Dark Matter: Halo Mergers in $\Lambda$CDM Cosmology

Chung-Pei Ma
(UC Berkeley)
Most of the Universe is Dark

- Dark Energy: 73%
- Cold Dark Matter: 23%
- Atoms: 4%

You, me, stars, grey matter, white matter, antimatter...
Matter-energy budget surplus in the universe

\[ \Omega_{star + gas \ in \ gal} \ll 0.01 \]
\[ \Omega_{baryon} \approx 0.04 \rightarrow \text{“Dark” Baryons} \]
(e.g. dead stars, warm gas?)
\[ \Omega_{matter} \approx 0.27 \rightarrow \text{Dark Non-Baryons} \]
(e.g. neutralinos, axions, massive neutrinos)
\[ \Omega_{\Lambda} \approx 0.73 \rightarrow \text{Dark Energy} \]
(e.g. cosmological constant)
Growth of Cosmic Fluctuations

Primordial fluctuations are amplified by gravitational instability.
Basic Paradigm

The Smooth Universe (linear regime):
\[ \delta << 1 \]

Solve linearized, coupled Einstein and Boltzmann equations
\[ \Rightarrow \text{Time evolution of fluctuations in spacetime metric, dark matter, baryons, photons, neutrinos} \]
(Scalar) Metric Perturbations

\[ ds^2 = \alpha^2 \left[ (1 + 2\phi) d\tau^2 - (1 - 2\psi) dx_i dx^i \right] \]

Density Perturbations

\[ \delta(\vec{x}, t) = \frac{\rho(\vec{x}, t)}{\bar{\rho}} - 1, \quad \bar{\rho}_m \approx 2 \times 10^{-30} \text{ g/cm}^3 \]

Velocity Perturbations

\[ \theta = \nabla \cdot \nu \]
**Einstein Equation**

\[ G_{\mu \nu} = 8\pi G T_{\mu \nu} \]

\[ \{\phi, \psi, \dot{\phi}, \dot{\psi} \ldots\} = \{\delta_{\text{cdm}}, \delta_{\text{baryon}}, \delta_{\gamma}, \delta_{\nu}, \theta_{\text{cdm}} \ldots\} \]

**Boltzmann Equation**

- **collisionless**: cold dark matter, neutrinos
- **collisional**: baryons, photons (e.g. Thomson scattering)

**Take velocity moments** of phase space distribution functions

\[ \dot{\delta}_i = \{\delta_i, \theta_i, \phi, \psi \ldots\} \quad i = c, b, \gamma, \nu \]

\[ \dot{\theta}_i = \{\delta_i, \theta_i, \sigma_i, \phi, \psi \ldots\} \]

\[ \dot{\sigma}_i = \ldots \ldots \ldots \]
Linear Evolution of the Nearly Smooth Universe

Ma & Bertschinger (1995)
Statistics of Clustering

Two-point correlation function
\[ \xi(r) = \langle \delta(\vec{x})\delta(\vec{x} + \vec{r}) \rangle \]

Fourier transform

Power Spectrum
\[ P(k)\delta_D(\vec{k} + \vec{k}') = \left\langle \delta(\vec{k})\delta(\vec{k}') \right\rangle \]
Statistics of Clustering

Three-point correlation function
\[ \zeta(r_1, r_2, r_3) \]

Fourier transform

Bispectrum
\[ B(k_1, k_2, k_3) \delta_D(k_1 + k_2 + k_3) = \langle \delta(k_1) \delta(k_2) \delta(k_3) \rangle \]
Shape and Growth of Linear Matter Power Spectrum

\[ P(k) = Ak^n T^2(k) \]

**Shape:** depends on \( \Omega_m \), nature of dark matter etc

**Growth rate:** depends on \( \Omega_m, \Omega_\Lambda \) etc

\[ P(k) \sim k^n \]
\[ k_{eq} \sim \Omega_m h^2 \]
\[ P(k) \sim k^{-3} \] (for CDM)
Statistics of Clustering
Temperature Power Spectrum

\[ \Omega_m + \Omega_\Lambda \approx 1 \]
Cold vs Warm Dark Matter Power Spectrum

(a) Linear theory, $z=3$

$\log P(k) [h^{-3} \text{ Mpc}^3]$ vs $\log k [h \text{ Mpc}^{-1}]$

- CDM
- WDM 200 eV
- WDM 1 keV

Linear vs Nonlinear Power Spectrum

$P(k) \ (h^{-3} \text{ Mpc}^3)$

$k \ (h \text{ Mpc}^{-1})$

- Linear
- Nonlinear

$z=0$
$z=1.5$
$z=3$