Lecture 2

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•Dynamics of structure formation

•Gravity
general relativity, but
Newtonian approximation
in expanding space usually
sufficient

•Hydrodynamics
shock waves
radiation processes
star formation
supernovae
black holes, etc...

•dark matter is collisionless
(described by Vlasov equation)

•Monte-Carlo integration as
an **N-body system**

•3N **coupled**, non-linear differential
equations of second order

•Problems:

- N is very large
- All equations are coupled to each other

•DYNAMIC RANGE IN COSMOLOGICAL SIMULATIONS

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•Two conflicting requirements complicate the study of **hierarchical** structure formation

•Want **small particle mass** to resolve internal structure of halos

•Want **large volume** to obtain respresentative sample of universe

•*need large* ***N***

•*where N is the particle number*

•Problems due to a small box size:

- •Fundamental mode goes non-linear soon after the first halos form. Simulation cannot be meaningfully continued beyond this point.
- •No rare objects (the first halo, **rich** galaxy clusters, etc.)

•Problems due to a large particle mass:

- •Physics cannot be resolved.
- •Small galaxies are missed.

•At any given time, halos exist on a large range of mass-scales !

Moore's Law for Cosmological N-body Simulations

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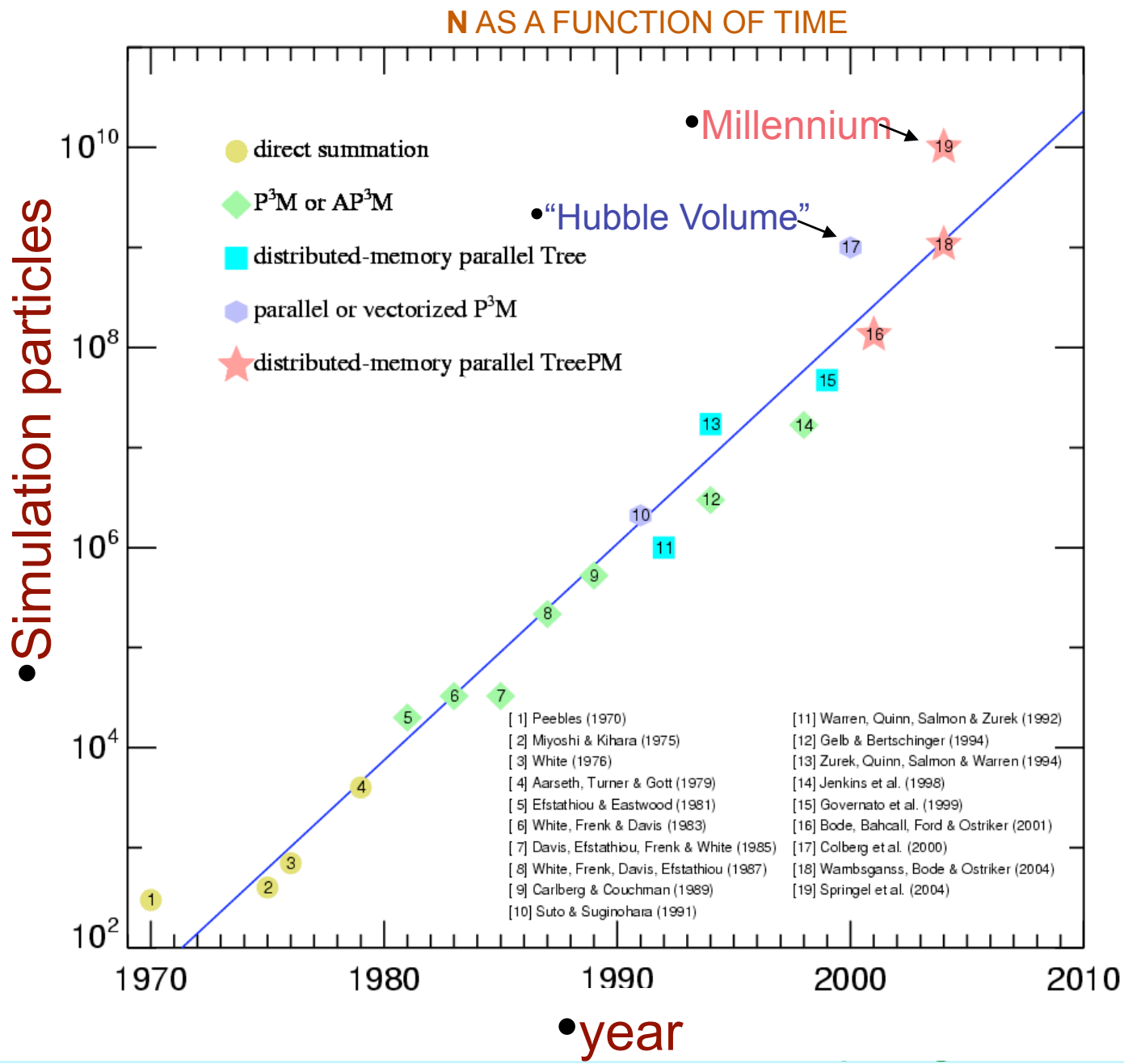
- Computers double their speed every 18 months
- A naive N-body force calculation needs N^2 op's
- Simulations double their size every 16.5 months
- $N = 10^{10}$ should have been reached in 2008



2005

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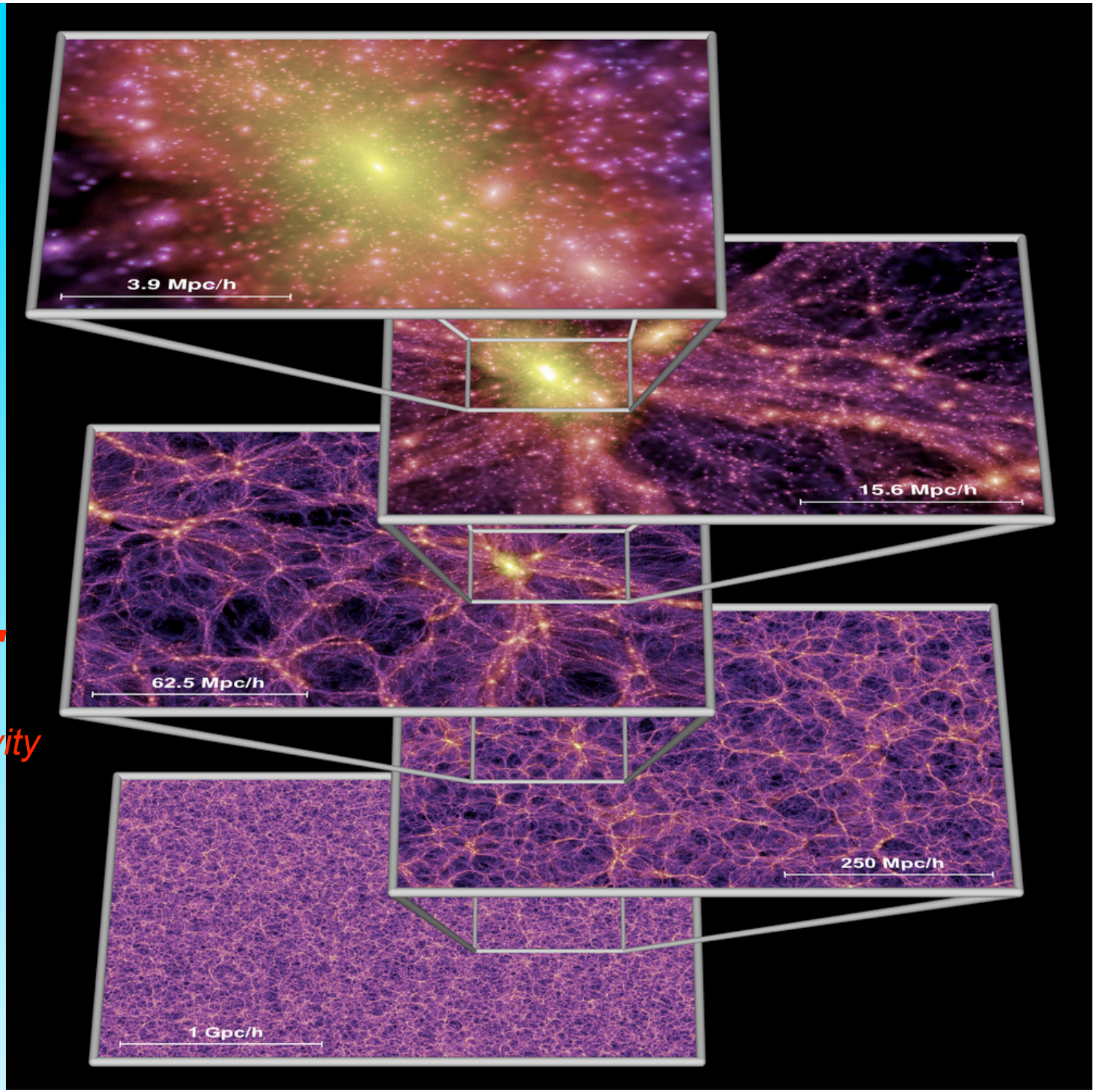
- The Universe has structure on all scales...

- *need large* ***L***

- *and spatial adaptivity*



•Hierarchical tree algorithms

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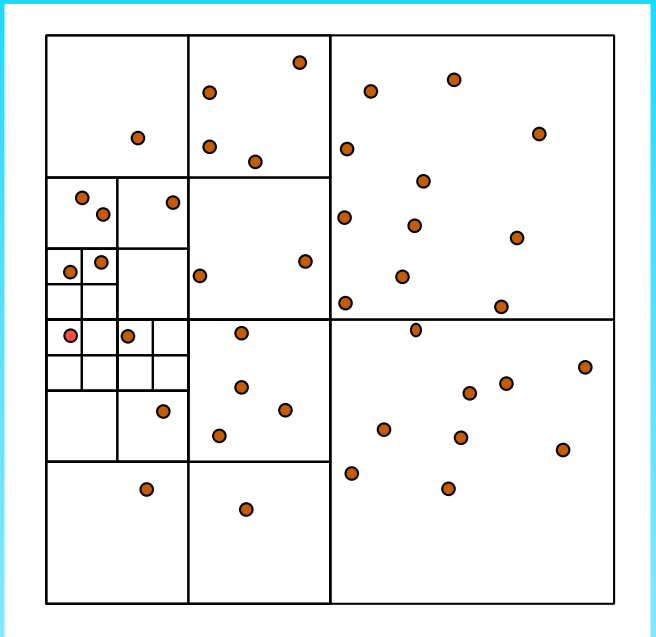
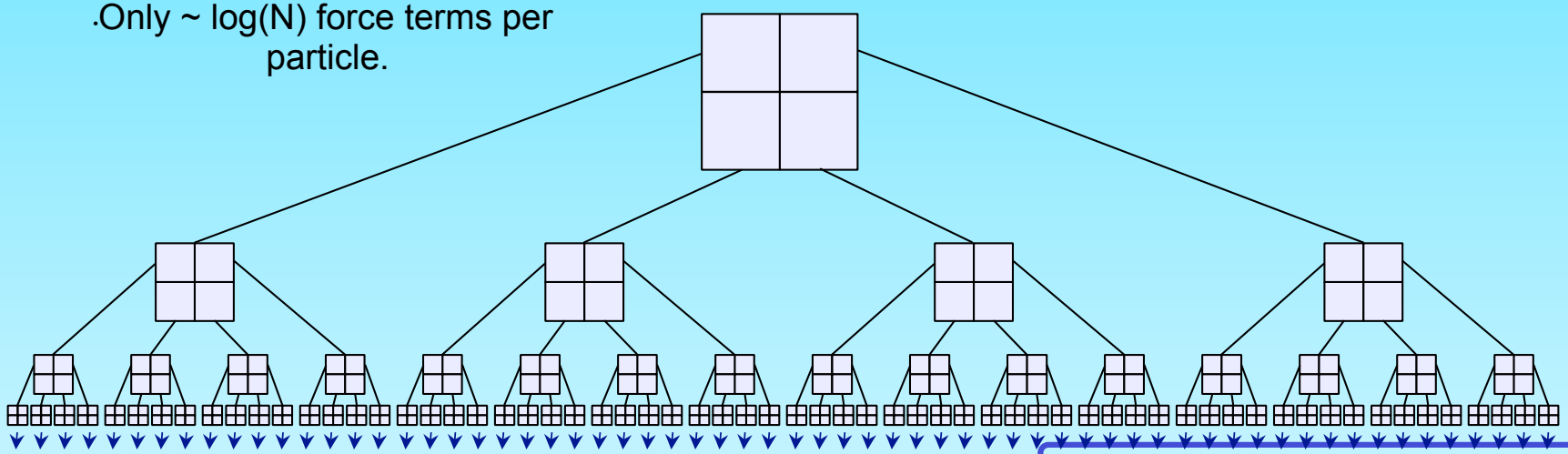
- The N^2 - scaling of direct summation puts serious limitations on N...



- But we want N ~ 10^6-10^{10} for collisionless dynamics of dark matter!
- **Idea:** Group distant particles together, and use their mulipole expansion.

→

- Only ~ log(N) force terms per particle.



•The Tree-PM method (Gadget-2)

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•Periodic peculiar potential

$$\nabla^2\phi(\mathbf{x}) = 4\pi G[\rho(\mathbf{x}) - \overline{\rho}] = 4\pi G \sum_{\mathbf{n}} \sum_i m_i \left[\tilde{\delta}(\mathbf{x} - \mathbf{x}_i - \mathbf{n}L) - \frac{1}{L^3}\right]$$

•**Idea:** Compute the long-range force with the PM algorithm, and only a **local** short-range force with the tree.

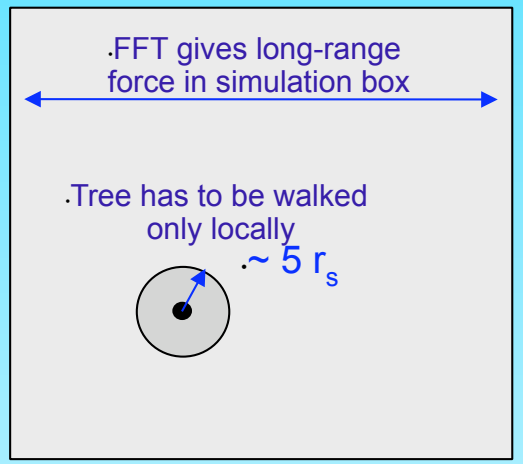
•Let's split the potential in Fourier space into a long-range and a short-range part:

$$\phi_{\mathbf{k}}^{\text{long}} = \phi_{\mathbf{k}} \exp(-\mathbf{k}^2 r_s^2)$$

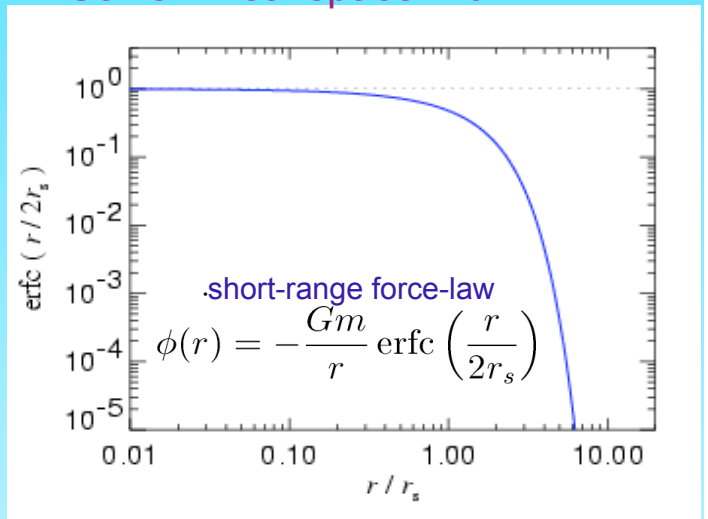
$$\phi_{\mathbf{k}}^{\text{short}} = \phi_{\mathbf{k}} \left[1 - \exp(-\mathbf{k}^2 r_s^2)\right]$$

•Solve with PM-method

- •CIC mass assignment
- •**FFT**
- •multiply with kernel
- •**FFT backwards**
- •Compute force with 4-point finite difference operator
- •Interpolate forces to particle positions



•Solve in real space with TREE



•Volker Springel

•Advantages of this algorithm include:

- •Accurate and fast long-range force
- •No force anisotropy
- •Speed is insensitive to clustering (as for tree algorithm)
- •No Ewald correction necessary for periodic boundary conditions

•Cosmological hydrodynamics

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•The baryons in the universe can be modelled as an *ideal* gas

•BASIC JATIONS

•**Euler equation:**

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{\nabla P}{\rho} - \nabla\Phi$$

•**Continuity equation:**

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\nabla\cdot\mathbf{v} = 0$$

•**First law of thermodynamics:**

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{P}{\rho}\nabla\cdot\mathbf{v} - \frac{\Lambda(u,\rho)}{\rho}$$

•**Equation of state of ideal monoatomic gas:**

$$P = (\gamma - 1)\rho u\ , \qquad \gamma = 5/3$$

•Cosmological hydrodynamics

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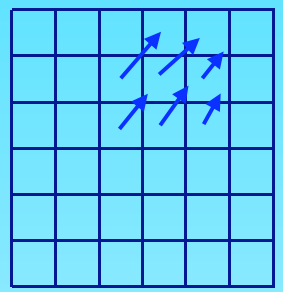
.DIFFERENT METHODS TO DISCRETIZE A FLUID

.Eulerian

.**discretize space**

.representation on a mesh (volume elements)



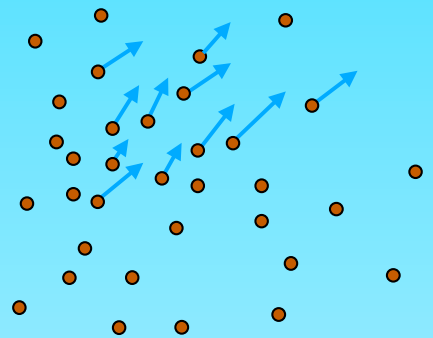
.<u>principal advantage:</u>

.high accuracy (shock capturing), low numerical viscosity

.Lagrangian

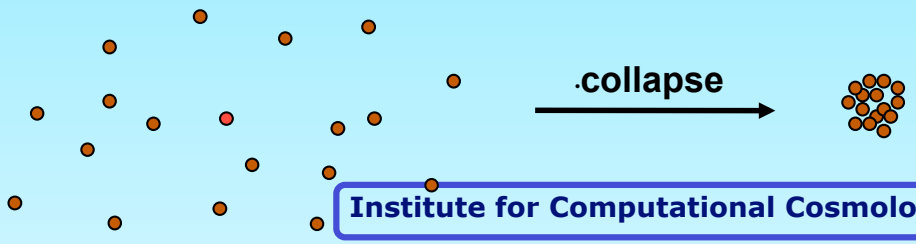
.**discretize mass**

.representation by fluid elements (particles)



.<u>principal advantage:</u>

.resolution adjusts automatically to the flow



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The origin of cosmic structure

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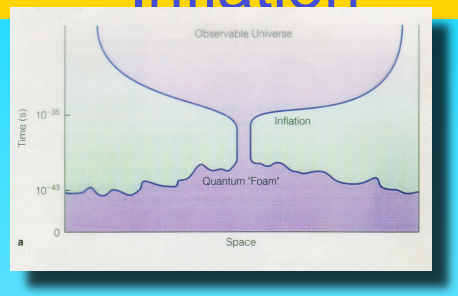
•Inflation



•→ QUANTUM FLUCTUATIONS:

$|\delta_k|^2 \propto k^n$ •n ≈1

Gaussian amplitudes

•

+

•Damping (nature of dark matter)

