 130,000 groups/clusters

2-halo term

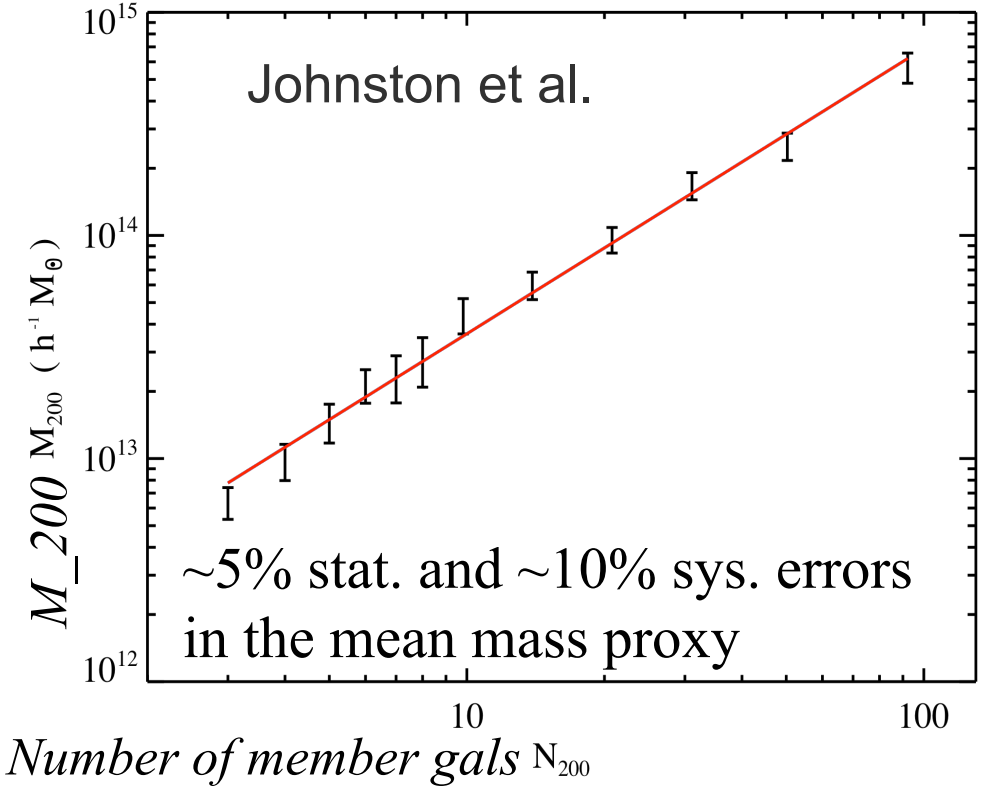
$$\langle \gamma_+ \rangle_{2h}(\theta) \propto b(M)\xi_m(d_A\theta)$$

