•The large-scale structure of the Universe, surveys and simulations

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•Goal: understand origin and evolution of large-scale structure

- Growth of small fluctuations (linear theory)
- Non-linear evolution (simulations)
- Use of galaxy surveys to test cosmological theory (CDM)

•Connection to three outstanding problems in 21st Physics:

- The identity of the dark matter
- The nature of the dark energy
- Origin of cosmic structure

•Galaxy Formation

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•http://star-www.dur.ac.uk/~csf/homepage/GalForm_lectures

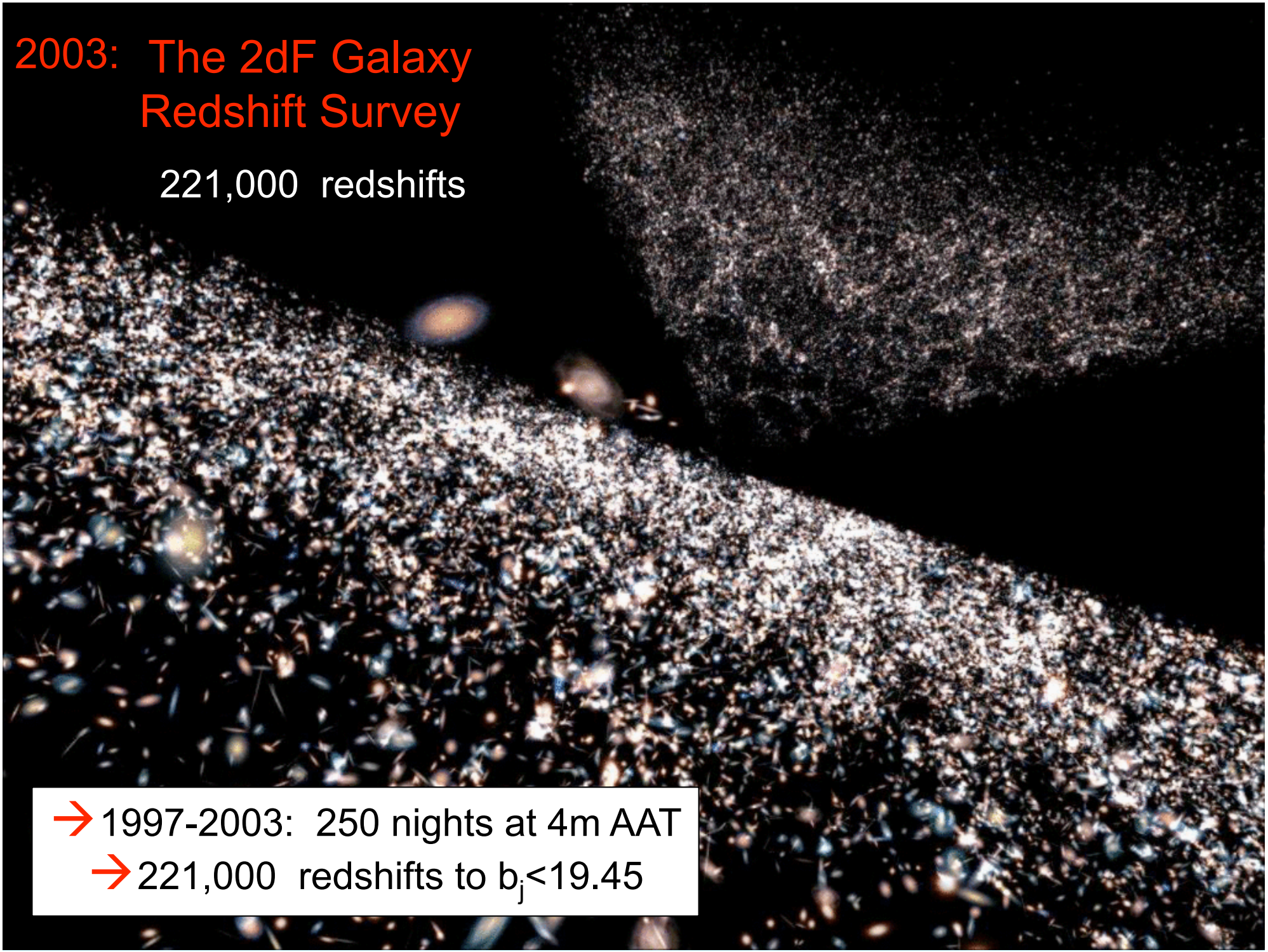
•Books:

•Cole & Lucchin: Cosmology

•Peacock: Galaxy Formation

•Linder First Principles of Cosmology

•Padhmanaban Large-scale structure


2003: The 2dF Galaxy Redshift Survey
221,000 redshifts
→ 1997-2003: 250 nights at 4m AAT
→ 221,000 redshifts to b_j<19.45