•→ FLAT UNIVERSE

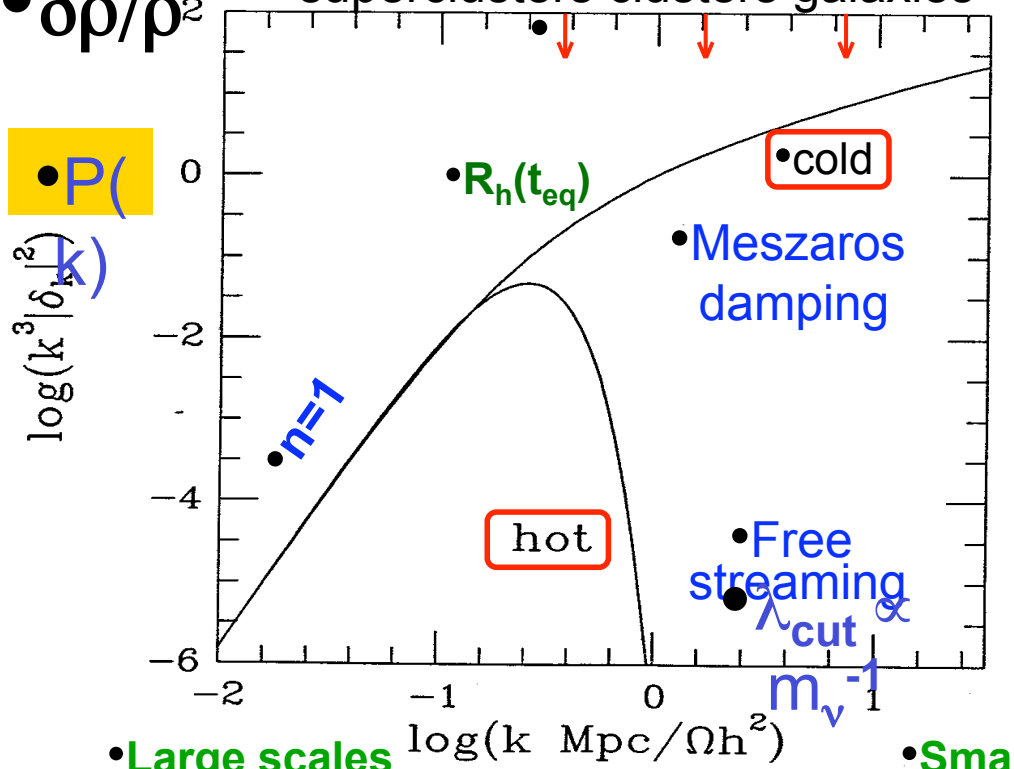
•$P(k)=Ak^n$

$T^2(k,t)$ → •Transfer function

•$\delta\rho/\rho^2$

•superclusters clusters galaxies

•P(k)



•Large scales

•Small scales

- Hot DM (eg ~30 ev neutrino)
  - Top-down formation
- Cold DM (eg ~neutralino)
  - Bottom-up (hierachical)

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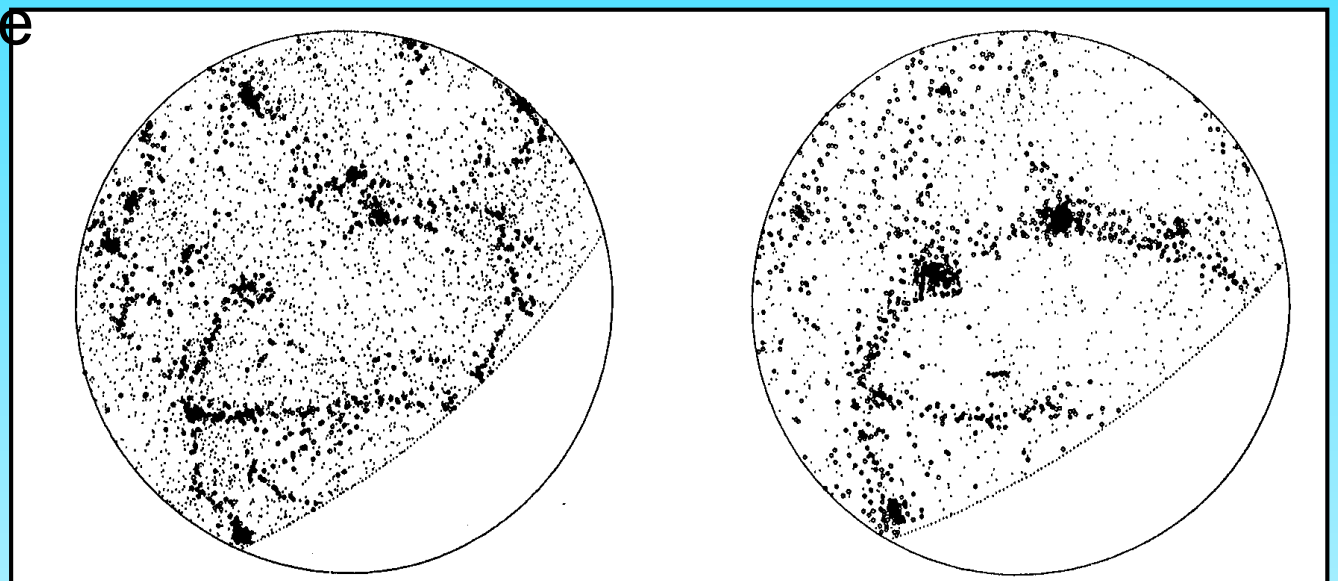
•Neutrino (hot) dark matter

•Free-streaming length so large that superclusters form first and galaxies are too young

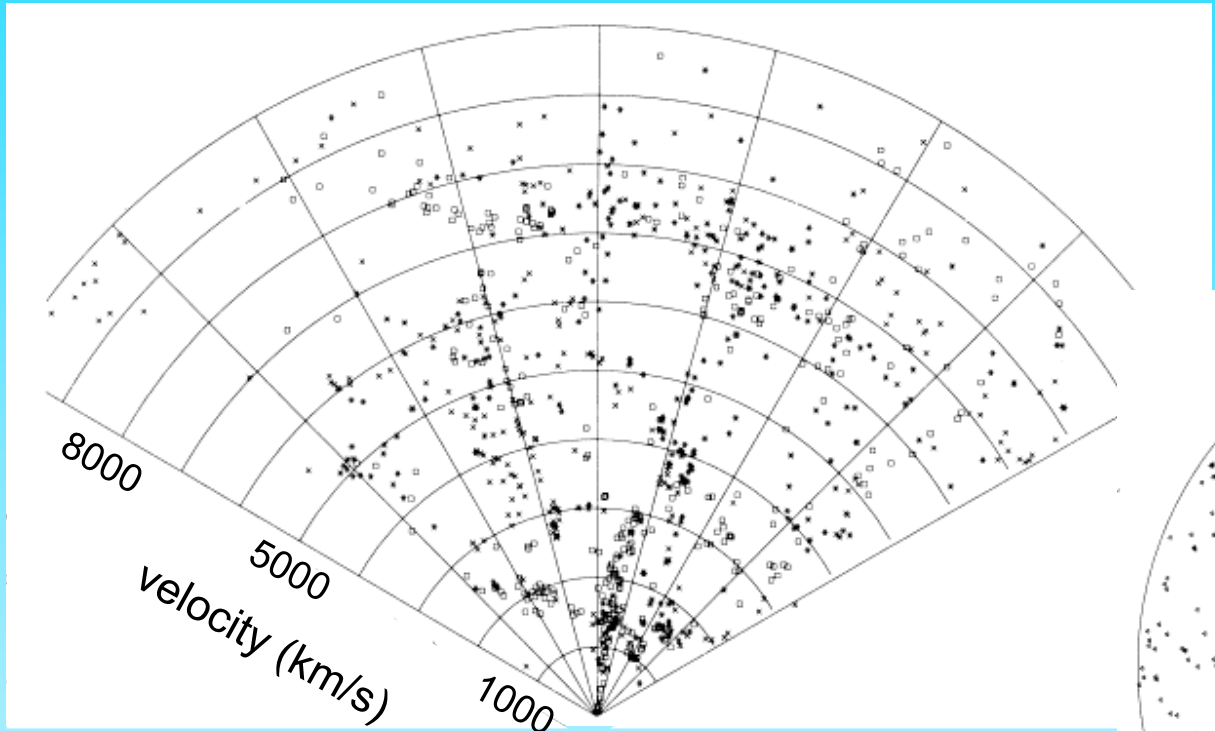
•Ω_ν=1 (m_ν = 30 ev)

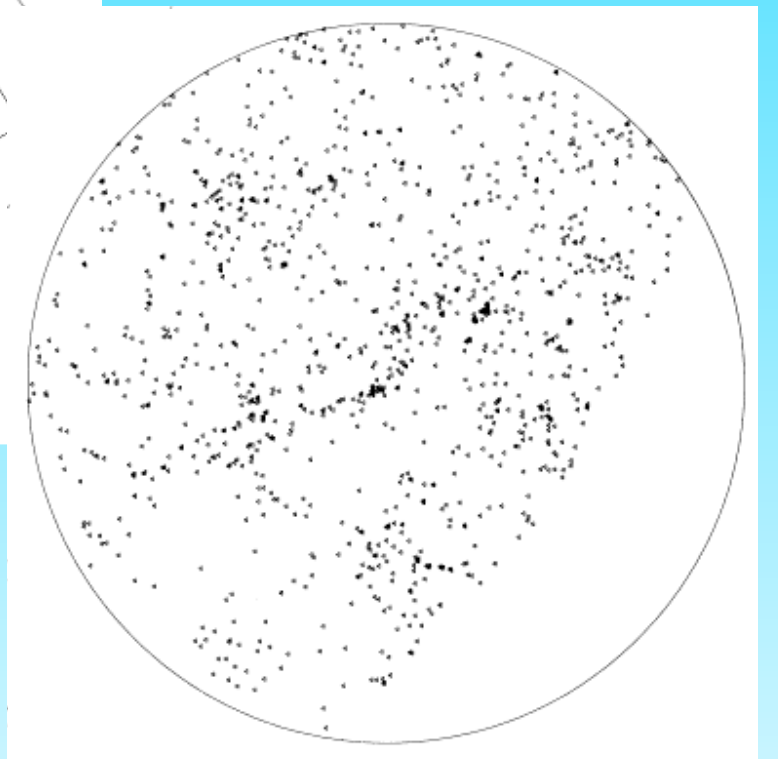


•Frenk, White & Davis '83

The CFA redshift survey

1982: 2543 galaxies, magnitude limited to $m_z = 14.5$





Davis, Huchra, Latham & Tonry '82

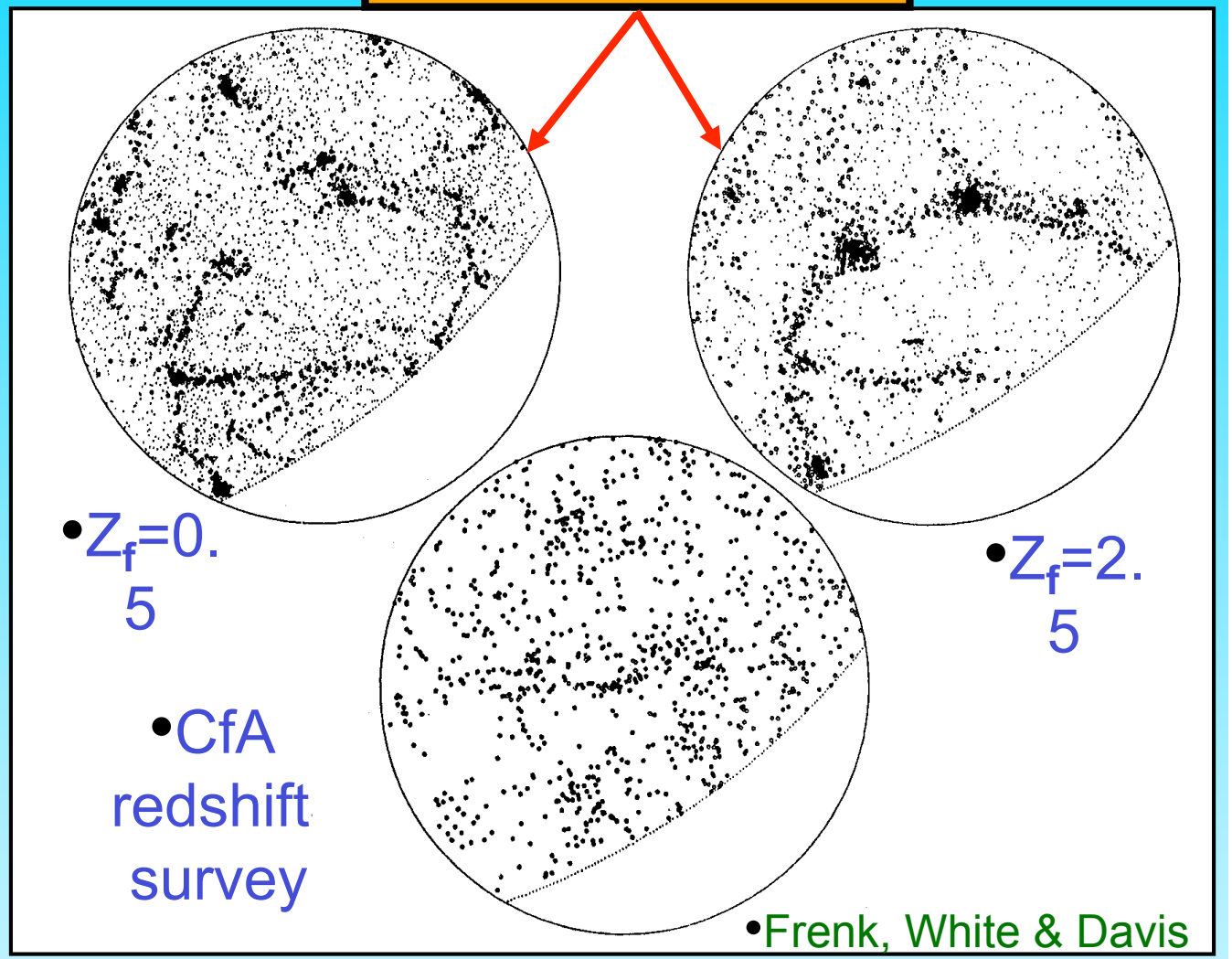
•Neutrino (hot) dark

•Ω_ν=1 (m_ν = 30 ev)

•Free-streaming length so large that superclusters form first and galaxies are too young

⇨

•Neutrinos cannot make an appreciable contribution to Ω and m_ν<< 10 ev



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The origin of cosmic structure

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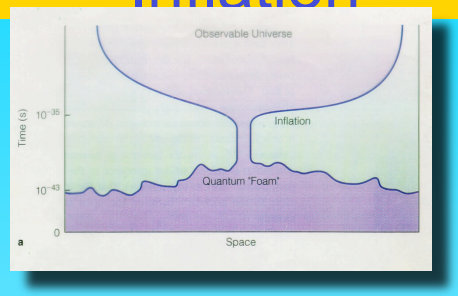

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Gaussian amplitudes

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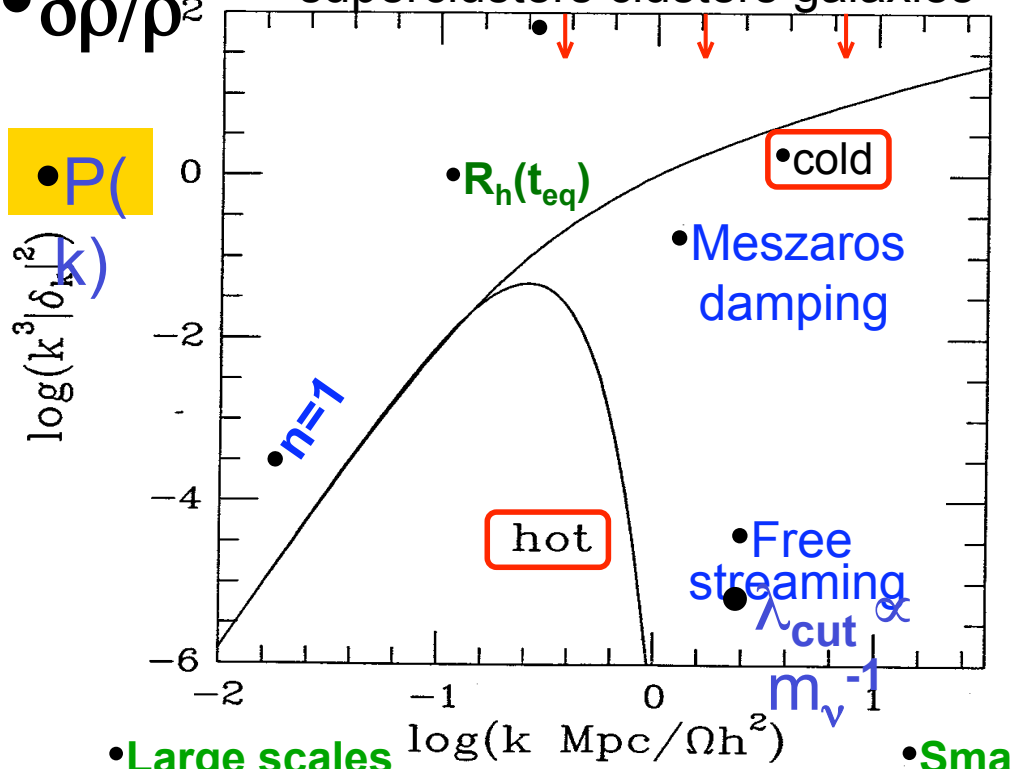

•Large scales

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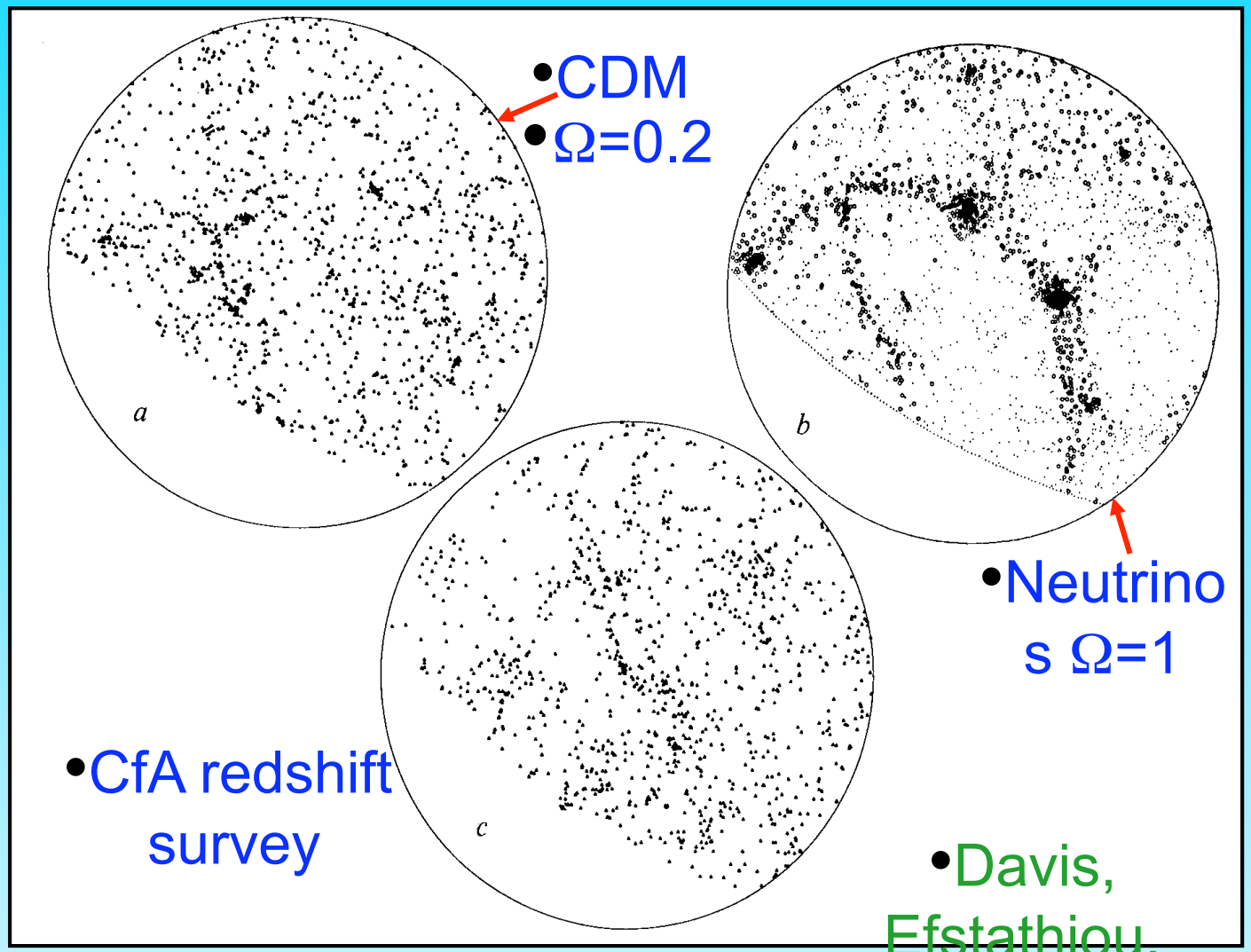
•Non-baryonic dark matter cosmologies

