Dark Matter:
Halo Mergers
in ΛCDM Cosmology

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Most of the Universe is Dark

You, me, stars,
grey matter,
white matter,
antimatter…

Matter-energy budget surplus in the universe

$$\Omega_{star+gas \text{ in gal}} << 0.01$$

$$\Omega_{baryon} \approx 0.04$$ → **"Dark" Baryons**

(e.g. dead stars, warm gas?)

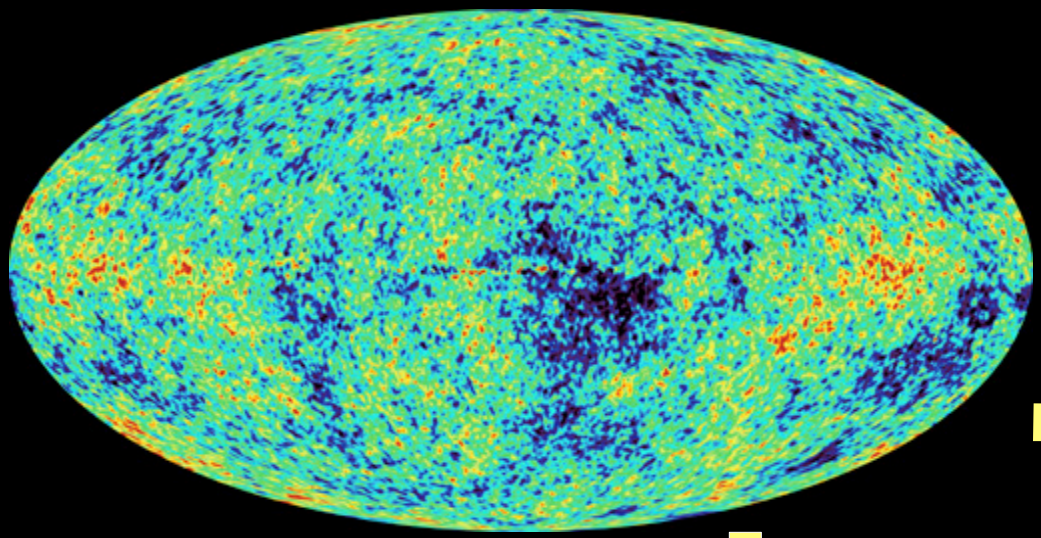
$$\Omega_{matter} \approx 0.27$$ → **Dark Non-Baryons**

(e.g. neutralinos, axions, massive neutrinos)

$$\Omega_{\Lambda} \approx 0.73$$ → **Dark Energy**

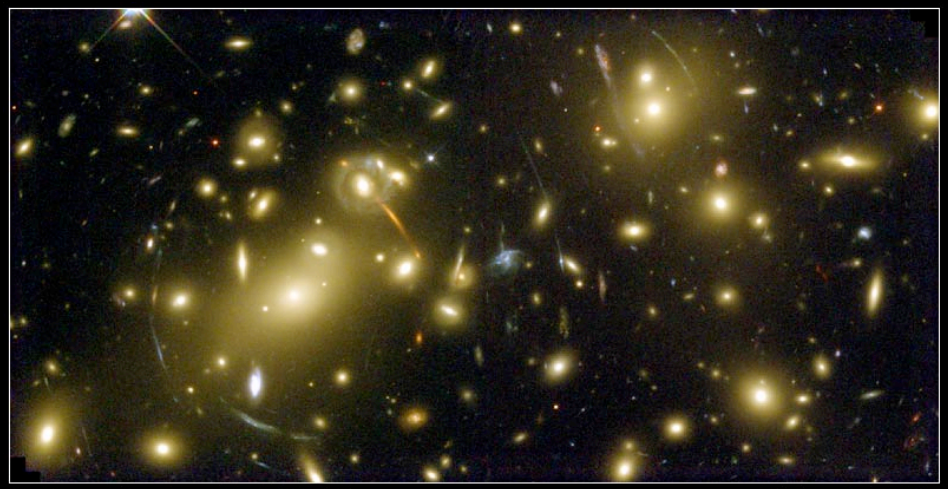
(e.g. cosmological constant)

Growth of Cosmic Fluctuations



Primordial fluctuations are amplified

by gravitational instability



Galaxy Cluster Abell 2218 HST • WFPC2

NASA, A. Fruchter and the ERO Team (STScI, ST-ECF) • STScI-PRC00-08

Basic Paradigm

The Smooth Universe (linear regime):

$$\delta << 1$$

Solve linearized, coupled Einstein and Boltzmann equations
⇒Time evolution of fluctuations in spacetime metric,
dark matter, baryons, photons, neutrinos