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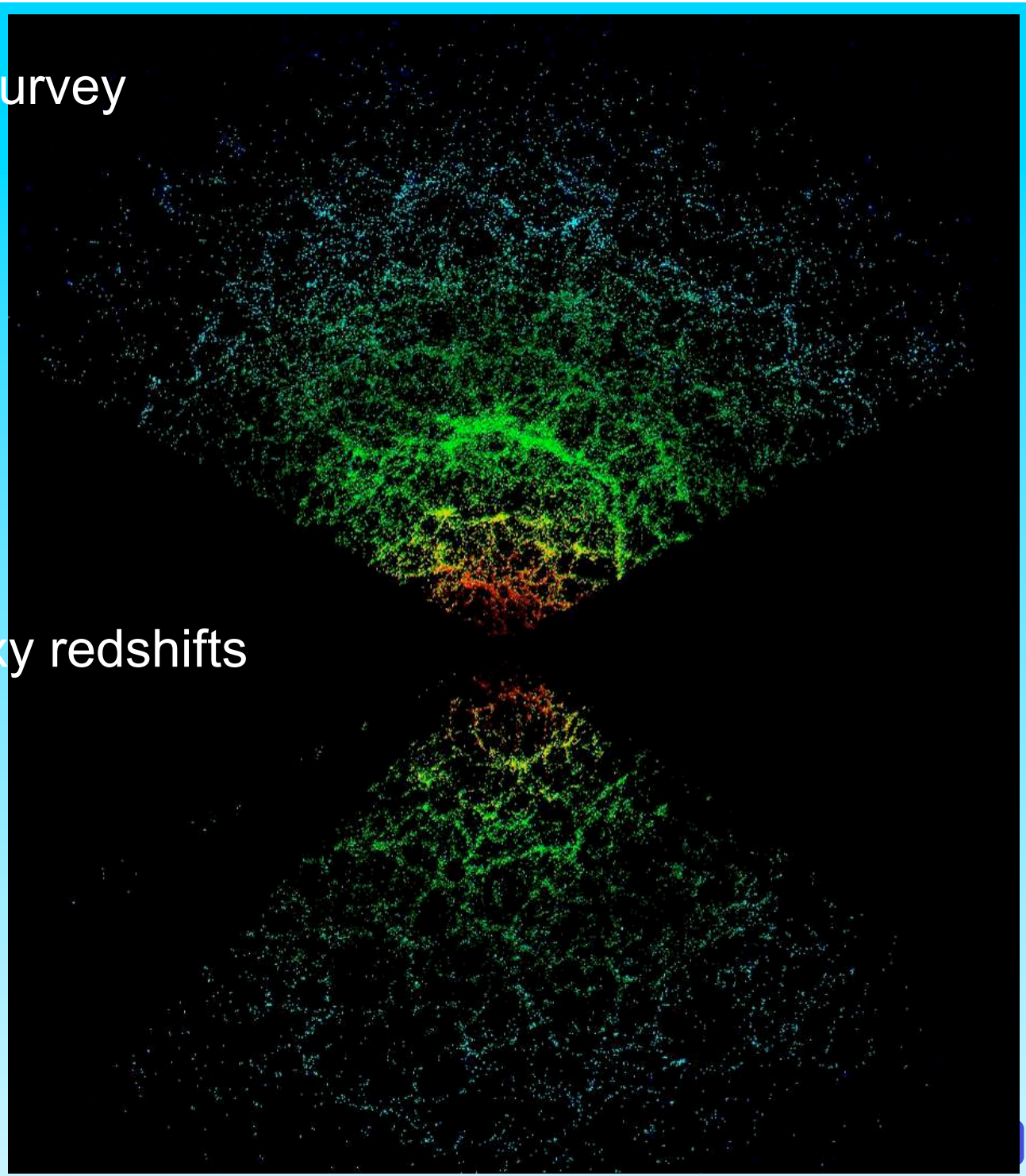
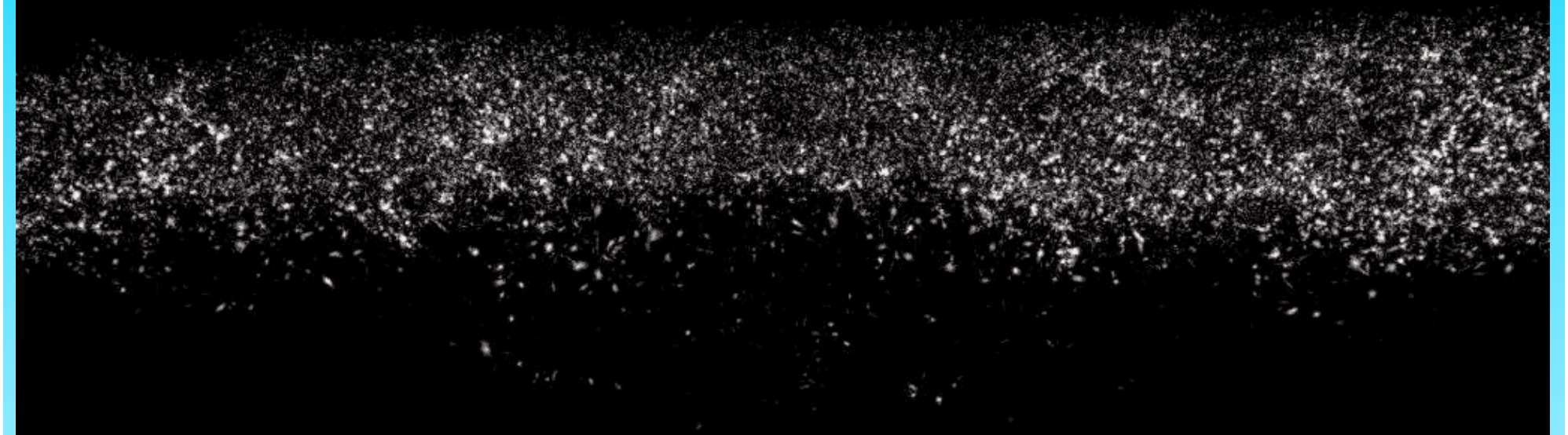
- Sloan Digital Sky Survey