•Neutrino dark matter produces unrealistic clustering

•Early CDM N-body simulations gave promising results

•In CDM structure forms hierarchically



•Davis, Efstathiou, Frenk & White '85

$\Omega = 1$ CDM

DEFW '85

Galaxies trace mass

Dark matter

pec vels too large

wrong clustering

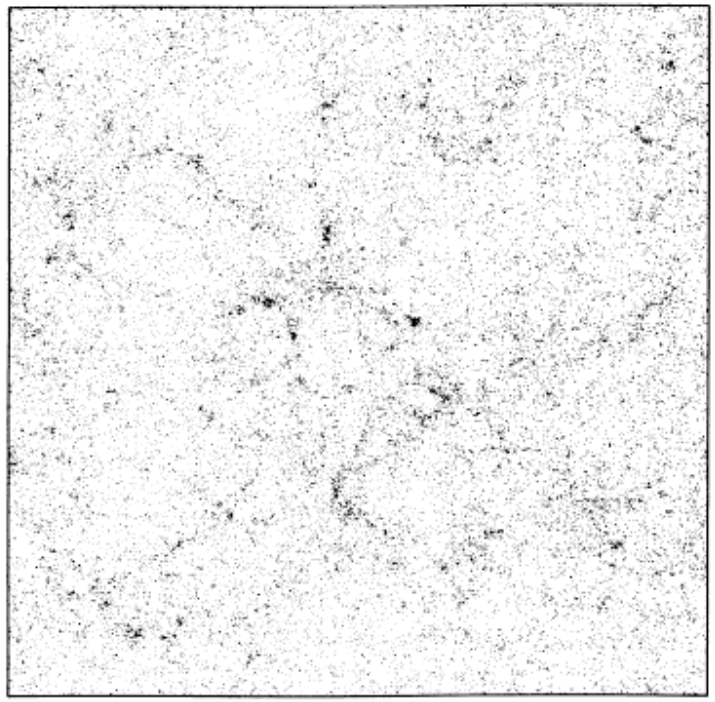


FIG. 16.—The projected distribution of all particles (*left*) and of the "galaxies" (*right*) the 2.5 σ peaks of the linear density distribution.

Biased galaxy formation

… or how to rescue Ω=1 ! DEFW '85

Dark matter

Galaxies

- High peaks of density field

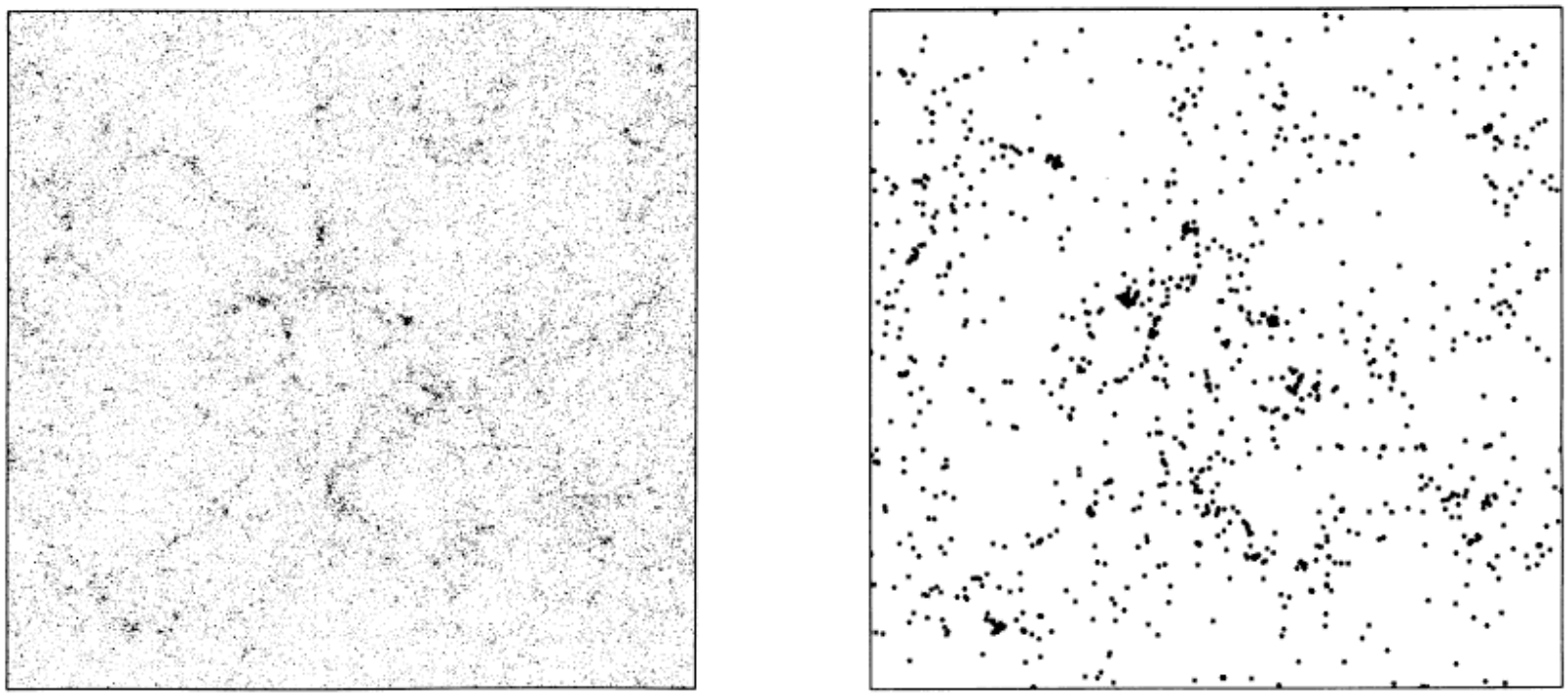
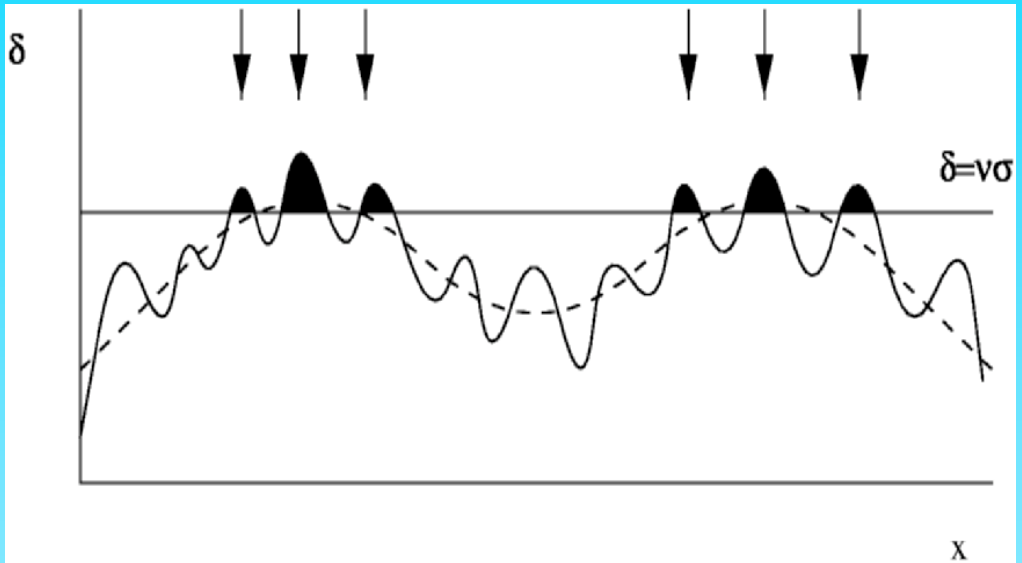


FIG. 16.—The projected distribution of all particles (*left*) and of the "galaxies" (*right*) in EdS1 at $a = 1.4$. The side of the box is $32.5h^{-1}$ Mpc. "Galaxies" are assumed to form only at the 2.5 σ peaks of the linear density distribution.

- Are more strongly clustered than the mass

Biased galaxy formation



- Cole-Kaiser-Mo-White:

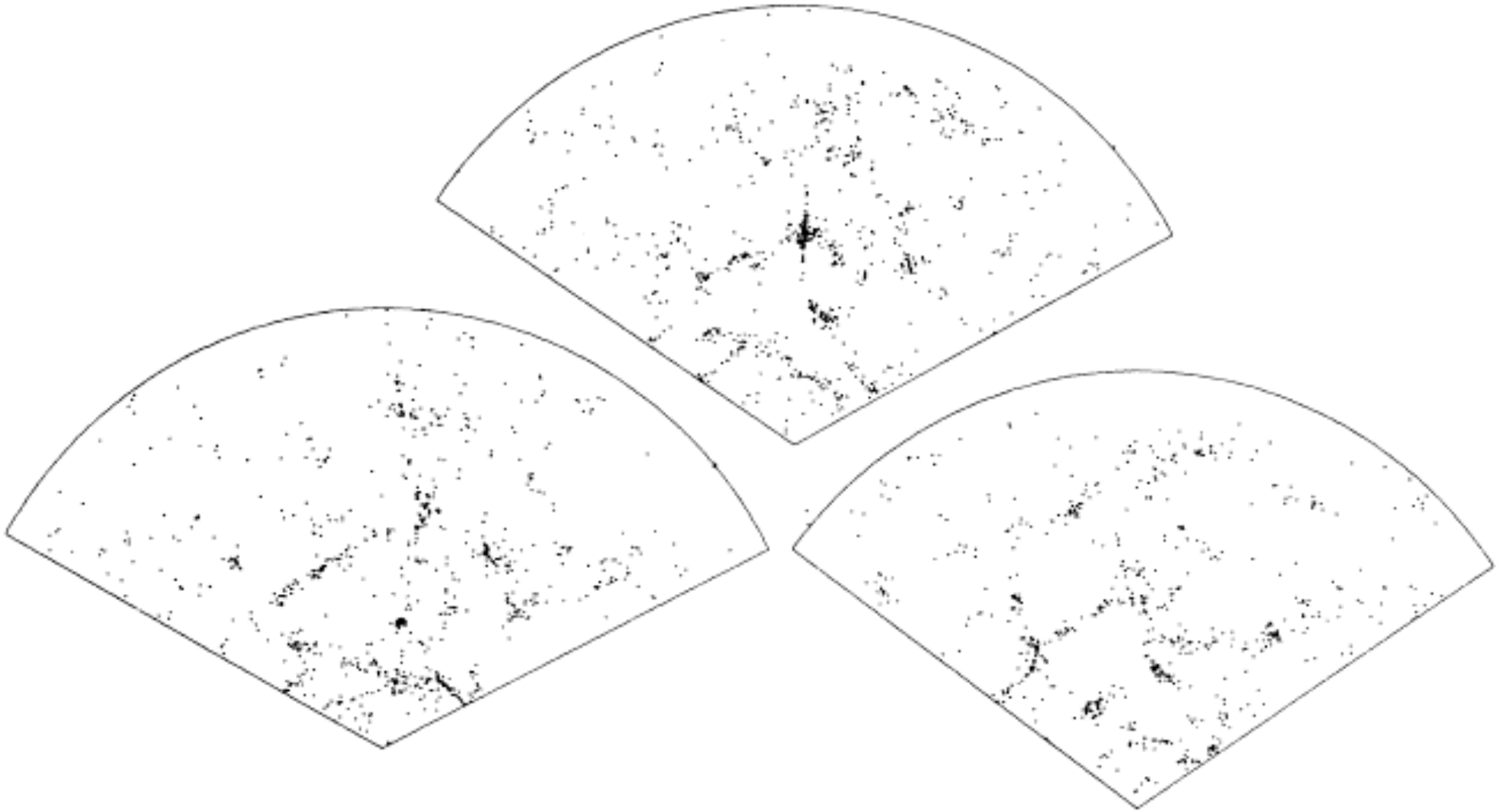
$$b(M) = 1 + (\nu^2 - 1)/\delta_c$$

$$\nu \equiv \delta_c/\sigma(M)$$

Bias is inevitable for rare systems

- Peak-background split: $\delta_c \rightarrow \delta_c - \varepsilon$;
- $n(m) \rightarrow n(m) + (dn/d\nu)(d\nu/d\varepsilon)\,\varepsilon = n(m)\,[1+b\varepsilon]$
- bias: $\xi \rightarrow b^2\xi$ depends on halo mass

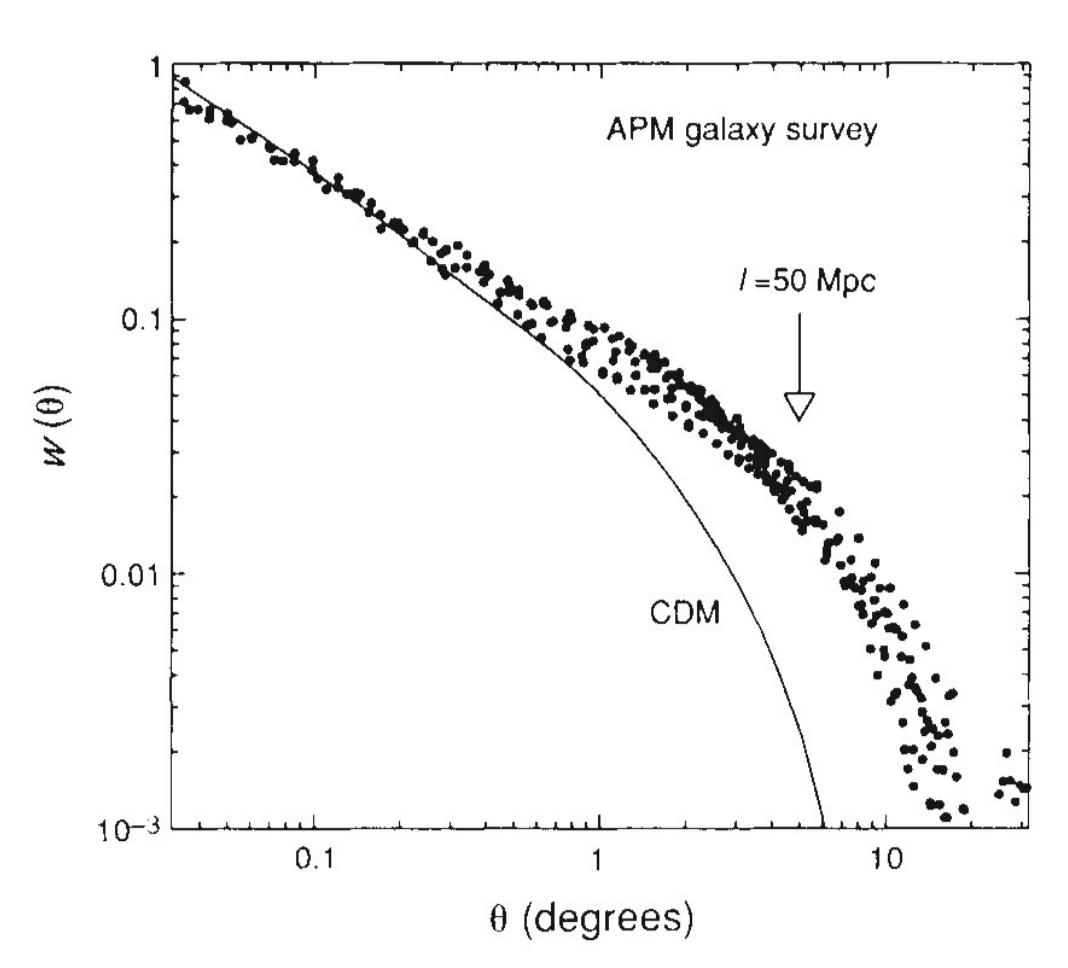
SCDM compared to CfA-2 z-survey



White, Frenk, Davis, Efstathiou '87

Ω = 1 CDM under strain

Angular 2-pt correlation function



Maddox, Efstathiou, Sutherland & Loveday '90

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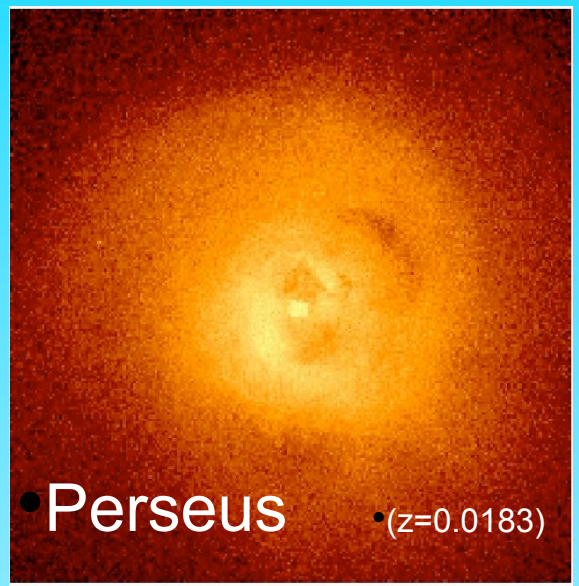
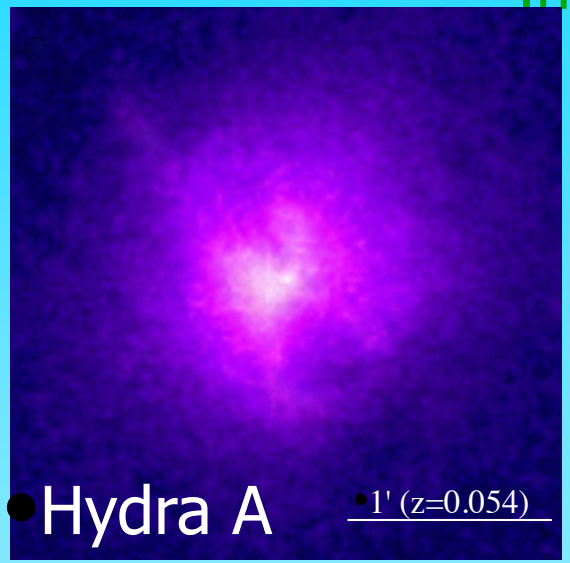
- The end of standard (Ω_{matter} =1) CDM … or why Ω_{matter} cannot be 1