(Scalar) Metric Perturbations

$$ds^2 = a^2[(1+2\phi)d\tau^2 - (1-2\psi)dx_i dx^i]$$

Density Perturbations

$$\delta(\vec{x},t) = \frac{\rho(\vec{x},t)}{\bar{\rho}} - 1, \qquad \bar{\rho}_m \approx 2\times 10^{-30}\,\mathrm{g/cm^3}$$

Velocity Perturbations

$$\theta = \nabla \cdot v$$

Einstein Equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\rightarrow \{\phi, \psi, \dot{\phi}, \dot{\psi} \ldots\} = \{\delta_{cdm}, \delta_{baryon}, \delta_{\gamma}, \delta_{\nu}, \theta_{cdm} \ldots\}$$

Boltzmann Equation

collisionless: cold dark matter, neutrinos

collisional: baryons, photons (e.g. Thomson scattering)

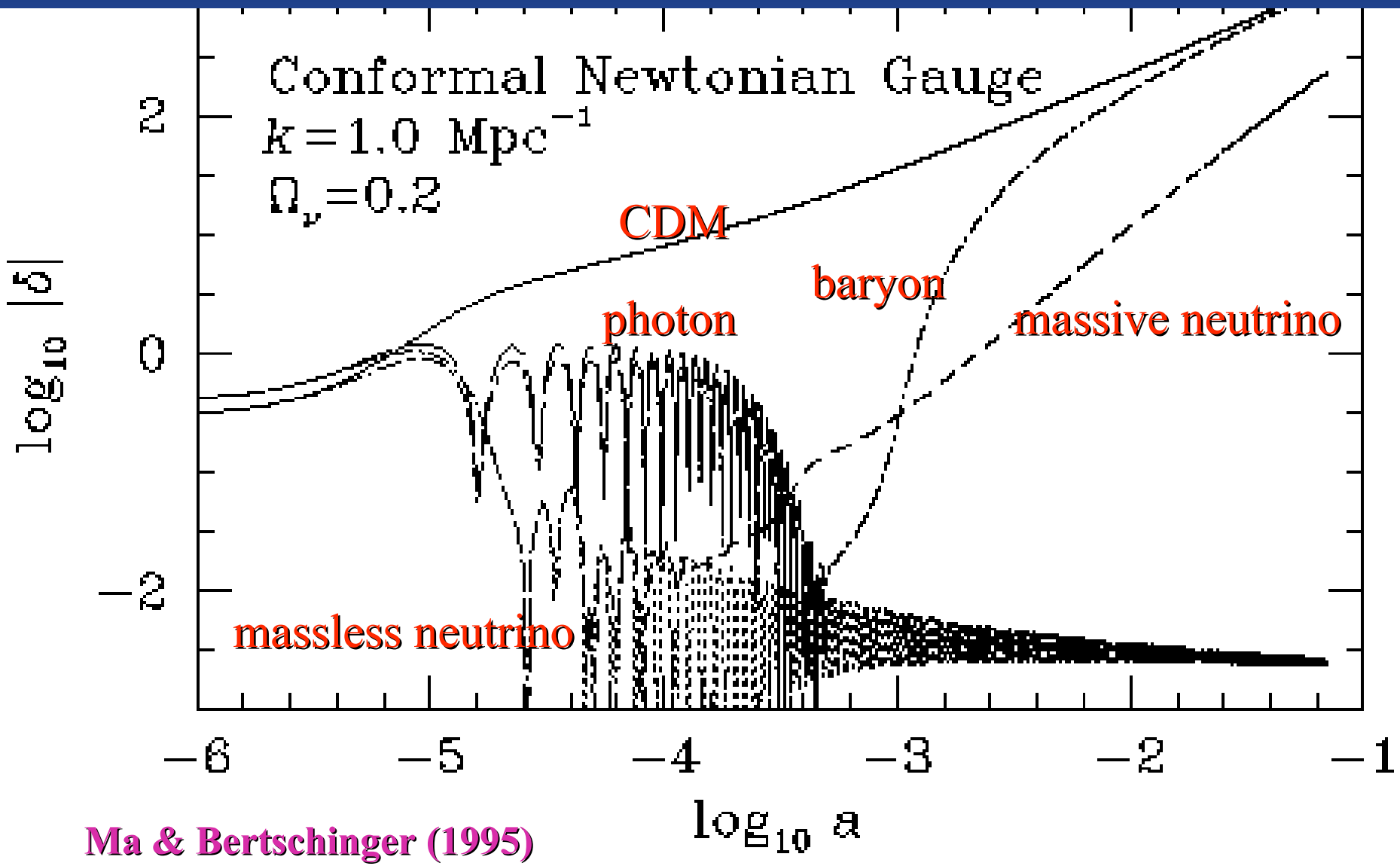
Take velocity moments of phase space distribution functions