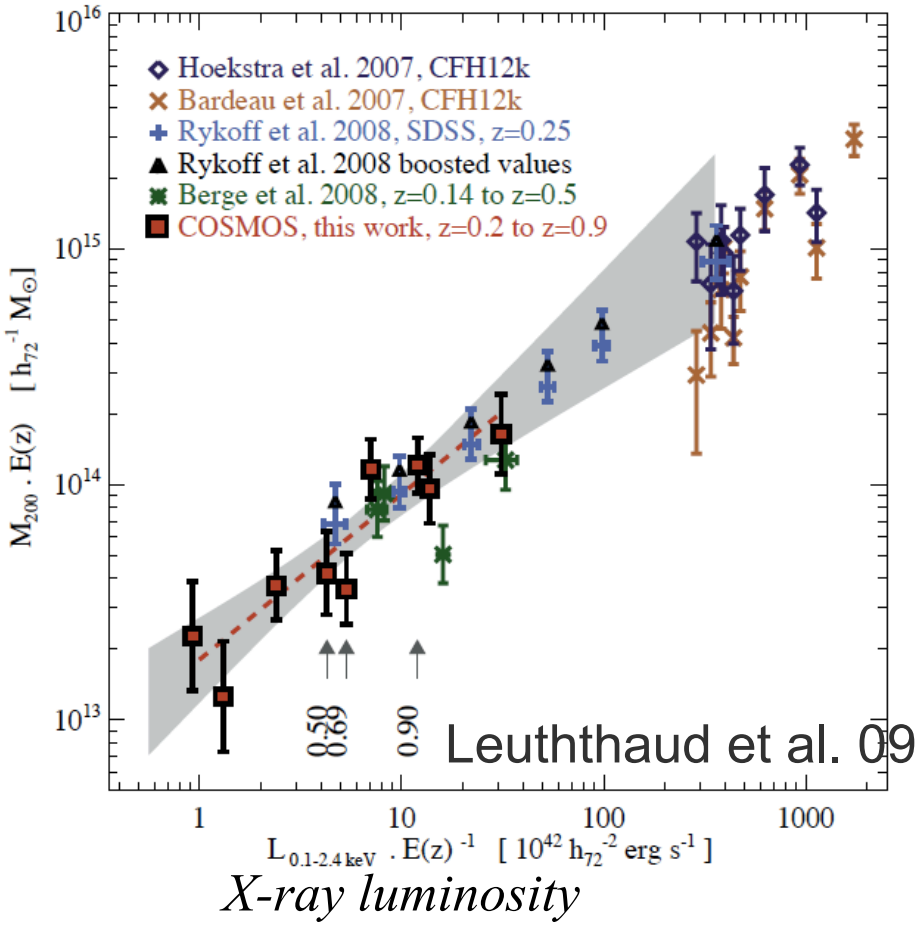
NFW

Cosmic webs

BCG

Mass proxy relation

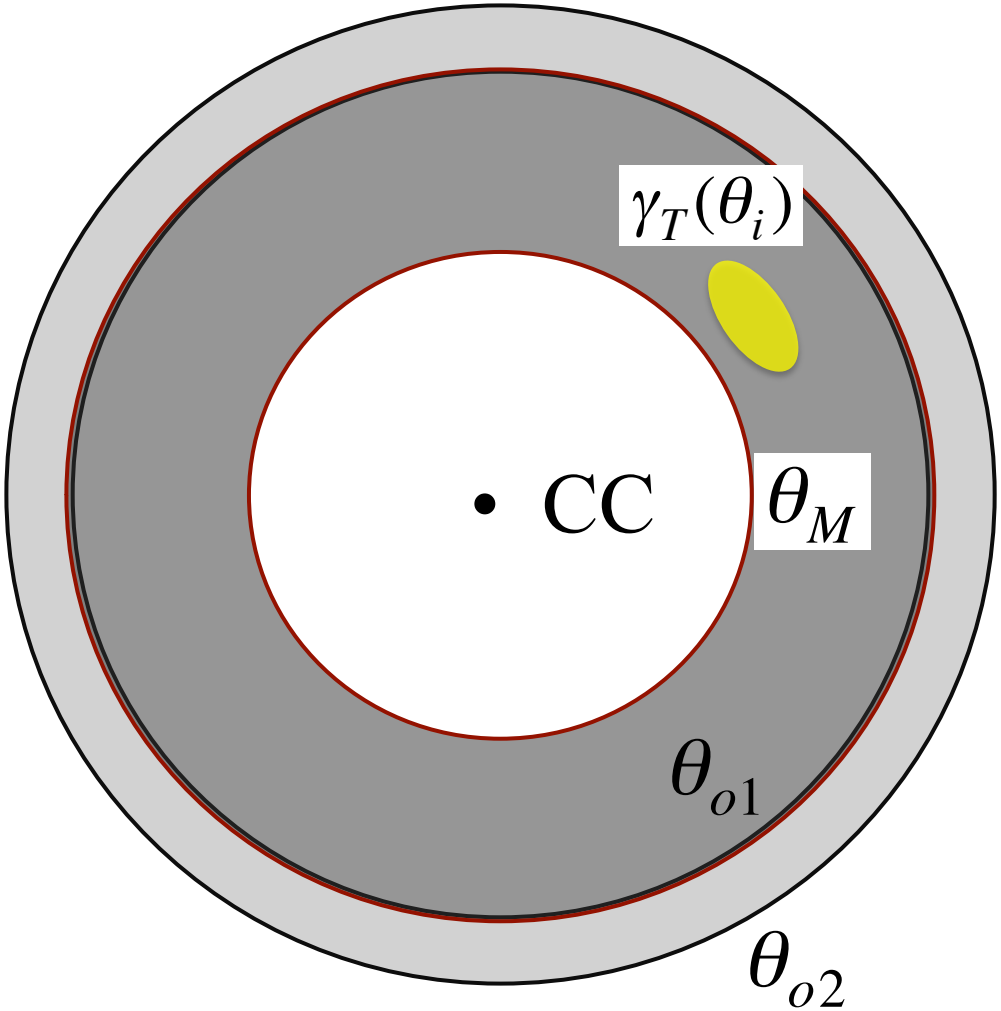




- Most of constraint on halo mass from the 1-halo term
- Note that the stacking method still suffers from the mass and concentration degeneracy

Aperture mass method

- model-independent estimate of 2D mass -



Use the measured shear profile at radii greater than θ_M (don't use the shear in the inner region)

$$\zeta(\theta_M) = 2\int_{\theta_M}^{\theta_{o1}} d\ln\theta \langle\gamma_T\rangle(\theta) - \frac{2}{1-\theta_{o1}^2/\theta_{o2}^2}\int_{\theta_{o1}}^{\theta_{o2}} d\ln\theta\langle\gamma_T\rangle(\theta)$$

$$\propto \frac{M_{2D}(<\theta_M)}{\pi\theta_M^2} - \frac{M_{2D}(\theta_{o1}<\theta<\theta_{o2})}{\pi(\theta_{o2}^2-\theta_{o1}^2)}$$

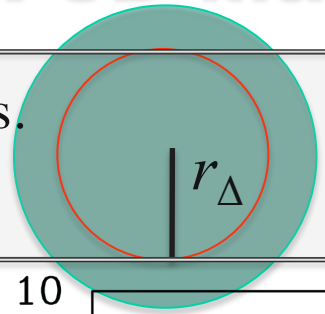
if $M_{2D}(\theta_{o1}<\theta<\theta_{o2}) \approx 0$

$$M_{2D}(<\theta_M) \approx \pi\theta_M^2\zeta(\theta_M)\Sigma_{\rm cr}$$

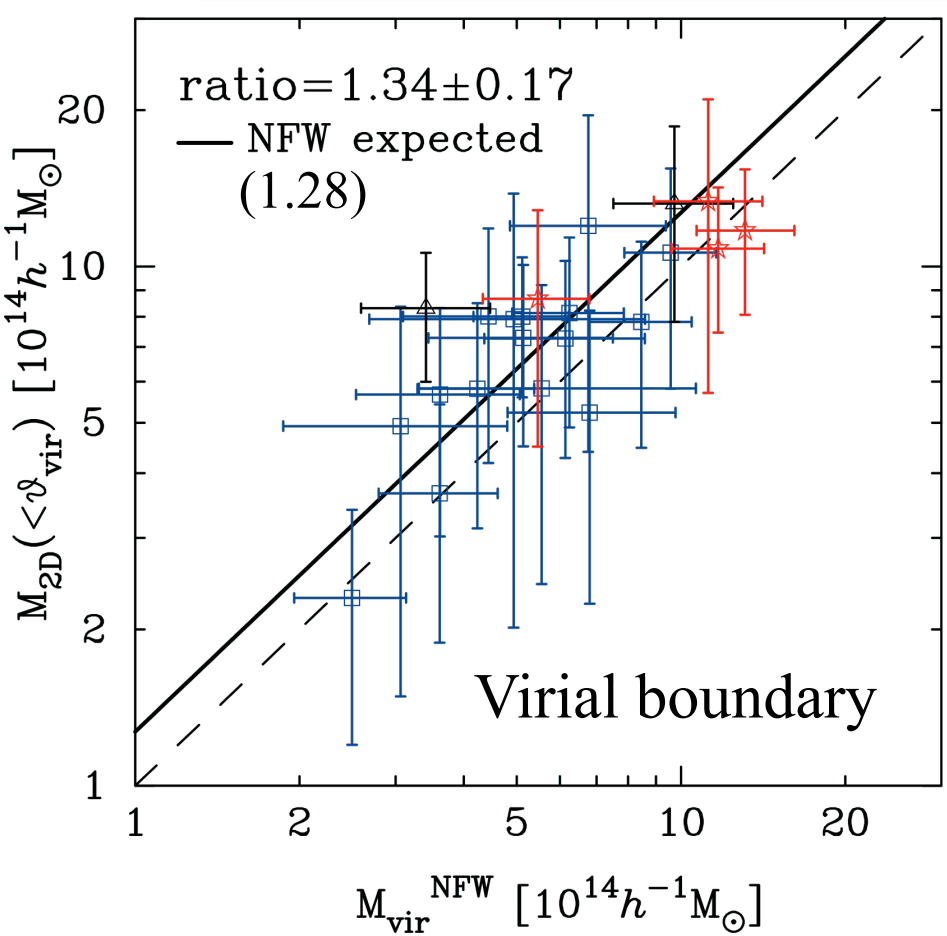
Model-independent mass estimate: 2D Mass vs. 3D Mass