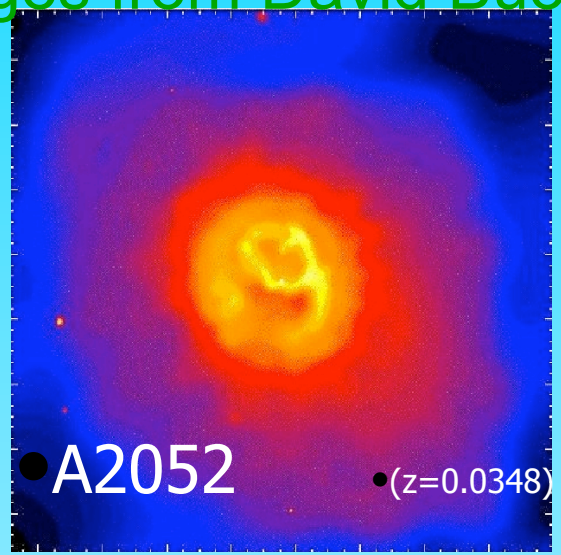
•Galaxy clusters

X-ray emission from hot plasma in clusters

•Images from David Buote







•About 90% of baryons in clusters are in hot gas

- X-rays ⇒ gas mass
- Photometry ⇒ stellar mass
- Gas in hydrostatic equilibrium so X-rays

} •⇒ Baryon fraction, f_b

- (or lensing) ⇒ total gravitating mass

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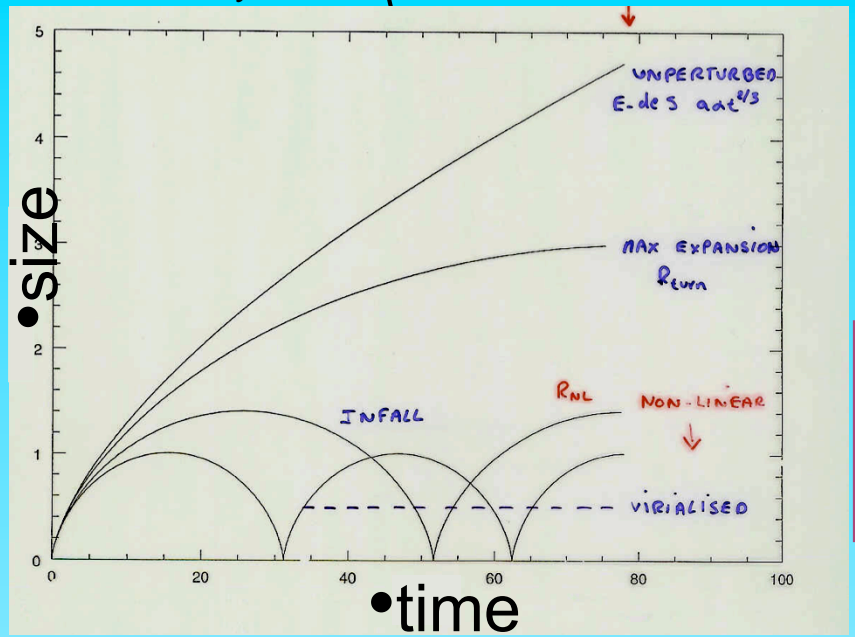
•Ω from the baryon fraction in clusters



- In clusters matter that has fallen in is still in the cluster ($r_{vir} \sim r_{non\text{-}linear}$)

- ⟹ baryon fraction in clusters ≈ baryon fraction of universe

$$f_{baryon} = \frac{\text{mass in baryons}}{\text{total mass}}$$

- $M_b = M_{gas} + M_{stars}$

$$f_b = \gamma \frac{\Omega_b}{\Omega}$$

- White, Navarro, Evrard & Frenk '93

- where γ=1 if f_b has the universal value

- Simulations ⟹ γ=0.9

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•Ω from the baryon fraction in clusters

•baryon fraction in clusters ≈ baryon fraction of universe

$$f_b = \frac{M_b}{M_{tot}} = \gamma \frac{\Omega_b}{\Omega_m}$$

•White, Navarro, Evrard & Frenk Nature 1993

•where γ=1 if f_b has the universal value

•simulations → $\gamma = 0.9 \pm 10\%$

•X-rays+lensing → $f_b = (0.060h^{-3/2} + 0.009) \pm 10\%$

•BBNS, CMB → $\Omega_b h^2 = 0.019 \pm 20\%$

•HST → $h = 0.7 \pm 10\%$

→ $$\Omega_m = \frac{\Omega_b \gamma}{f_b} = 0.31 \pm 0.12$$

•Allen etal '04

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• $\Omega < 1$: open or flat universe?

• $\Omega_{matter} < 1 \Rightarrow$

BBNS is wrong or

• Universe is open or

• Need cosmological constant if $\Omega_{tot}=1$

• White et al 1993

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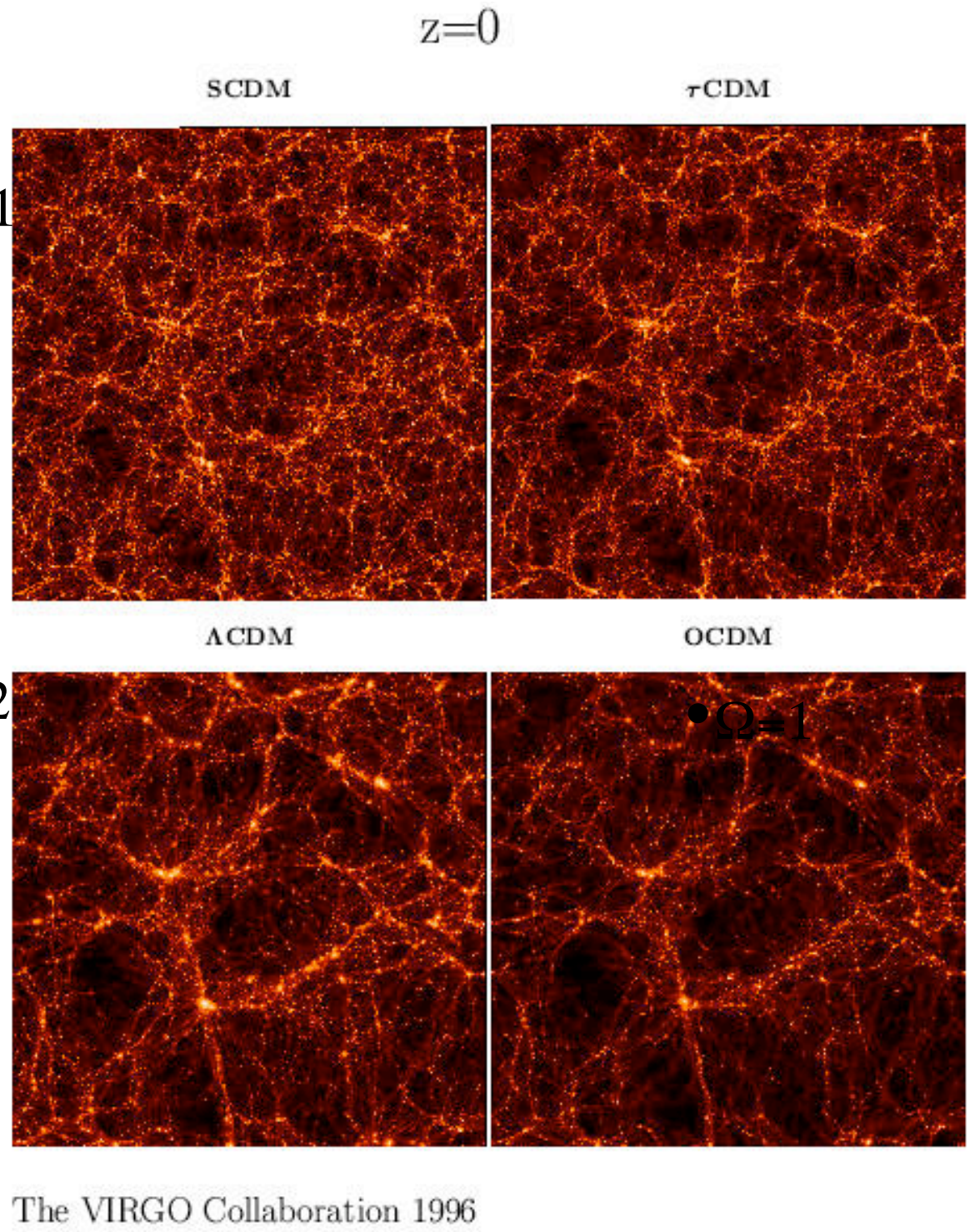
The Virgo consortium

Jenkins et al 1998

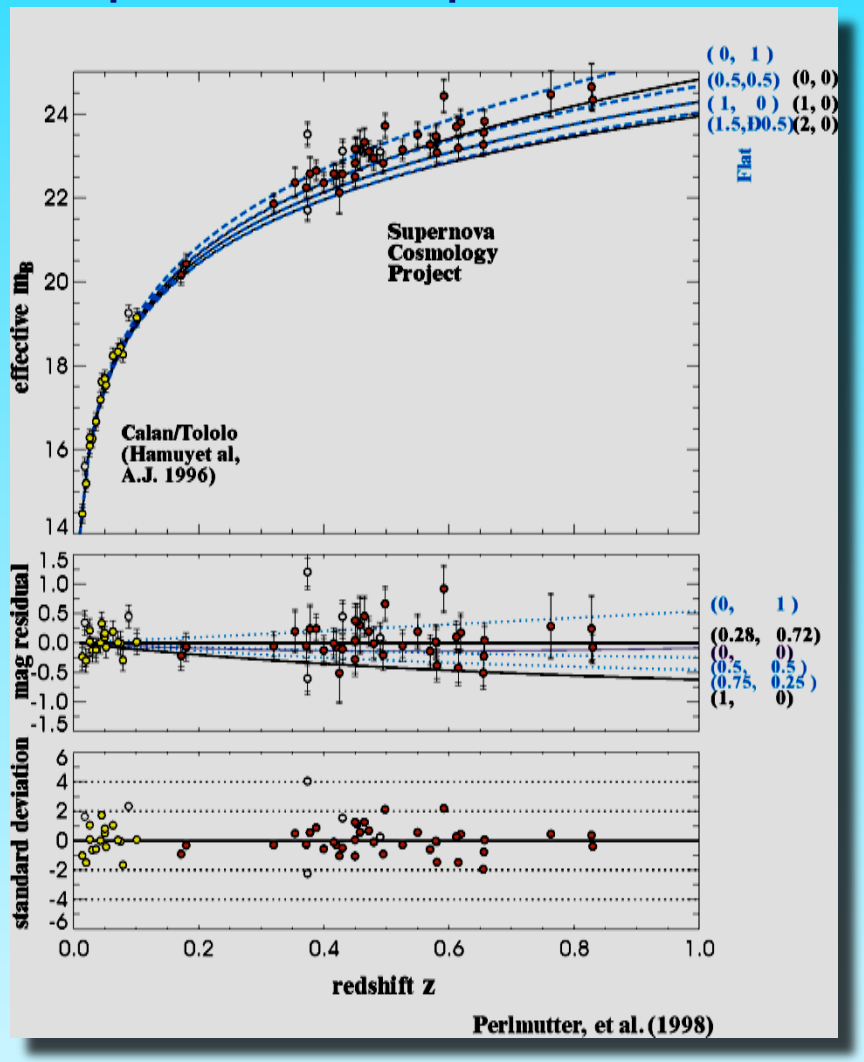


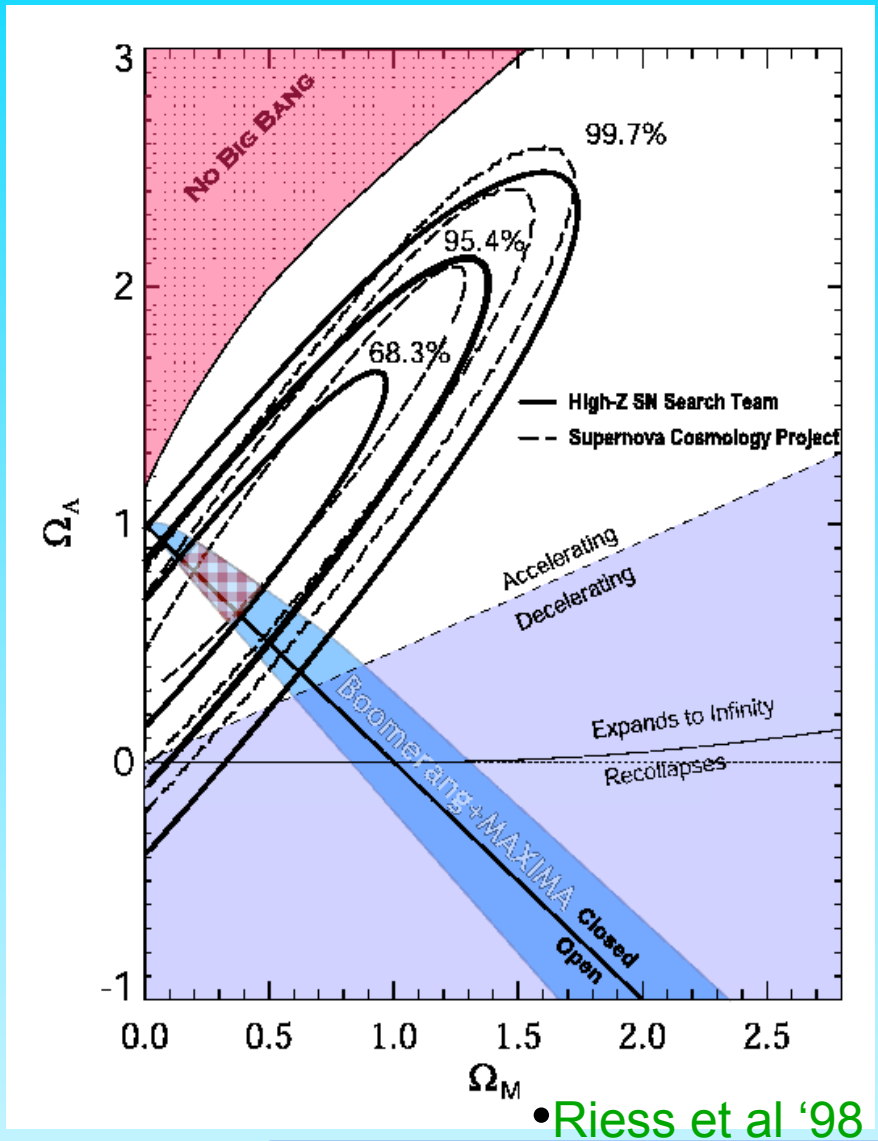
- Ω_m=1
- Ω_m=0.2
- Ω=1

- **(Some) evidence for dark energy**

•Evidence for Λ from high-z supernovae

•Distant SN are fainter than expected if expansion were





•Riess et al '98

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The Virgo consortium

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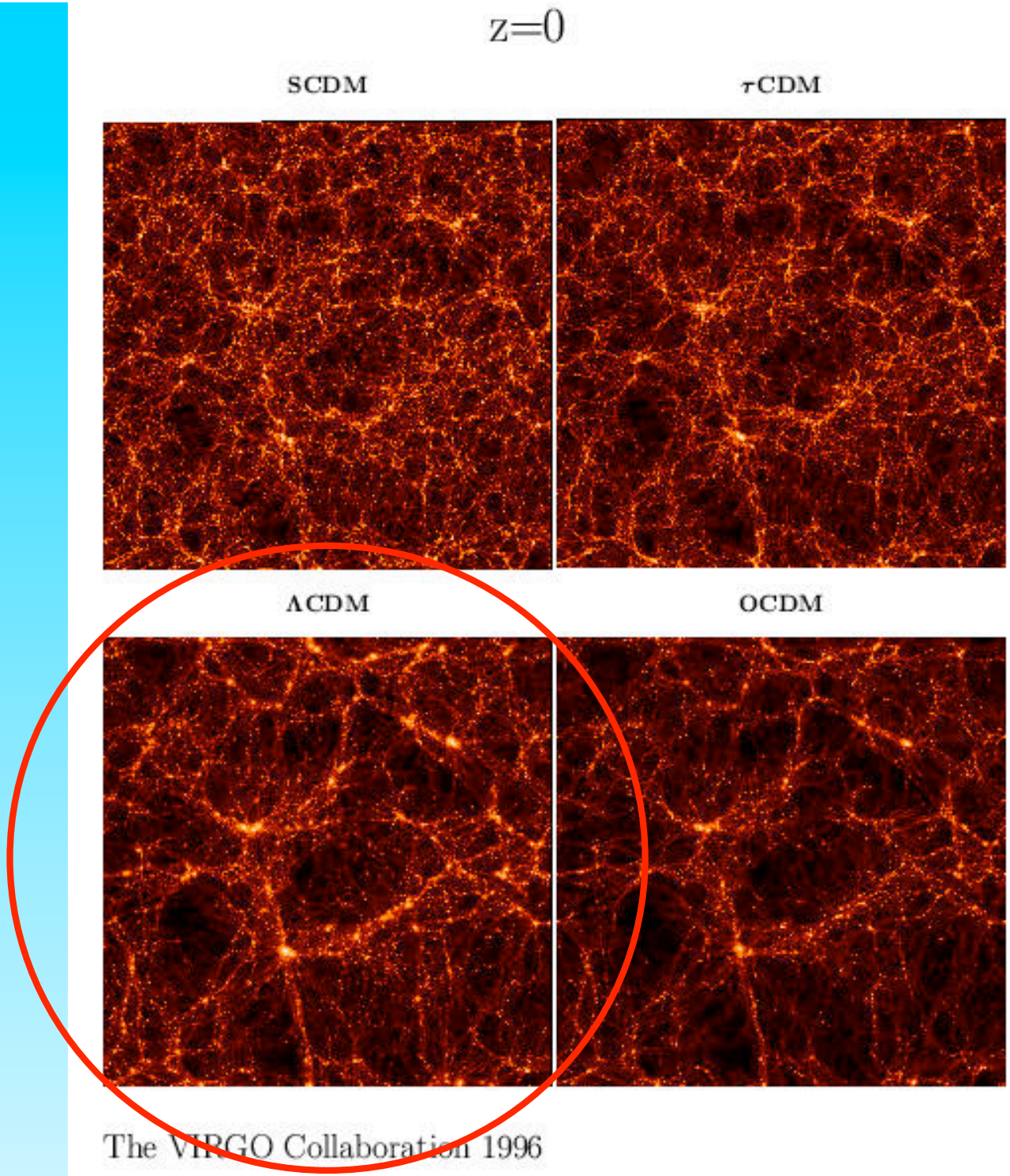