$$\rightarrow \quad \dot{\delta}_i = \{\delta_i, \theta_i, \phi, \psi \ldots\} \qquad i = c, b, \gamma, \nu$$

$$\dot{\theta}_i = \{\delta_i, \theta_i, \sigma_i, \phi, \psi \ldots\}$$

$$\dot{\sigma}_i = \ldots\ldots\ldots\ldots$$

Linear Evolution of the Nearly Smooth Universe



Ma & Bertschinger (1995)

Statistics of Clustering

Two - point correlation function

$$\xi(r) = \left\langle \delta(\vec{x})\delta(\vec{x}+\vec{r}) \right\rangle$$

Fourier transform

Power Spectrum

$$P(k)\delta_D(\vec{k}+\vec{k}') = \left\langle \delta(\vec{k})\delta(\vec{k}') \right\rangle$$

Statistics of Clustering

Three - point correlation function

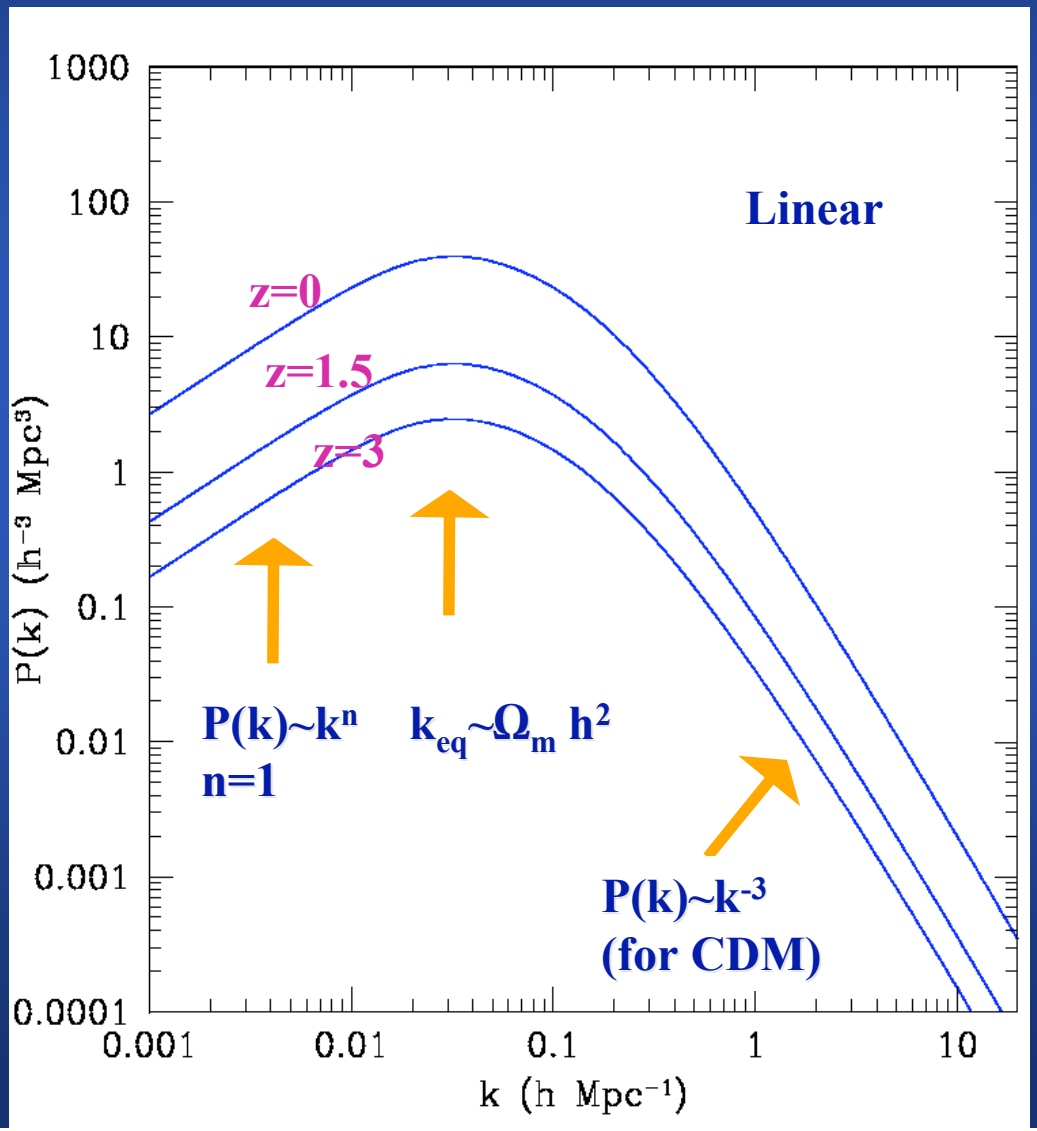
$$\zeta(r_1, r_2, r_3)$$

$\Updownarrow$ Fourier transform

Bispectrum

$$B(k_1, k_2, k_3)\delta_D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) = \left\langle \delta(\vec{k}_1)\delta(\vec{k}_2)\delta(\vec{k}_3) \right\rangle$$