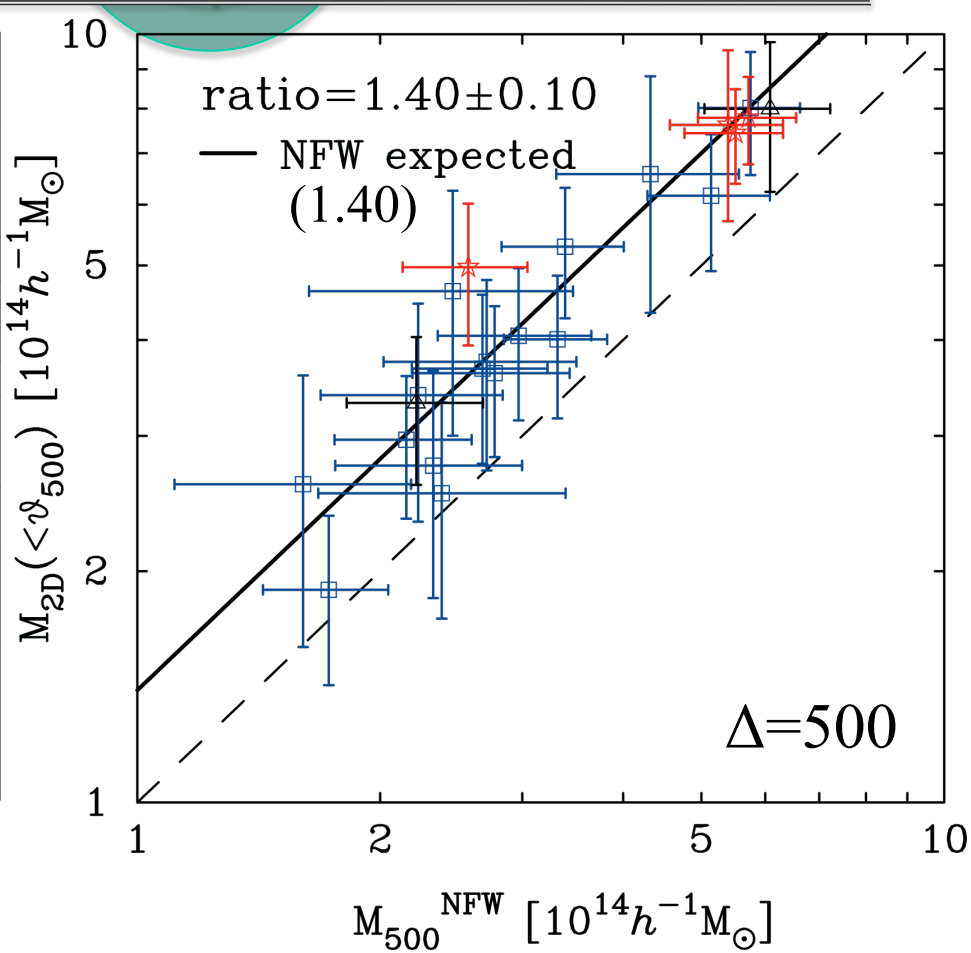
2D Mass = Projected mass along the l.o.s.

