Dark Matter: Halo Mergers in $\Lambda$CDM Cosmology

Chung-Pei Ma
(UC Berkeley)
Lecture Outline

1. **Halo merger rate** from Millennium simulation
   - “Universal” global rate
   - Environmental dependence

2. **Mass accretion** history

3. **Theoretical** merger models:
   - Extended Press-Schechter
   - Coagulation equation
Merger Rate vs Mass Accretion History

Mass Growth $M(z)$

Merger rate $B(M,\xi,z)/n(M,z)$
\[
\frac{dM(z)}{dz} \leftrightarrow \int d\xi \frac{B(M, \xi, z)}{n(M, z)} M_2 + C(M, z)
\]

Mass gained from **mergers**

“diffuse” accretion
Fitting Forms for $M(z)$

\[ M(z) = M_0 e^{-\alpha z} \]

\[ M(z) = M_0 (1 + z)^\beta e^{-\gamma z} \]

\[ \frac{d \ln M(z)}{dz} = \beta - \gamma + O(z) \]

van den Bosch (2002)
McBride, Fakhouri, Ma (2009)
Mass Accretion History

McBride, Fakhouri, Ma (2009)

I. exponential growth

II. steep late-time growth

III. slow late-time growth
Formation Redshift Distribution

McBride, Fakhouri, Ma (2009)

I. exponential growth

II. steep late-time growth

III. slow late-time growth

$M(z)/M_0$ vs. $z$

formation redshift
Formation Redshift Distribution

McBride, Fakhouri, Ma (2009)
Formation Redshift Distribution

McBride, Fakhouri, Ma (2009)

Median and dispersion of halo formation redshift
Mass Accretion History vs Environment

McBride, Fakhouri, Ma (2009)

Slow late-time growing halos live in denser regions (see also Maulbetsch et al 2007)
Mass Accretion History vs Mergers

Fraction of final halo mass due to major mergers

40% of final halo mass gained via major mergers
Only 20-30% of halos have exponential growth $M(z)$. 
$M(z)$ is specified by one parameter $z_t$. 
Median $z_t$ ranges from 1.3 (at $10^{12} \, M_{\odot}$) to 0.6 (at $10^{15} \, M_{\odot}$) 
Large dispersion.

20% of galaxy and 60% of cluster halos have steeper late-time growth. 
$M(z)$ is well fit by two parameters 
Median $z_t$ is only 0.5 (for all mass) 
The last major mergers occur at low redshifts 
High fraction of final mass due to major mergers

The rest have stunned late-time growth. 
$M(z)$ is well fit by two parameters. 
Median $z_t$ is high (1.5 to 0.8). 
Live in denser environments; stronger tidal effects?

[M(z) is applicable for people.]
1. **Halo merger rate** from Millennium simulation

2. **Mass accretion** history

3. **Theoretical** merger models:
   - Extended Press-Schechter
   - Coagulation equation
Extended Press-Schechter Model


Based on spherical tophat collapse
No explicit merger dynamics

In the excursion set picture, a random walk starting at
\( \sigma(M_0, z_0) \) crosses the density threshold \( \delta_c / D(z) \) at \( \sigma(M, z) \).

→ Conditional or progenitor mass function

\[ p(M, z|M_0, z_0) \]

(asymmetric in M and M- M_0)
Progenitor Mass Function at $z=0.24, 2, 7, 15$

of $z=0$ Cluster Halos

Zhang, Fakhouri, Ma (2008)
Merger Rates in EPS Model

Merger rate \( \leftrightarrow \) Conditional mass function with \( z-z_0 \ll 1 \)

\[
\frac{B(M_0, \xi, z)}{n(M_0, z)} \leftrightarrow \frac{d^2 p}{dMdz} (M \rightarrow M_0 \mid z)
\]

\[
= \sqrt{\frac{2}{\pi}} \frac{d\delta_c(z)}{dz} \frac{1}{\sigma(M)} \left| \frac{d\ln \sigma}{d\ln M} \right| \left[ 1 - \frac{\sigma^2(M_0)}{\sigma^2(M)} \right]
\]

(Exponential terms cancelled out)
Press-Schechter $n(M)$ is not accurate.

⇒ Not surprisingly, EPS merger rate is not accurate.

⇒ But galaxy formation models often use EPS

⇒ Improvement with ellipsoidal model & moving barrier? (need accurate barriers at small $z-z_0$)

**Ratio of EPS to Nbody**

Fakhouri & Ma (2008)
Ellipsoidal Collapse Model

Ellipsoidal model provides some improvement over spherical model.
(see also Jorge Moreno’s poster)

Zhang, Ma, Fakhouri (2008)
Based on spherical tophat model
No merger dynamics

Cosmological Simulations:
Halos formed by mergers and accretion

How does $n(m,t)$ evolve into $n(m, t+dt)$?
Inspiration from other fields: aerosols, asteroids, interstellar grains, colloids etc.
**Coagulation Equation**

Smoluchowski (1916)

\[
\frac{dn(m,t)}{dt} = \frac{1}{2} \int_{0}^{m} dm' A(m - m', m', t)n(m - m', t)n(m, t) - \int_{0}^{\infty} dm' A(m, m', t)n(m, t)n(m', t)
\]

(ignoring fragmentation)

**mergers into mass bin m**

**mergers out of mass bin m**

Intriguing case: for \( n=0 \) \( P(k) \), additive kernel \( A=m+m' \)
gives exactly the Press-Schechter \( n(m,t) \)
Coagulation Equation

How is coagulation merger kernel related to our merger rate?

\[ A(m,m') \leftrightarrow \frac{B(m,m')}{{n(m)n(m')}} \]

To-do list:
Coagulation equation must be modified to account for non mass conserving mergers in simulations

Does A contain local merger physics ( \( \propto \sigma v \))?

Benson et al (2005, 08): A for P(k) \( \propto k^n \)
Summary: Theoretical Models

Extended Press-Schechter is not accurate.

Ellipsoidal model (moving barrier) is more accurate than spheroidal model (constant barrier), but there is still discrepancy from N-body merger rates, and the model lacks environmental dependence (non-Markovian?).

Integrate coagulation equation with a merger kernel forward:
  Obtain halo mass function \( n(m,t) \)?
  Subtleties: non-binary mergers from 3-body,
  “diffuse” accretion, tidal mass loss
  Can \( A(m_1,m_2,t) \) be interpreted as \(<\text{cross section} \times \text{velocity}>\)?