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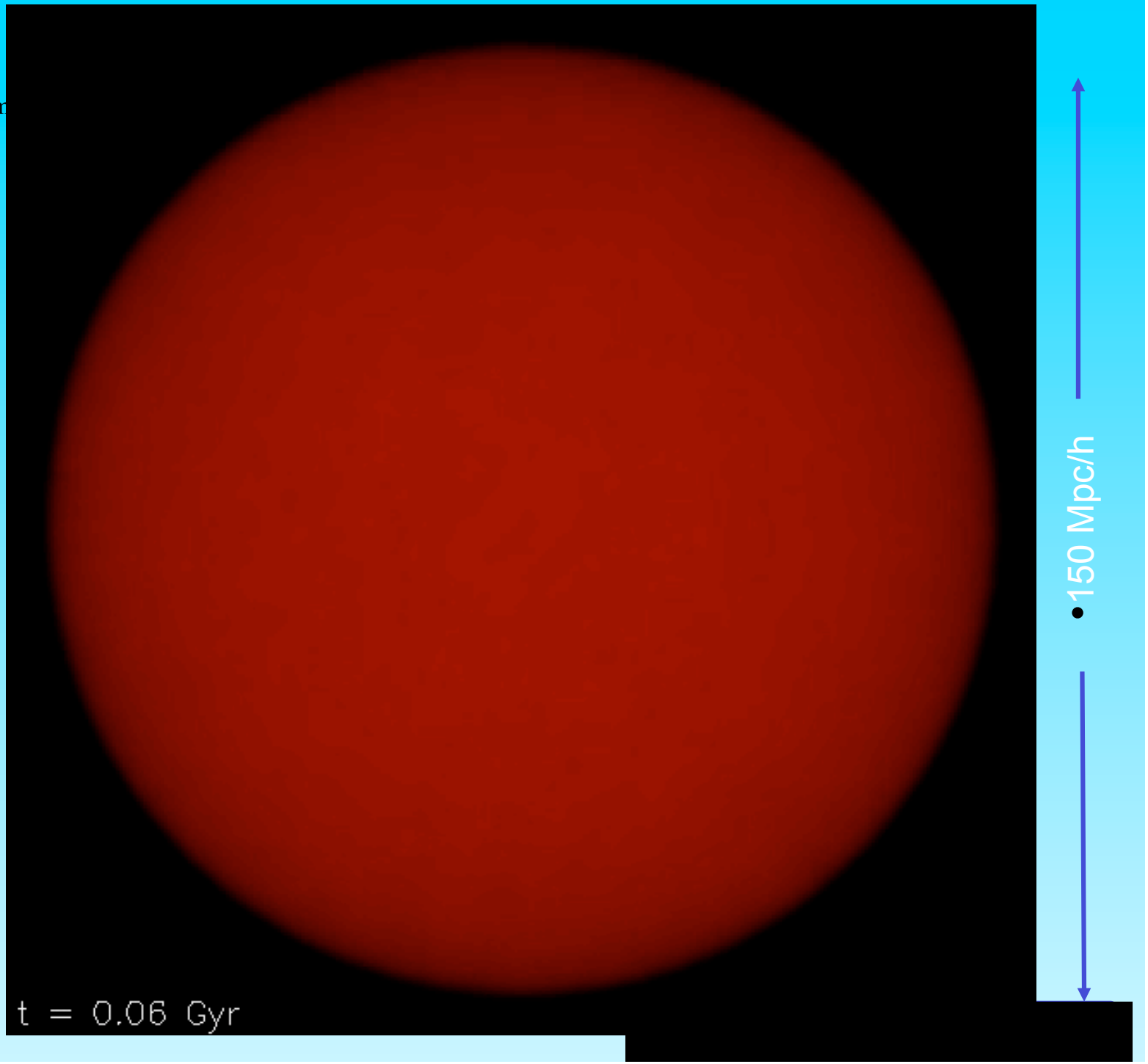
- dalla Vechia, Jenkins & Frenk



- 150 Mpc/h

- Comoving coordinates

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- Testing the CDM model

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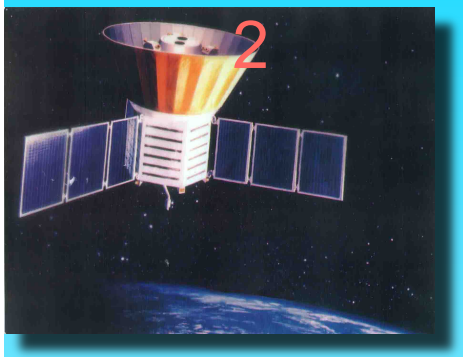
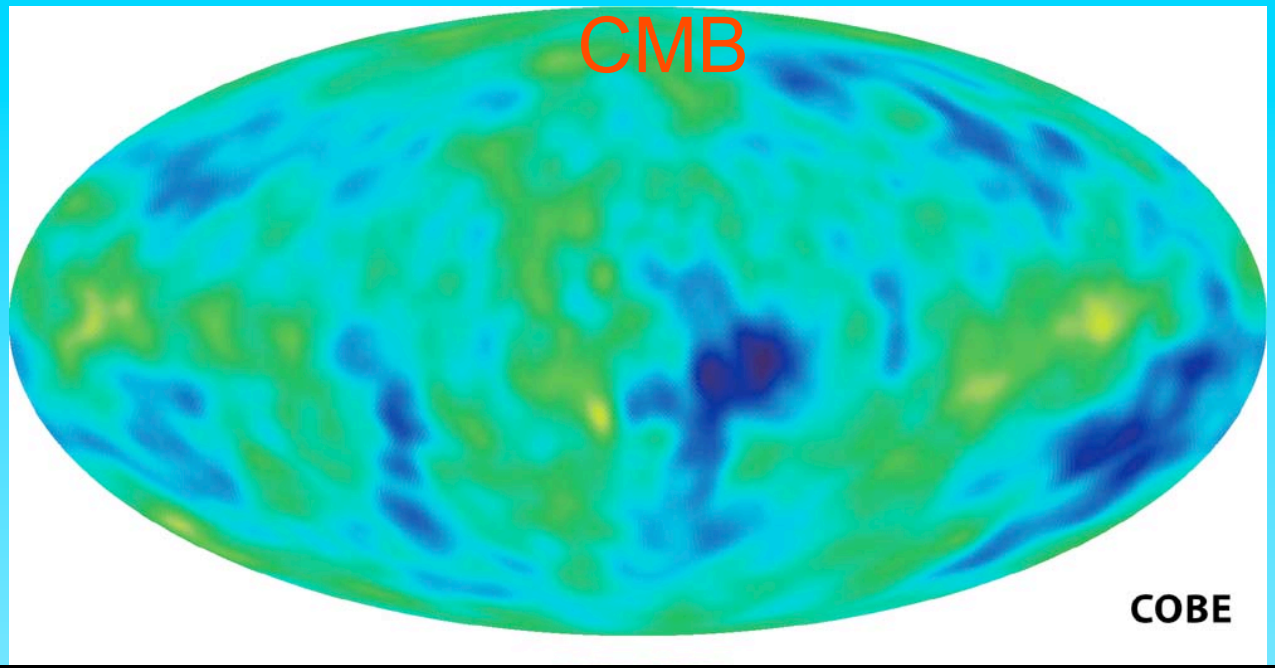
- The CMB

- 1992





The cosmic microwave background radiation (CMB) provides a window to the universe at $t \sim 3 \times 10^5$ yrs

In 1992 COBE discovered temperature fluctuations ($\Delta T/T \sim 10^{-5}$) consistent with inflation predictions

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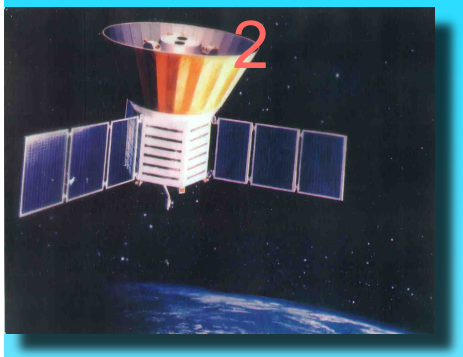
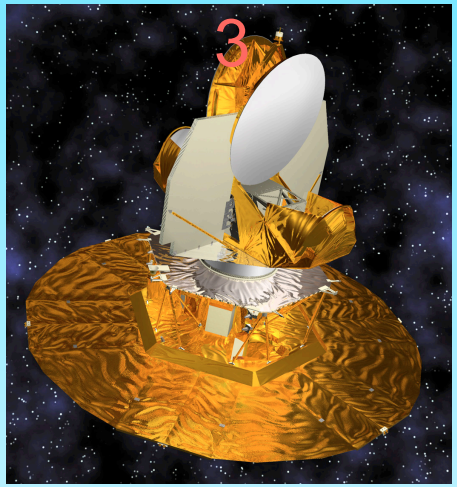
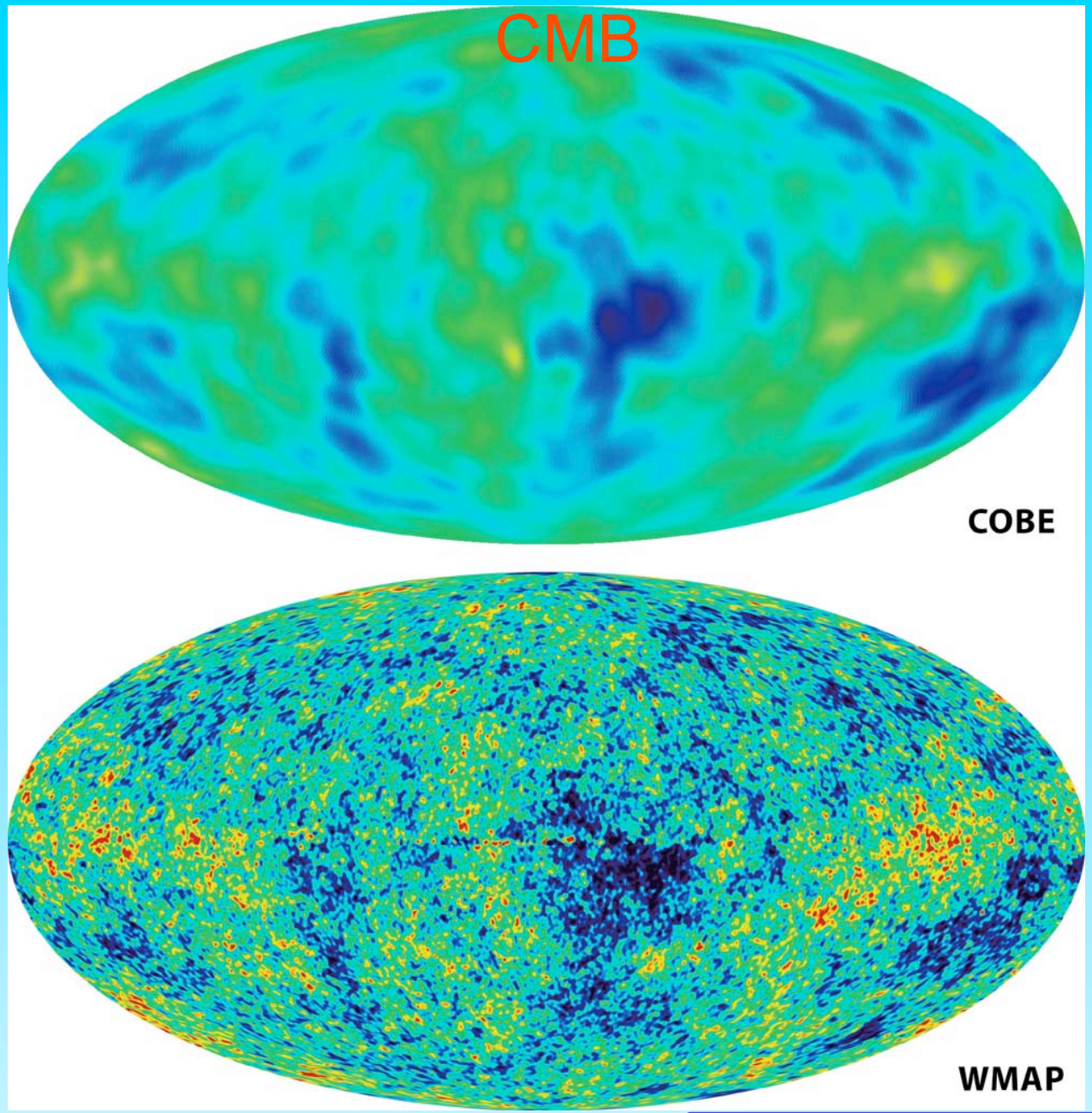
- 1992



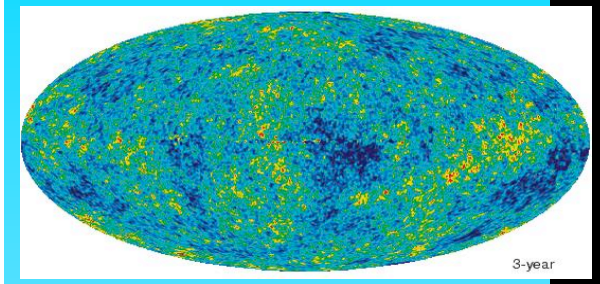
- 2003

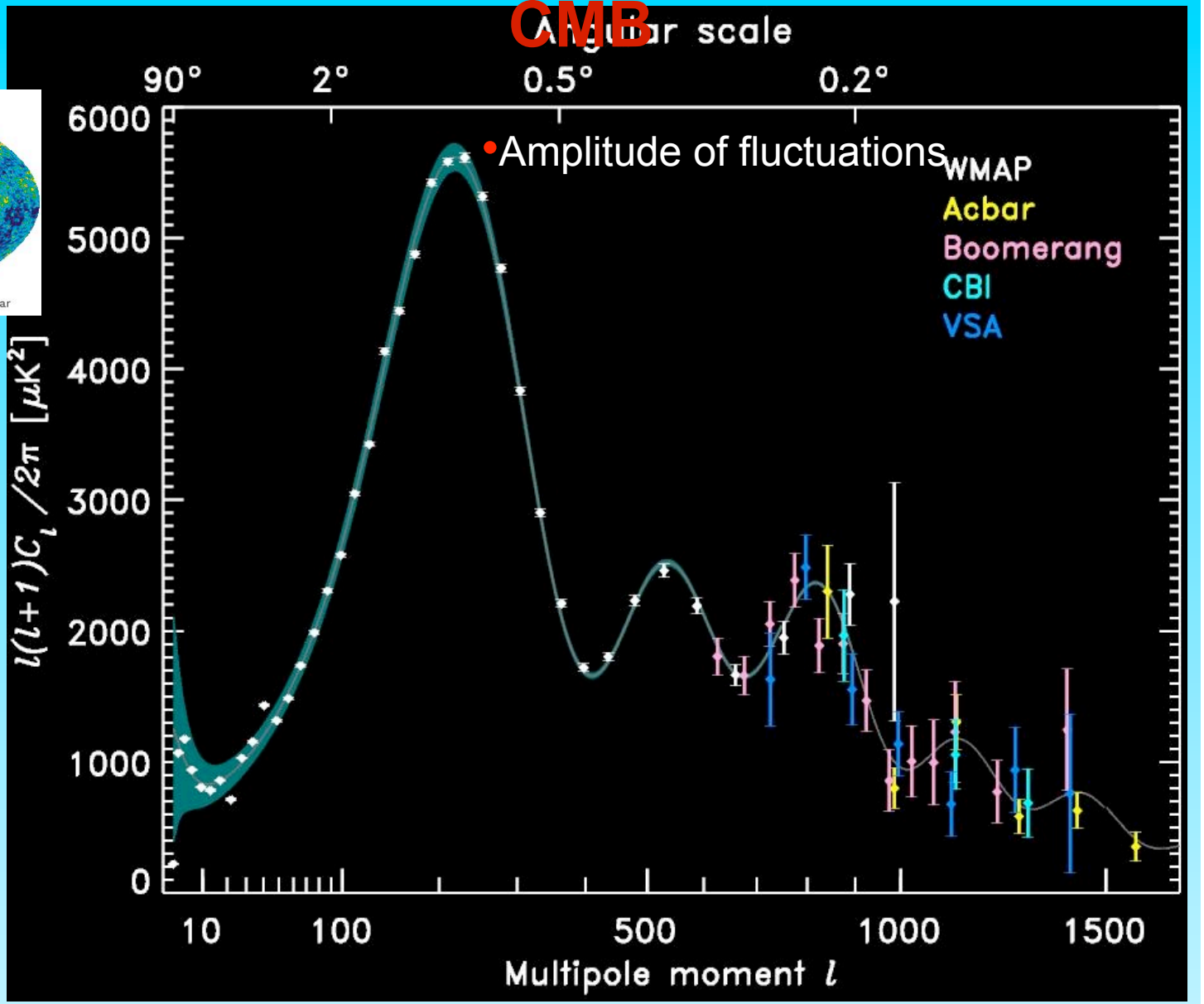


- The CMB



•WMAP temp anisotropies in CMB





- The amplitude of the CMB ripples is exactly as predicted by inflationary cold dark matter theory
- The position of the first peak
  - Hinshaw etal '06
- → FLAT UNIVERSE

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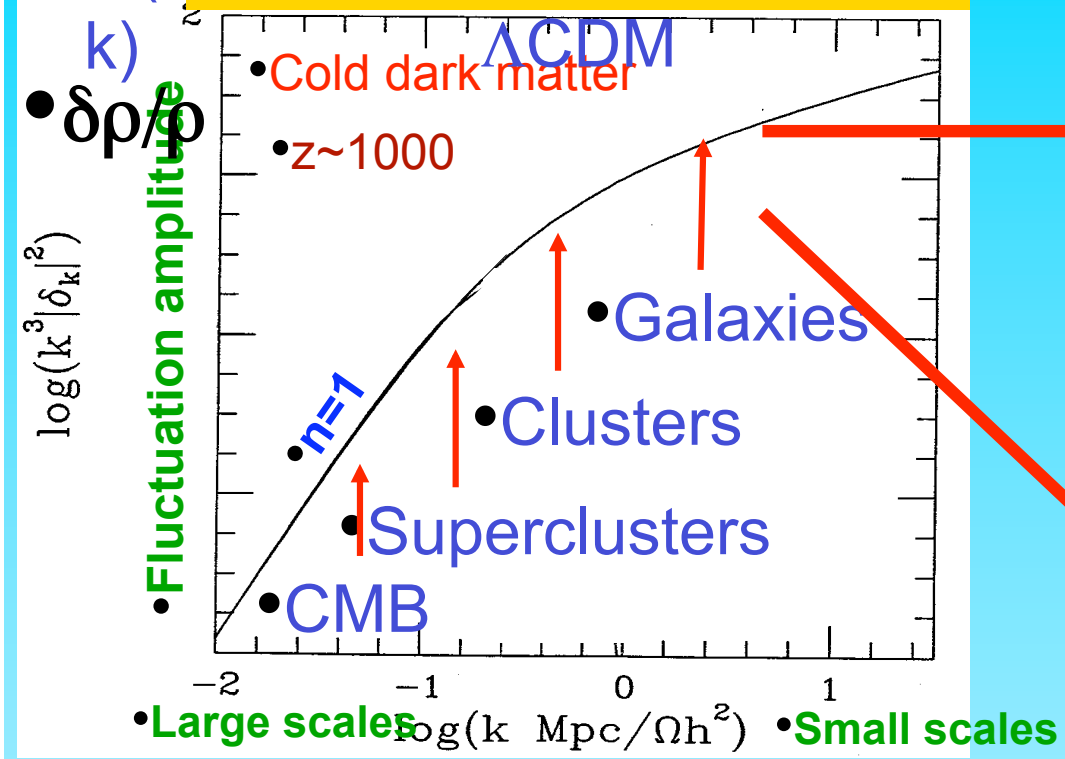
•Testing the CDM paradigm

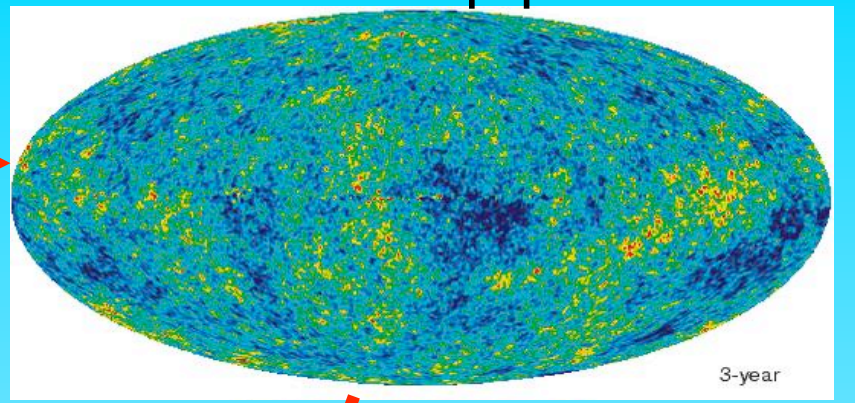
•P(•Initial conditions :

k)

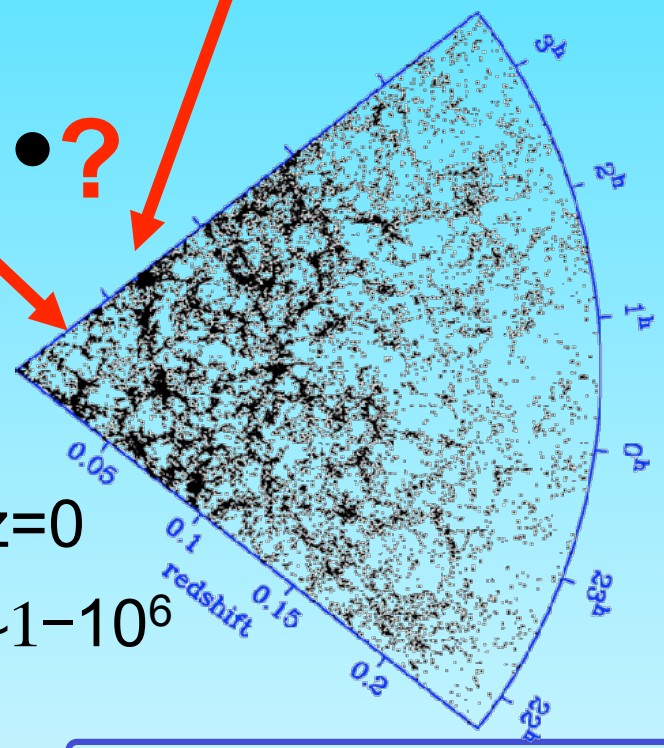
•δρ/ρ



•z=1000 •$\delta\rho/\rho \sim 10^{-5}$



•?