$M_{2D}(< r_\Delta) \sim \pi r_\Delta^2 \langle g_T \rangle (\theta_\Delta)$



3D mass: $M_{\rm NFW}(<r_\Delta)$





A new statistic: Mandelbaum et al.

Annular differential surface density (ADSD)

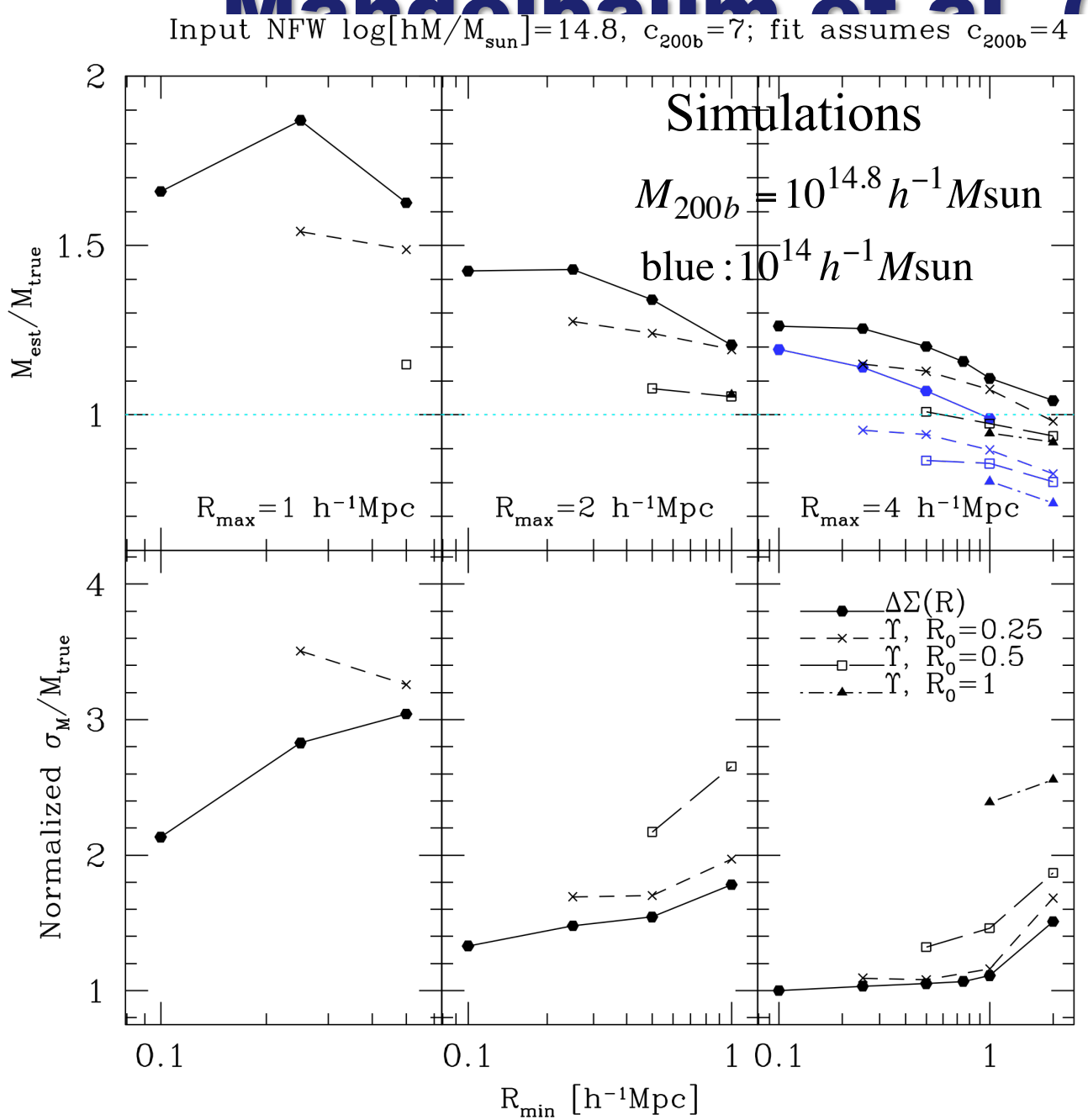
- Make the measured shear signal less sensitive to the inner region (that is sensitive to halo concentration)