Shape and Growth of Linear Matter Power Spectrum



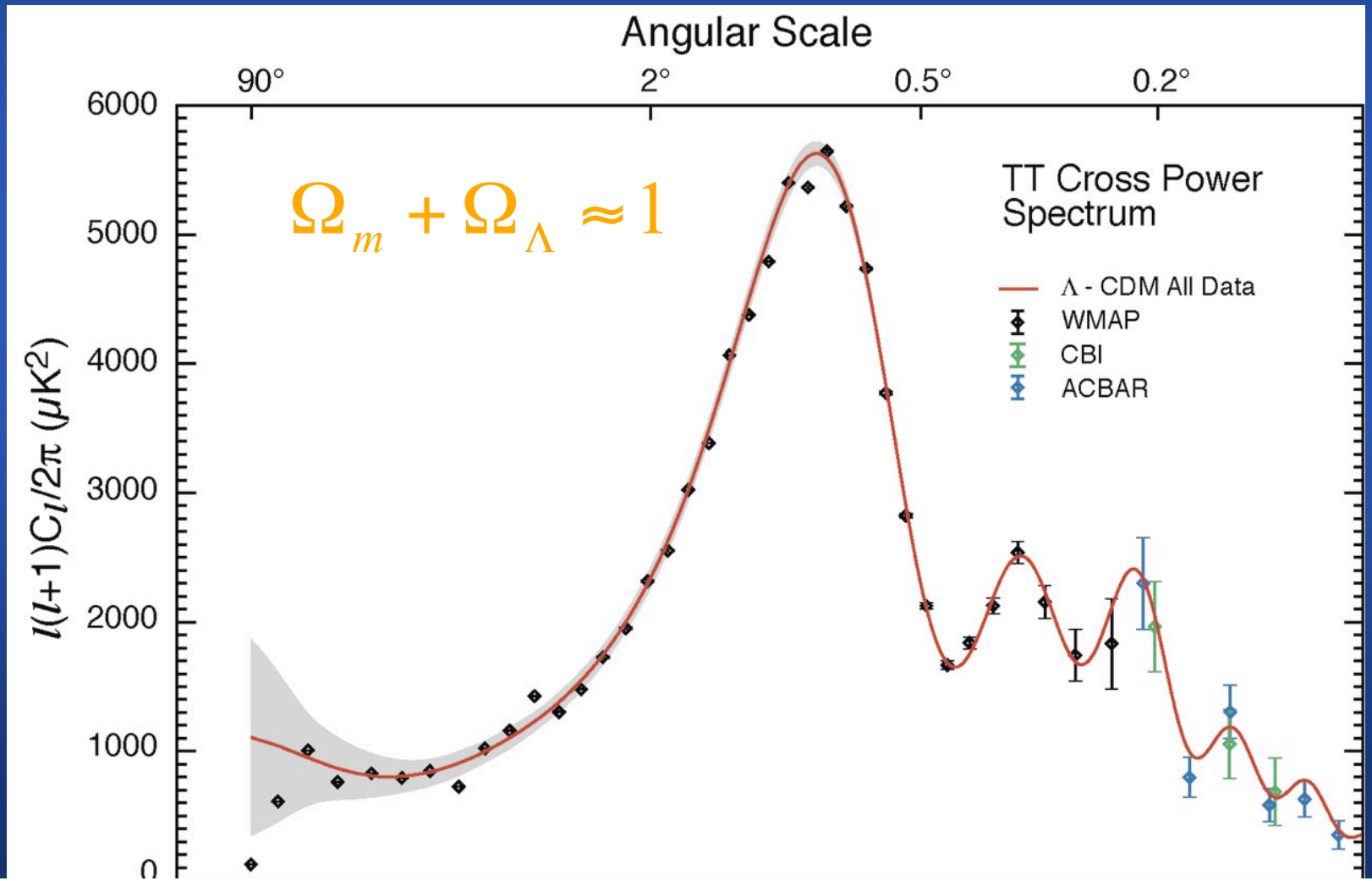
$$P(k) = Ak^n T^2(k)$$

Shape: depends on Ω_m, nature of dark matter etc