•z=0

•$\delta\rho/\rho \sim 1-10^6$

• The galaxy distribution evolves from fluctuations seen in CMB by gravitational amplification.

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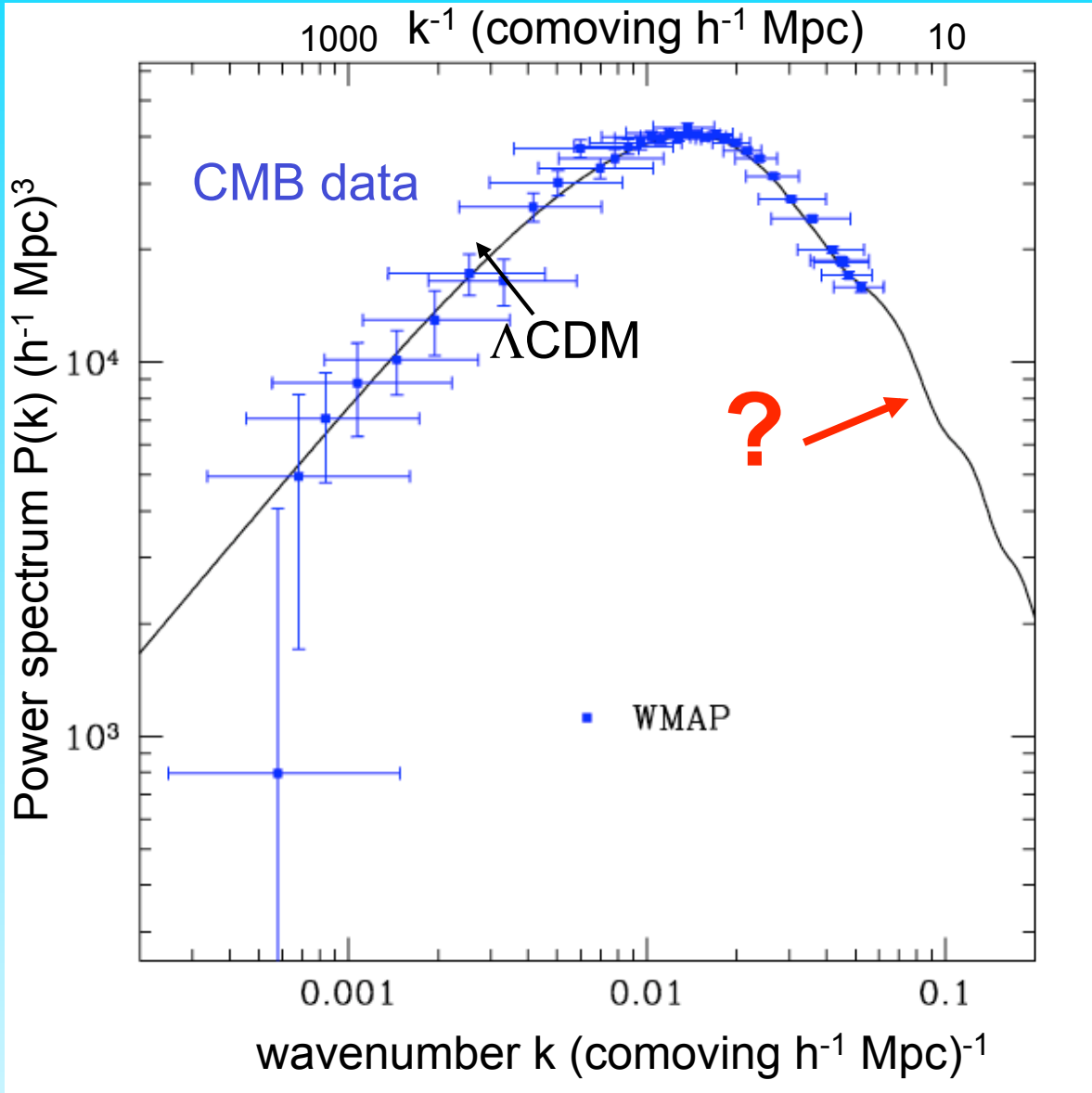
The cosmic power spectrum

CMB:

- Convert angular separation to distance (and k) assuming flat geometry
- Extrapolate to z=0 using linear theory



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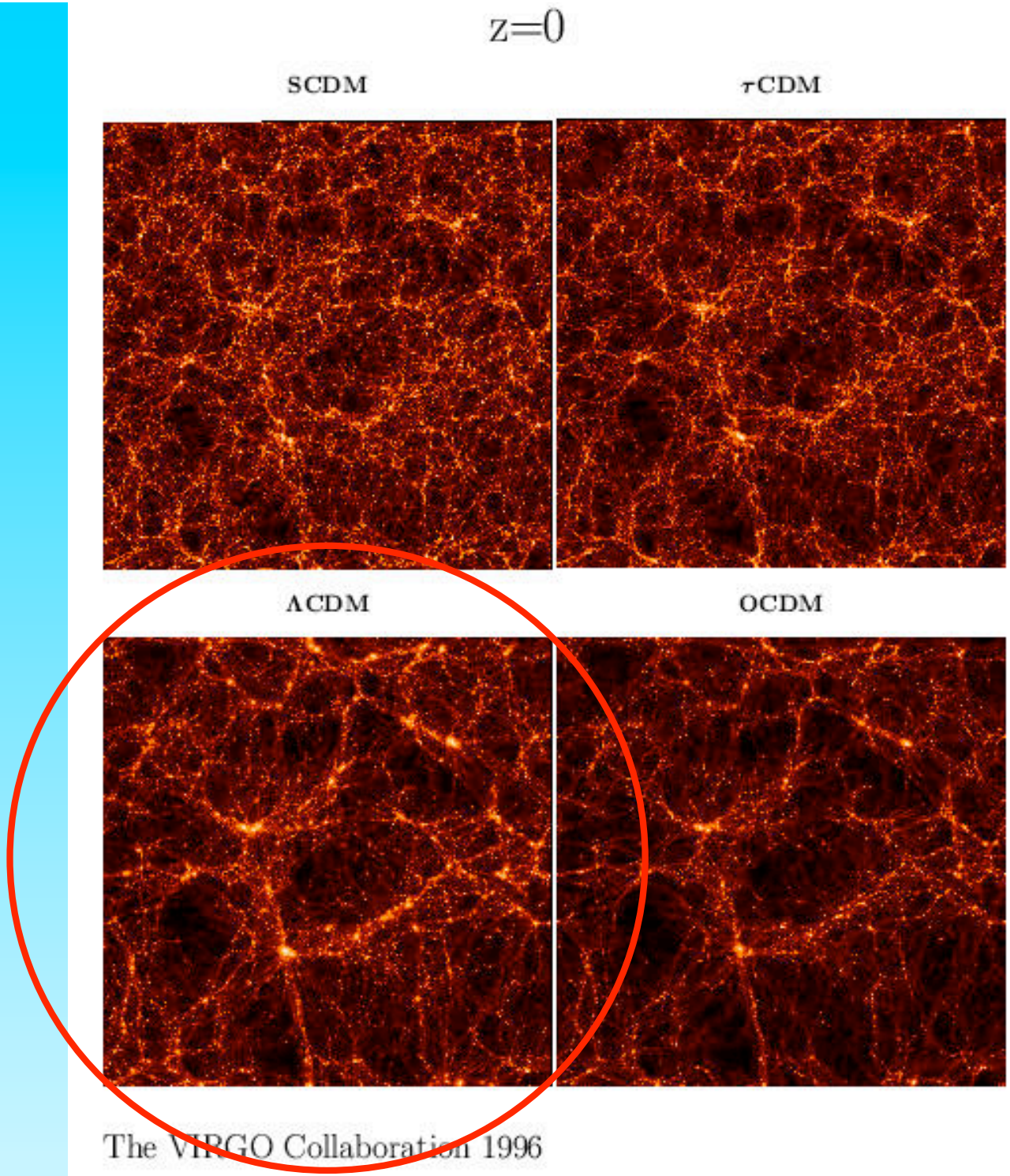
The Virgo consortium

Jenkins et al 1998



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•Testing the ΛCDM paradigm with galaxy surveys

•Three tests:

- Origin of fluctuations, nature of DM, cosm.parms
  (Power spectrum of galaxy distribution)
- Gravitational instability
  (z-space distortions)
- Hierarchical galaxy formation
  (non-linear clustering)

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  (non-linear clustering)

•The Millennium simulation

•UK, Germany, Canada, US collaboration

•Simulation data available at:

• http://www.mpa-garching.mpg.de/Virgo

•Pictures and movies available at:

• www.durham.ac.uk/virgo

•Nature, June/05

•Cosmological N-body simulation

- 10 billion particles
- 500 h^{-1} Mpc box
- $m_p = 8\times10^8\ h^{-1}\ M_o$
- Ω =1; Ω_m=0.25; Ω_b=0.045; h=0.73; n=1; σ_8 =0.9
- 20 $\times10^6$ gals brighter than LMC

•Carried out at Garching using L-Gadget by V. Springel

•(27 Tbytes of data)