$$\Upsilon(\theta,\theta_0) \equiv \gamma_+(\theta) - \left(\frac{\theta_0}{\theta}\right)^2 \gamma(\theta_0)$$

θ_0: a control radius that needs to be chosen by user
the recommendation is about 0.2 of the virial radius

- Then can estimate halo mass by fitting the measured profile of $\Upsilon(\theta, \theta_0)$ to the NFW predictions. The key is this fitting is less sensitive to the assumed halo concentration
- The price for this is less statistical accuracies and we need to measure the profile over a wider range of radii (up to ~4Mpc)

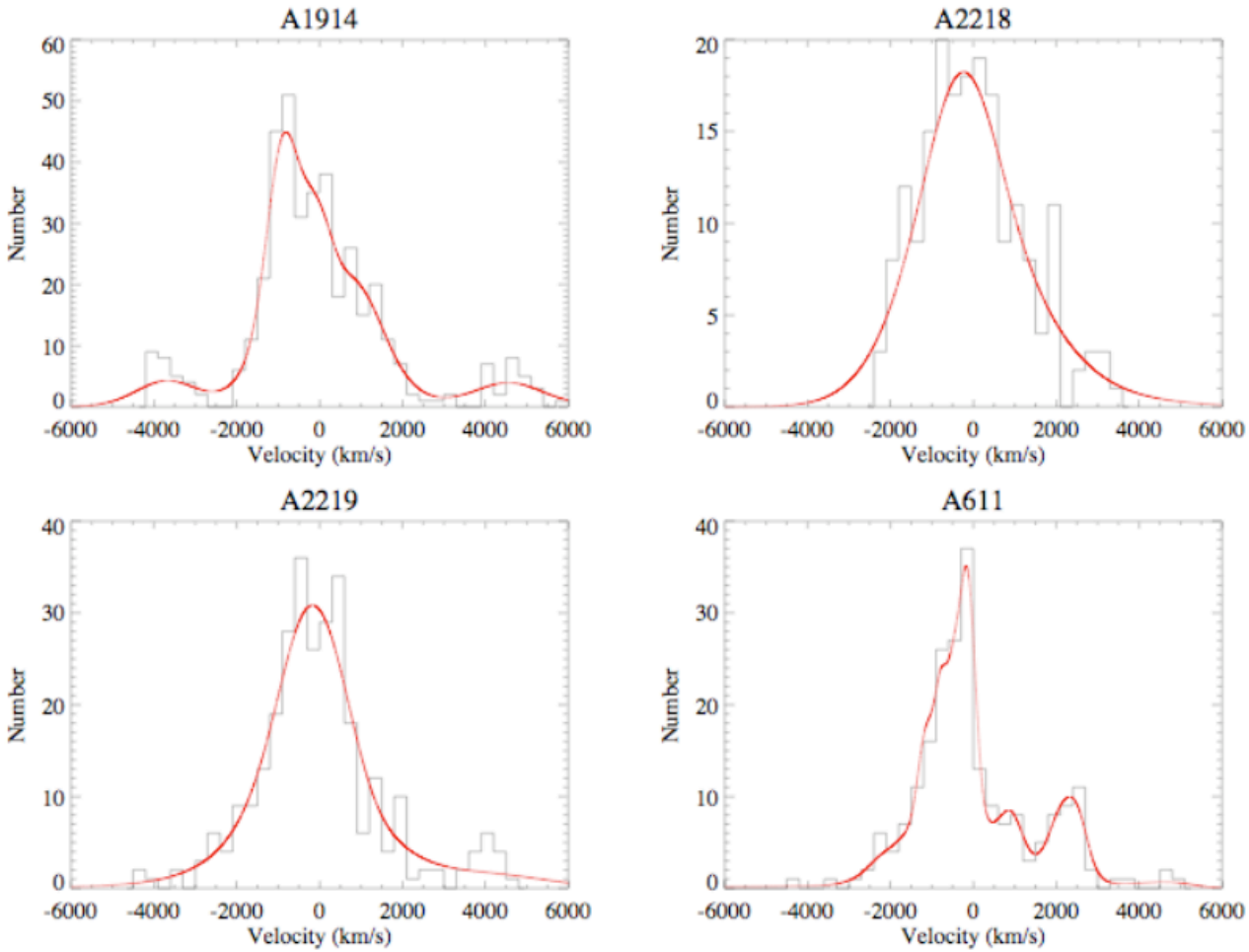
Mandelbaum et al. (contd.)