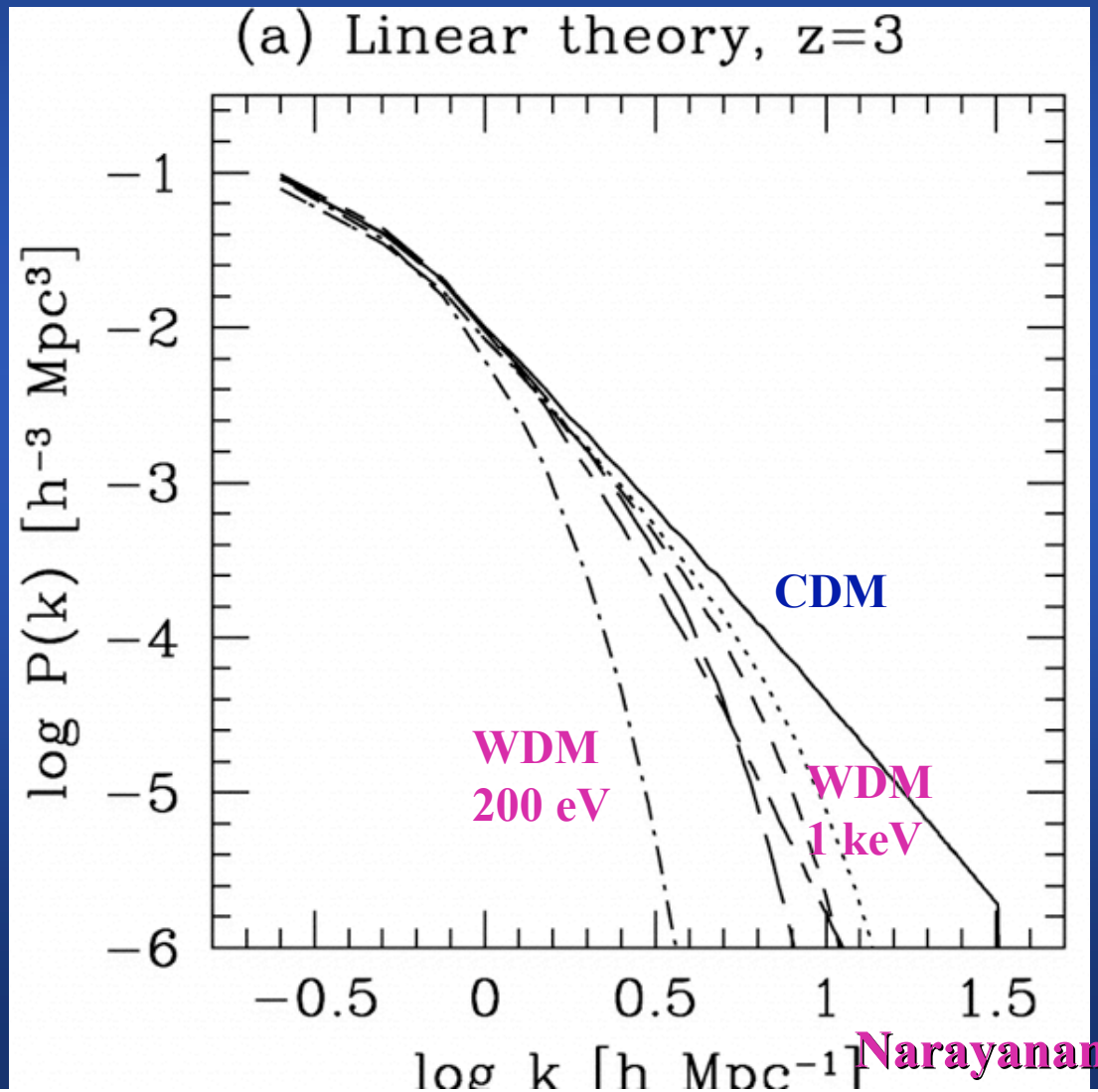
Growth rate: depends on Ω_m, Ω_Λ etc

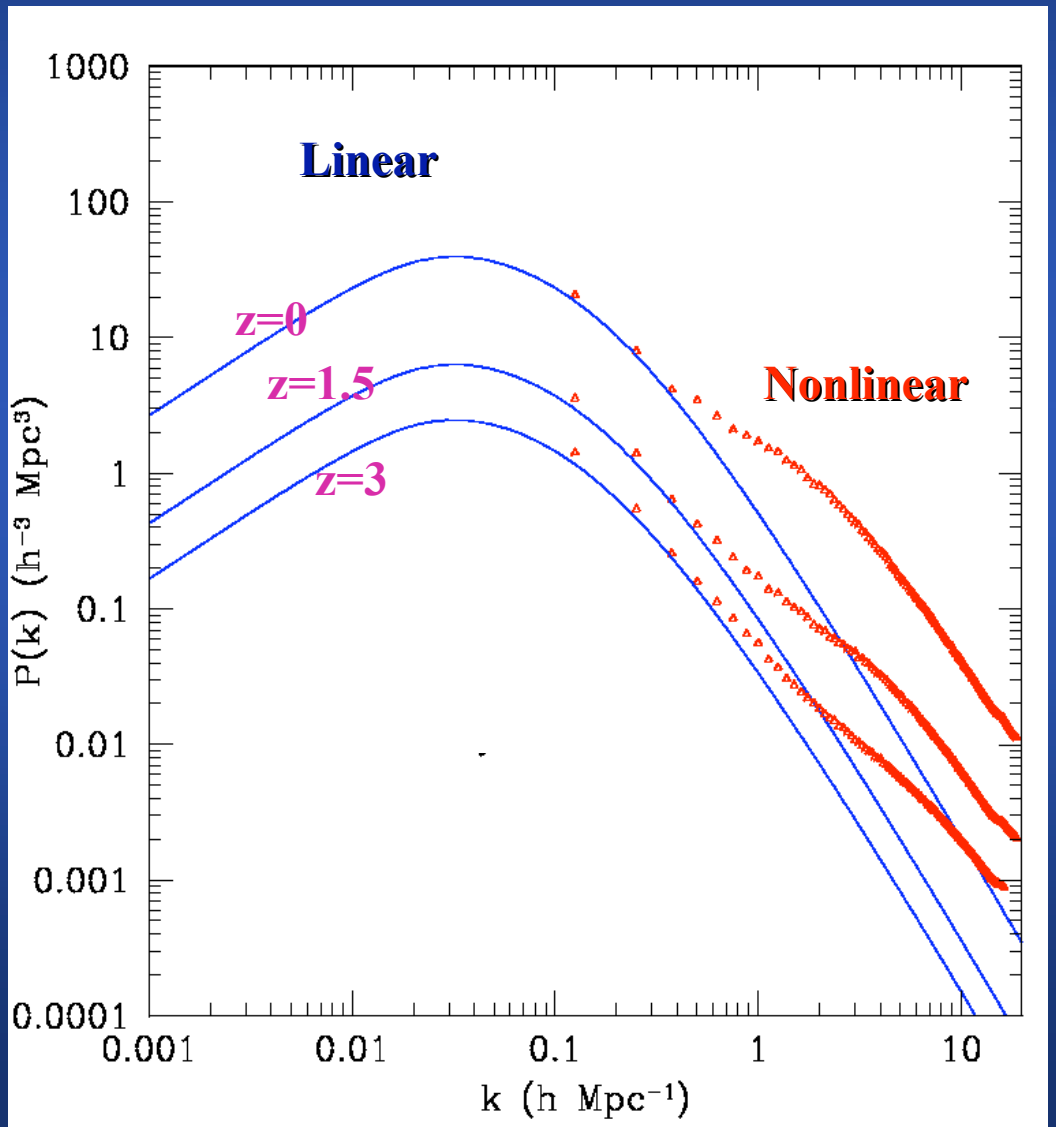
Statistics of Clustering

Temperature Power Spectrum



Cold vs Warm Dark Matter
Power Spectrum



Narayanan et al (2000)

Linear vs Nonlinear Power Spectrum