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•The Millennium simulation

Regatta supercomputer of the RZG

2005

January 2005

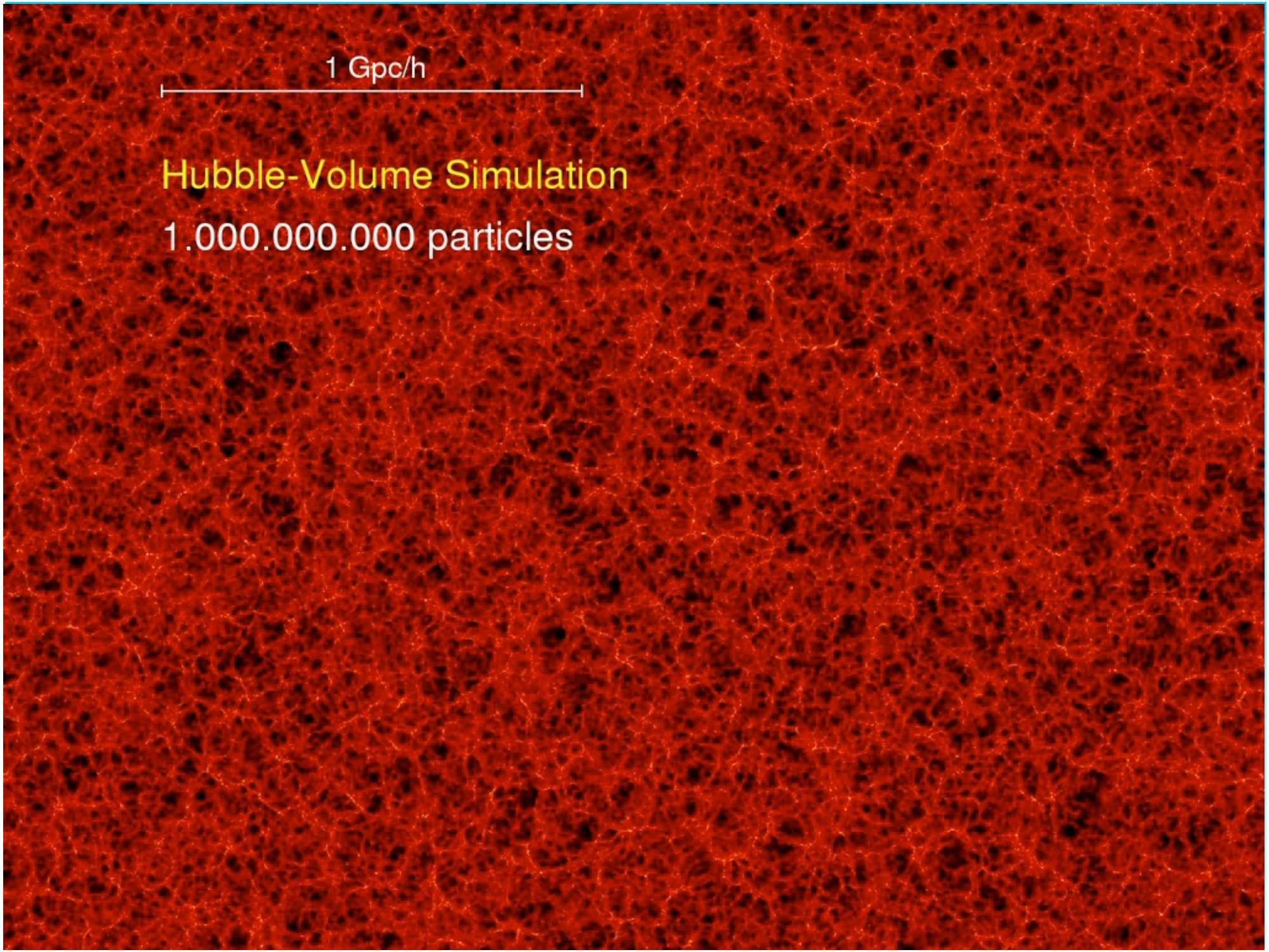
- 10 billion particles

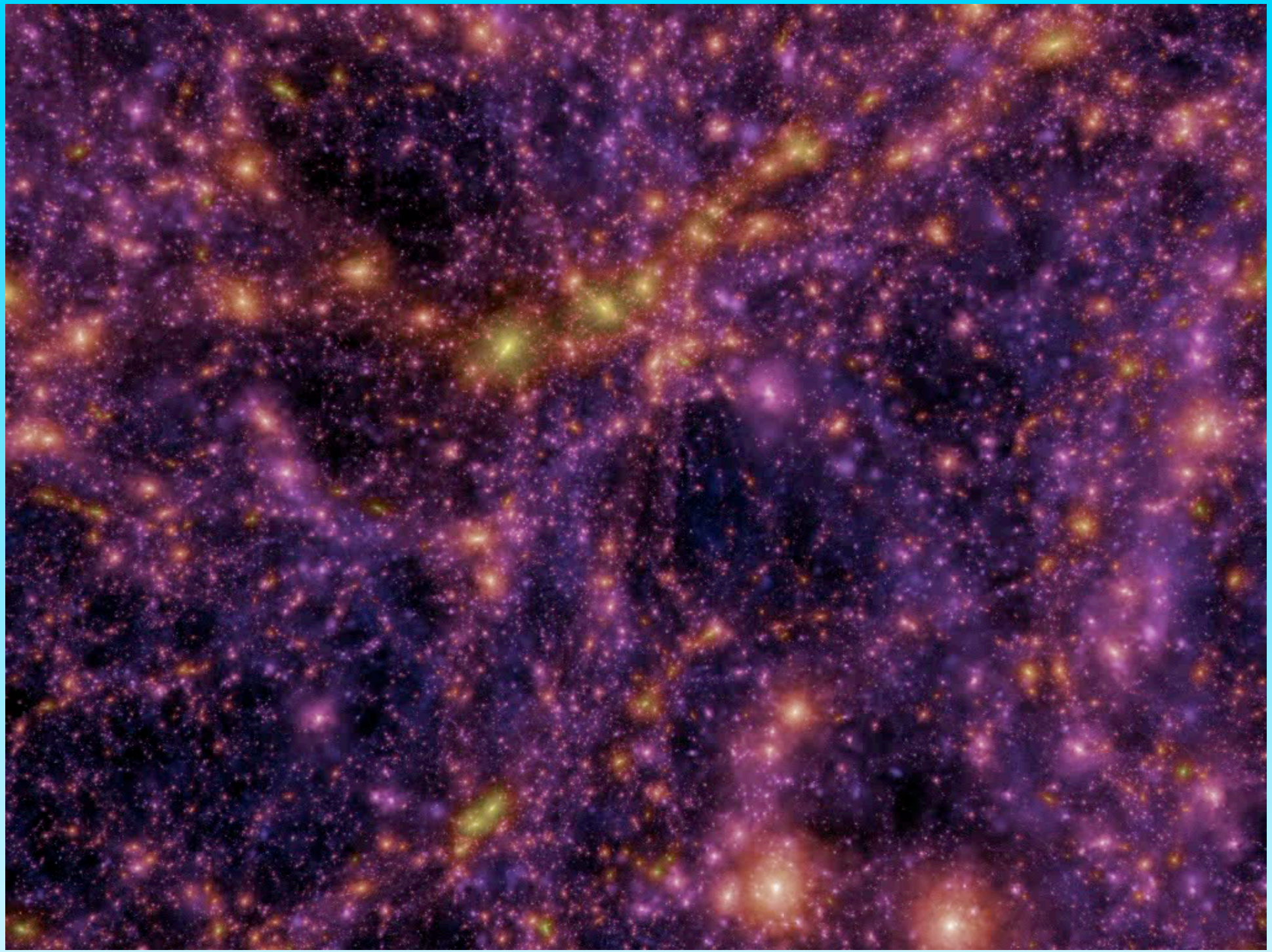
•The Millennium simulation

Regatta supercomputer of the RZG

2005

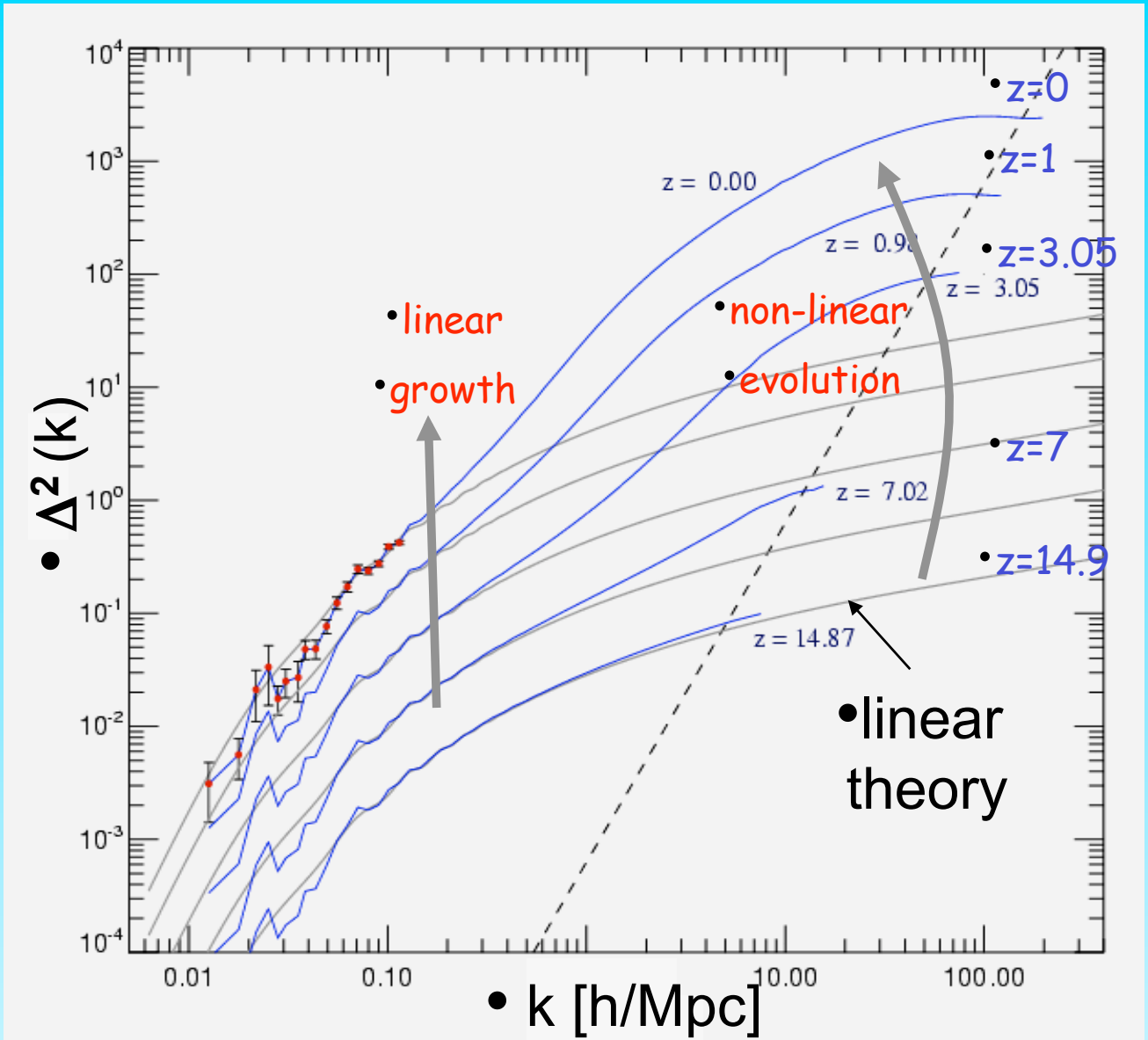
1983


1 Gpc/h
Hubble-Volume Simulation
1.000.000.000 particles



•The mass power spectrum

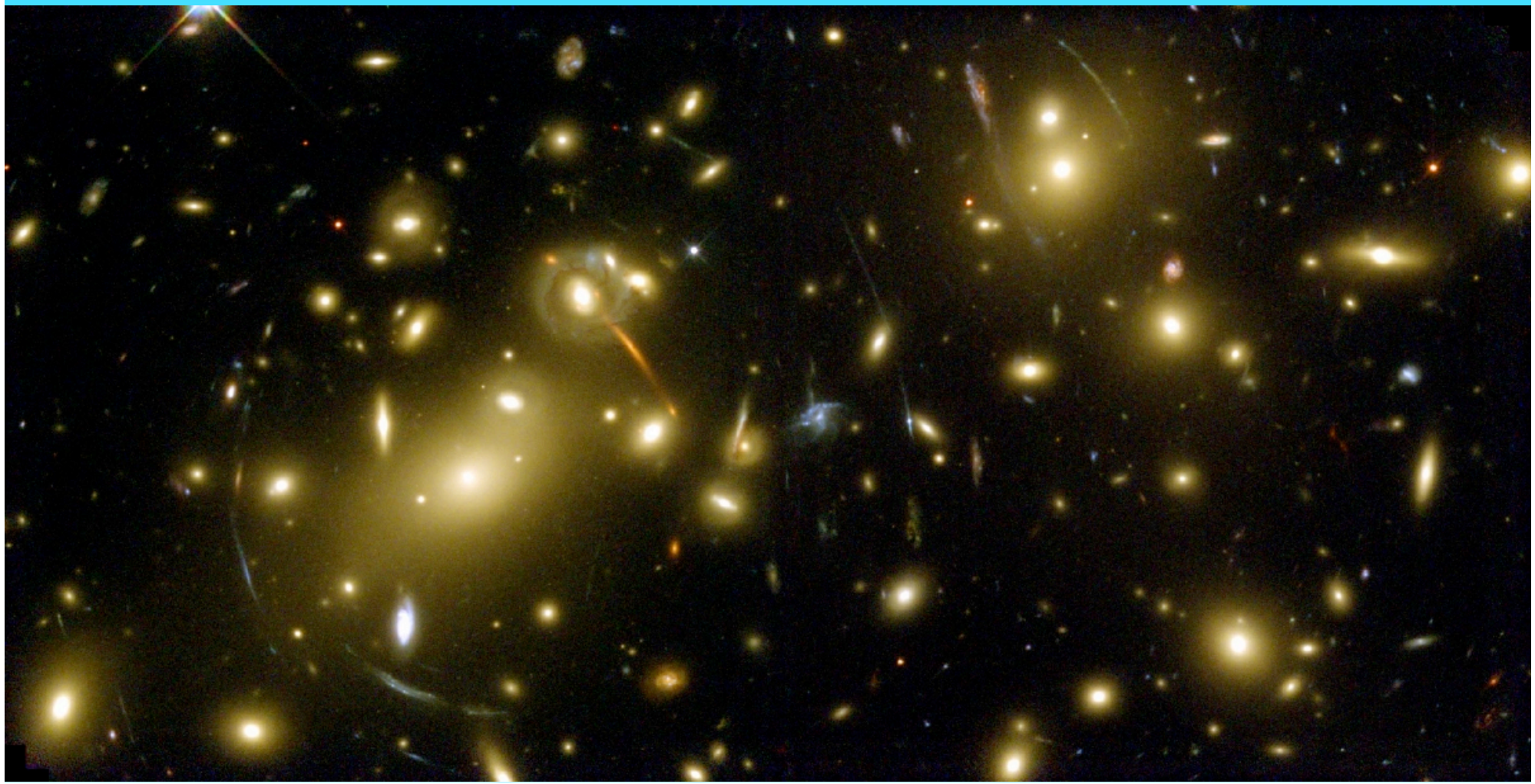
The non-linear mass power spectrum is accurately determined by the Millennium simulation over large range of scales



Gravitational lensing

- Dark matter concentrations amplify images of background galaxies and distort them into arc-like features

Abell 2218 z=0.17



A measurement of dark matter clustering

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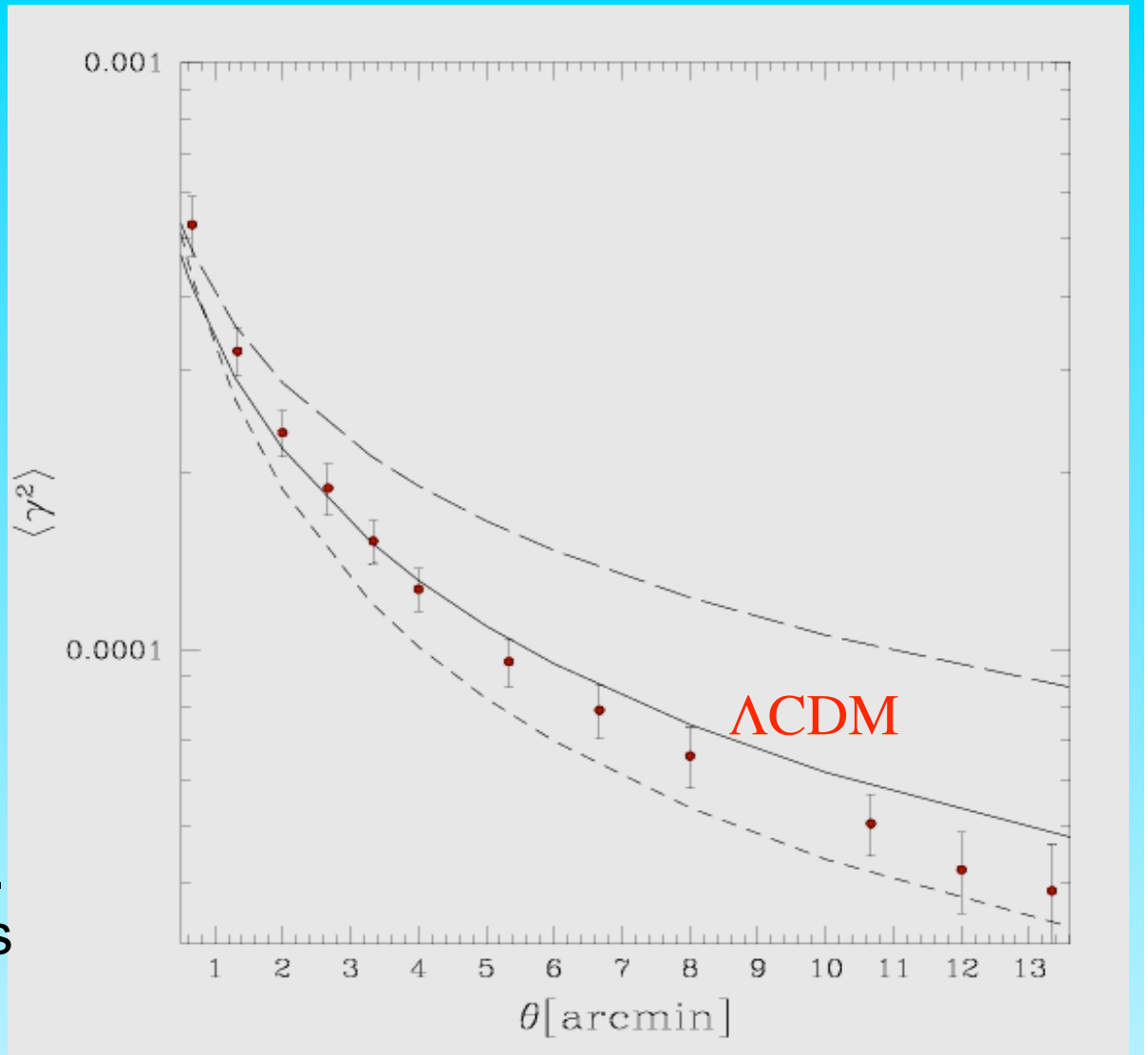
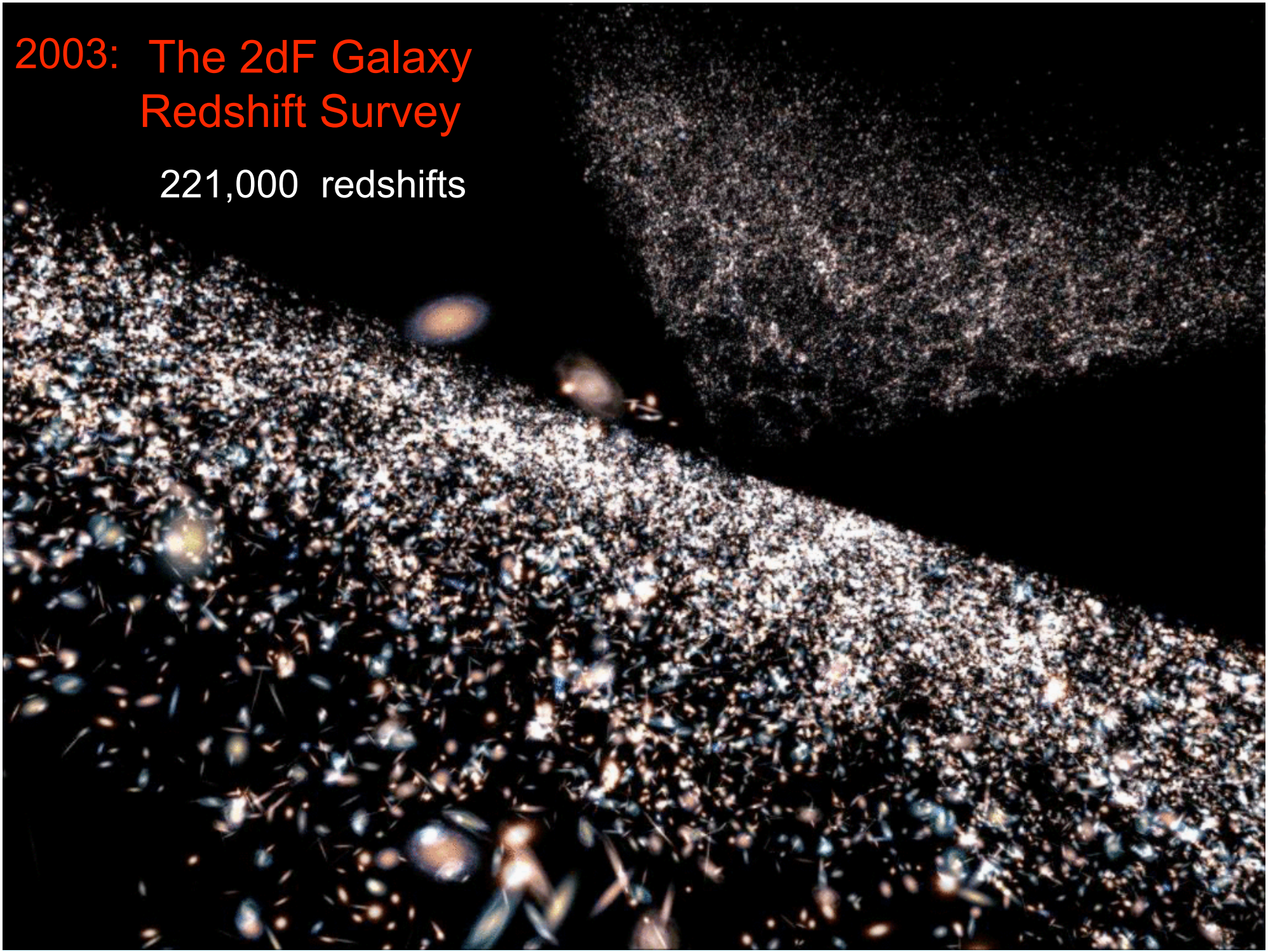
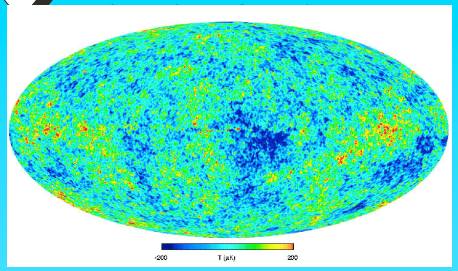
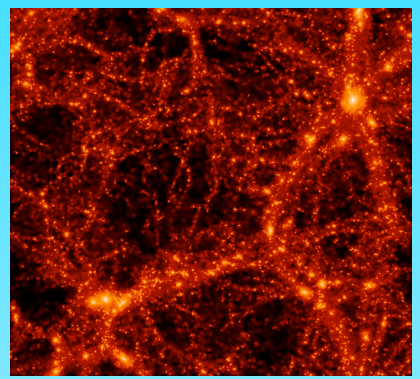
- $\langle\gamma^2\rangle$ is the mean square gravitational shear of background galaxy images within circles of radius θ.
- It is proportional to the mean square lensing mass within these circles
- On scales of a few arcmin the signal is dominated by *nonlinear* DM clustering, i.e. by the dark halos of galaxies and galaxy groups



Van Waerbeke et al 01


2003: The 2dF Galaxy Redshift Survey
221,000 redshifts





- **Cosmological model**
- (Ω_m, Ω_Λ, h); dark matter

↓

- Primordial fluctuations
- **$\delta\rho/\rho$(M, t)**

↓

- Dark matter halos
- (N-body simulations)

↓

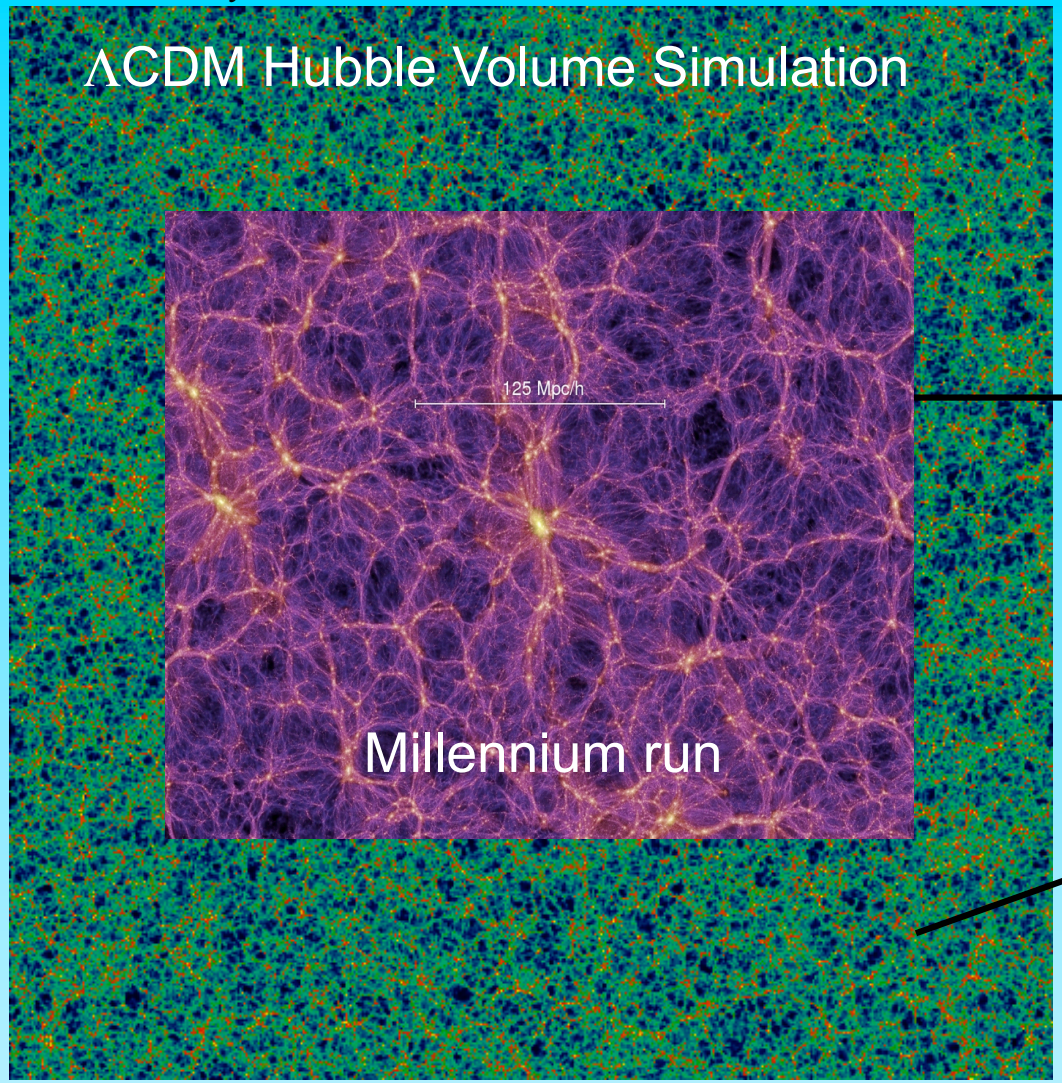
- Gas processes
- (cooling, star formation, feedback)