Bottom line is R0~0.25Mpc/h, Rmax~4Mpc are a good choice to obtain less biased halo mass estimate with reasonable statistical accuracy

Test of gravity on cluster scales?

Egami et al.: MMT (6.5m, 300 fibers) ~200-300 members/cluster for 30 clusters (17 clusters as of May 2009)



$$\gamma \sim \nabla^2(\Phi + \Psi)$$

$$\mathbf{v} \sim \nabla\Psi$$

or

$$M_{\rm wl} \text{vs.} M_{\rm dyn}$$

Systematic errors of cluster lensing

- Shape measurements: but perhaps less demanding compared with cosmic shear, because we look for the tangential shear components (rather than all the shear fields)
- Source redshifts: cluster members have accurate photo-z's
- Dilution effect: including member and foreground, i.e. unlensed, galaxies in source galaxy sample causes a dilution of the shear signal
  - The dilution significantly affects the halo concentration estimation
- Cluster center uncertainty
- Defining a fair good sample of clusters in each mass bins
- Need more studies to quantify the power of stacking cluster lensing for doing cosmology (MT, Oguri, + in prep.)

Summary: Lecture II

- Clusters are very powerful probe of cosmology
  - Need well-calibrated *mass proxy relation*
  - Q: a small sample of very massive clusters (e.g. ~100 X-ray clusters in Vikhilinin et al. 09) vs. large cluster sample?
- The signal is *"linear"* in the weak lensing limit
  - *Less sensitive to the systematics*: intrinsic alignments (if background galaxies are separated), shape errors
- Individual cluster lensing
  - The detailed studies are important in order to understand the scatters of mass proxy relation
  - Can constrain *mass, profile, halo shape and its centroid position*
- Stacking cluster lensing: cross-correlation of cluster and background galaxy images
  - Extract *"spherical" mass distribution* around the cluster
  - More straightforward to estimate cluster masses in each cluster mass bin
  - Systematics: cluster centroids; binning of cluster richness

IPMU international conference (July 2010)

CL J2010: from Massive Galaxy Formation to Dark Energy

an IPMU/DENET international conference on galaxy clusters
6/28 - 7/2, 2010, Kashiwa, Japan

confirmed invited speakers

Stefano Borgani (Trieste)
James Bullock (UCI)
Sudeep Das (BCCP)
Gabriella De Lucia (INAF-OAT)
Richard Ellis (Caltech)
August Evrard (Michigan)
Anthony Gonzalez (Florida)
Takashi Hamana (NAOJ)
Henk Hoekstra (Leiden)
Gilbert Holder (McGill)
William Holzapfel (UCB)
Tesla Jeltema (UCSC)
Tetsu Kitayama (Toho)
Tadayuki Kodama (NAOJ)
Andrey Kravtsov (Chicago)
Paul Martini (OSU)
Daisuke Nagai (Yale)
Nikhil Padmanabhan (Yale)
Bianca Poggianti (Padova)
David Spergel (Princeton/IPMU)
Tommaso Treu (UCSB)
Alexey Vikhlinin (CfA)

Organizers
Yen-Ting Lin (Chair)
Masahiro Takada
Taddy Kodama
Masamune Oguri, +