- Well established
- Well understood

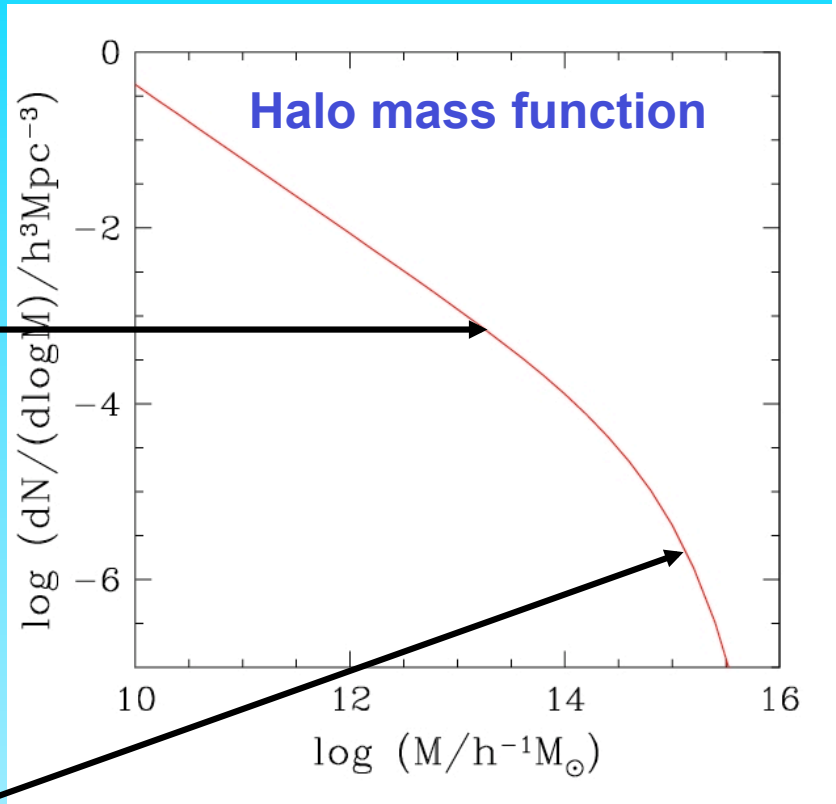
- Gasdynamic simulations
- Semi-analytics

- Formation and evolution of galaxies

The abundance of dark halos



ΛCDM



Jenkins etal '01

3000 Mpc/h

Virgo consortium

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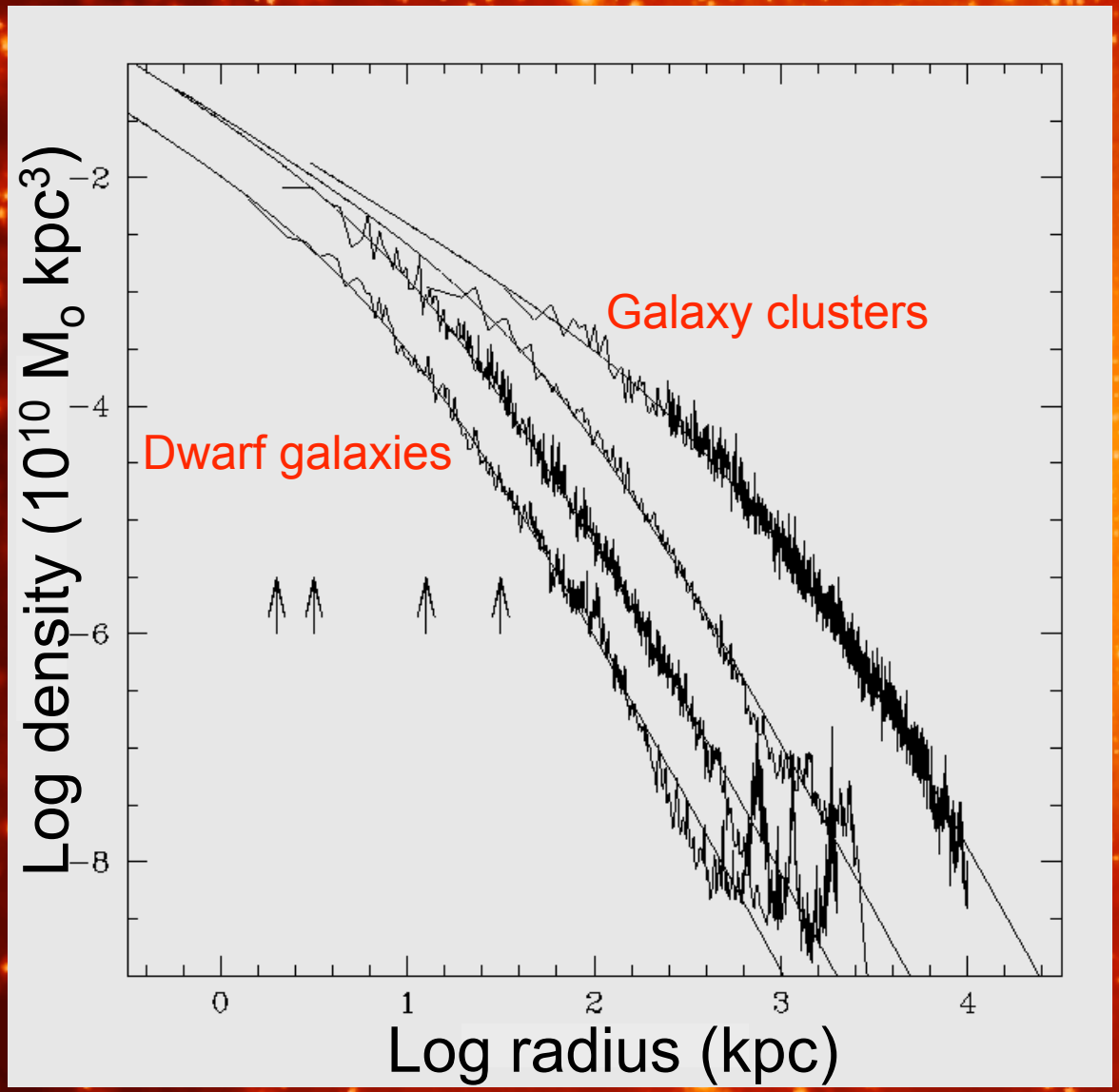
The Density Profile of Cold Dark Matter Halos



Halo density profiles are independent of halo mass & cosmological parameters

There is no obvious density plateau or `core' near the centre.

(Navarro, Frenk & White '97)

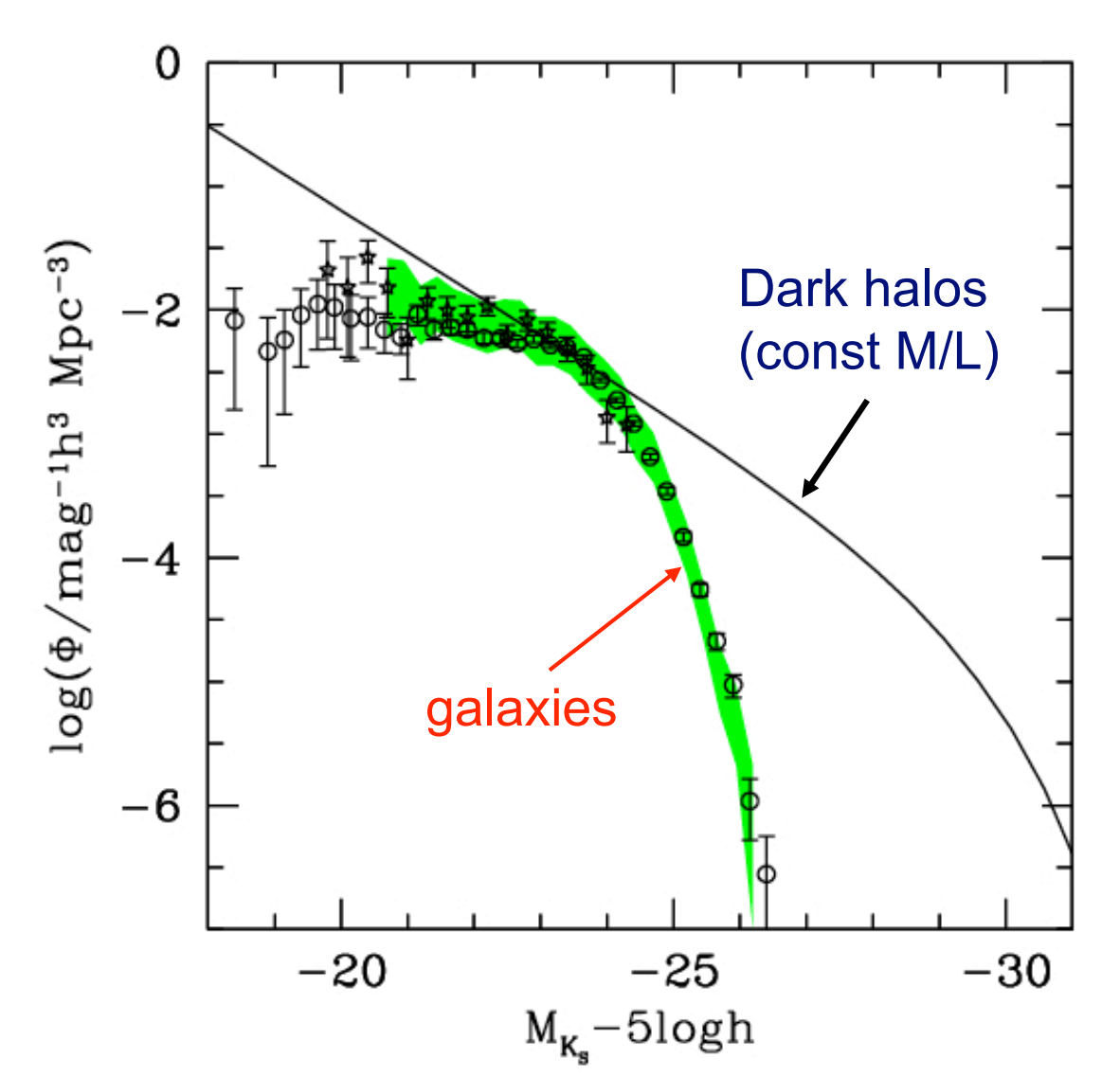
$$\frac{\rho(r)}{\rho_{crit}} = \frac{\delta_c}{(r/r_s)(1+r/r_s)^2}$$

More massive halos and halos that form earlier have higher densities (bigger δ)

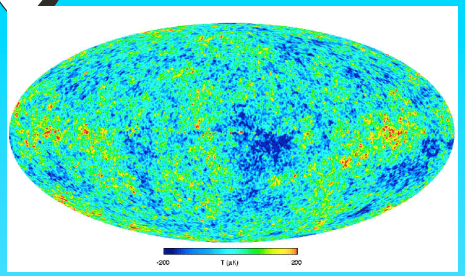
The galaxy luminosity function

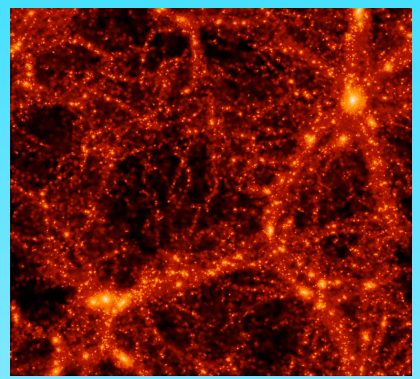
The halo mass function and the galaxy luminosity function have different shapes

⇨ Complicated variation of M/L with halo mass



Benson, Bower, Frenk, Lacey, Baugh & Cole '03





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Modelling galaxy formation

- **Aim:** follow history of galaxy formation *ab initio*, i.e starting from a cosmological model for structure formation so as to predict observables
- **Main Physical processes:**
  - Assembly of dark matter halos
  - Shock-heating and radiative cooling of gas within halos

Sub-grid physics

  - Star formation and feedback
  - Production & mixing of metals
  - Evolution of stellar populations
  - Dust extinction & emission
  - Black hole format'n, AGN feedback
  - Galaxy mergers

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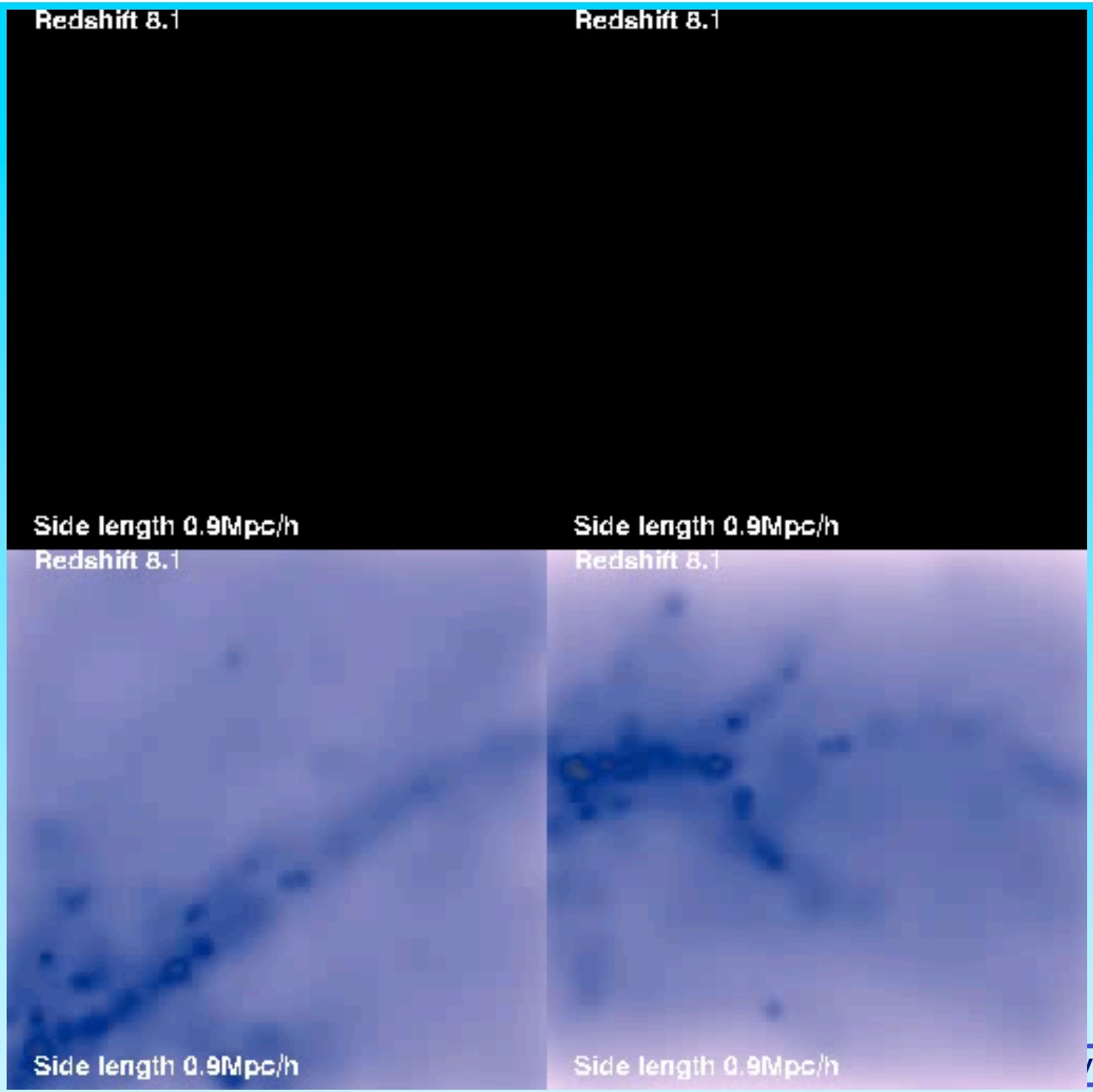
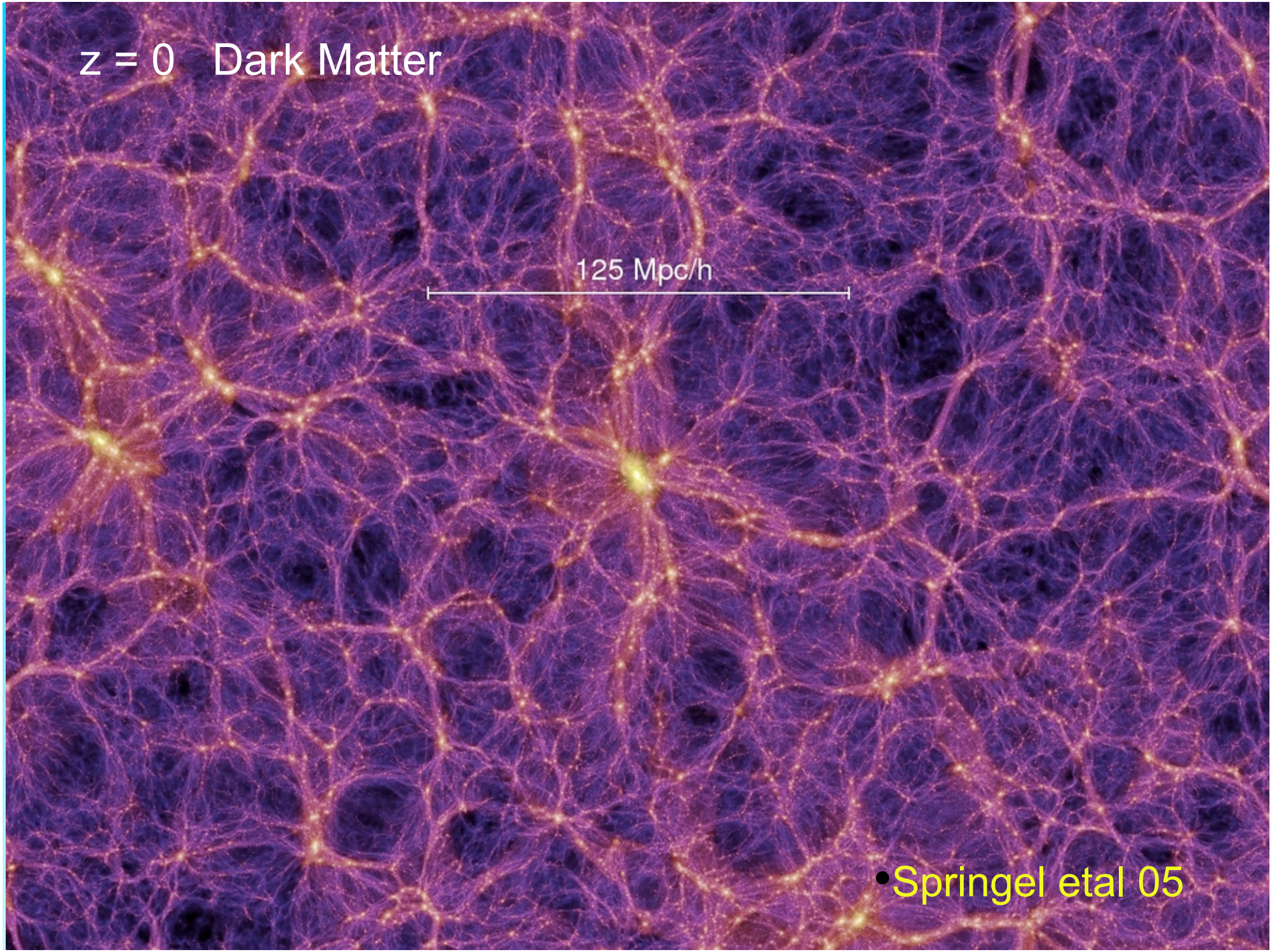
Shock-induced burst

stars

gas





•Springel etal 05

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Sub-grid physics

Semi-analytical model

– Parametrize physics

– Set of coupled differential equations

– Predict galaxy prop. (lum, mass, B/T, radius, metallicity etc) and evolution with redshift