- Over 500,000 galaxy redshifts

The 2dF galaxy redshift survey



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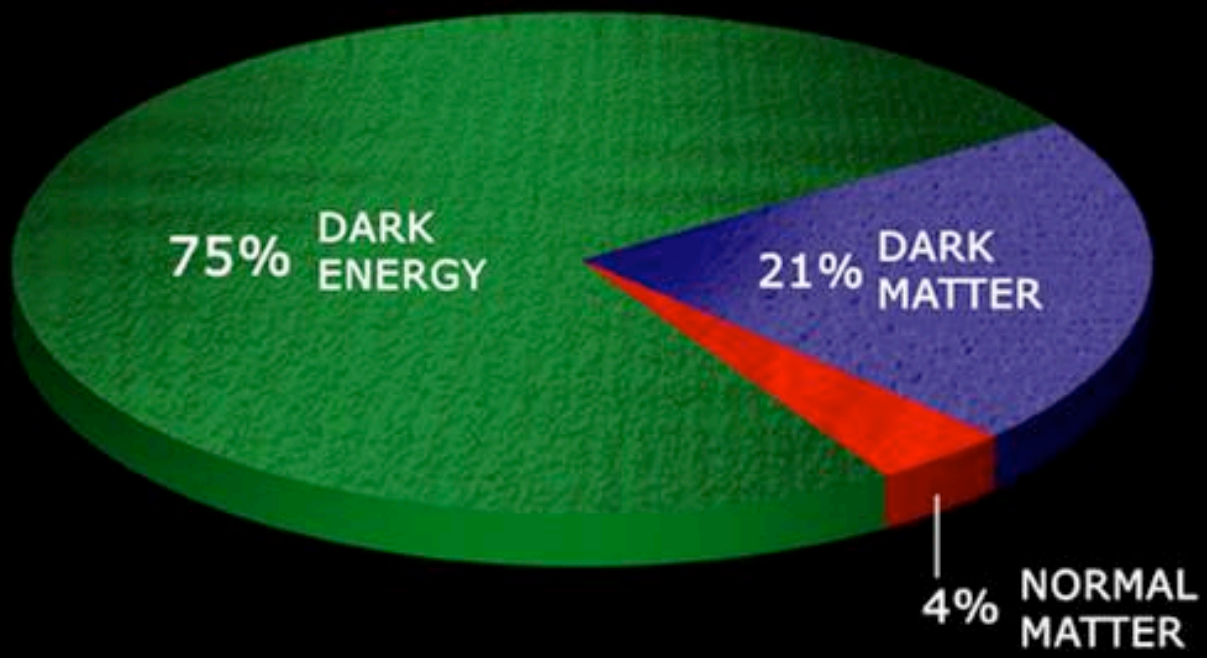
- **What is the Universe made of?**

•The content of our universe



- Dark matter ≡ matter that does not emit light at any wavelength

What is the universe made of?

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- $\Omega = \dfrac{\text{density}}{\text{critical density}}$
- $\rho = \rho_{mass} + \rho_{rel} + \rho_{vac}$
- critical density = density that makes univ. flat: (Ω = 1 for a flat univ.)

- Radiation (*CMB, T=2.726±0.005* °K) $\Omega_r = 4.7 \times 10^{-5}$
- Massless neutrinos $\Omega_\nu = 3 \times 10^{-5}$
- Massive neutrinos $\Omega_\nu = 6 \times 10^{-2}$ ($\langle m_\nu \rangle$/ev)
- Baryons $\Omega_b = 0.039 \pm 0.009$
  - (of which stars) $\Omega_s = 0.0023 \pm 0.0003$
- Dark matter (cold dark matter) $\Omega_{dm} = 0.26 \pm 0.03$
- Dark energy (cosm. const. Λ) $\Omega_\Lambda = 0.70 \pm 0.02$

- —> $\Omega = \Omega_b + \Omega_{dm} + \Omega_\Lambda \cong 1$
  - (assuming Hubble parameter h=0.73)

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•The visible and dark sides of the universe

- There is ~5 times more dark matter than there is ordinary (baryonic) matter

•(~90% of the mass of the Universe is dark matter)

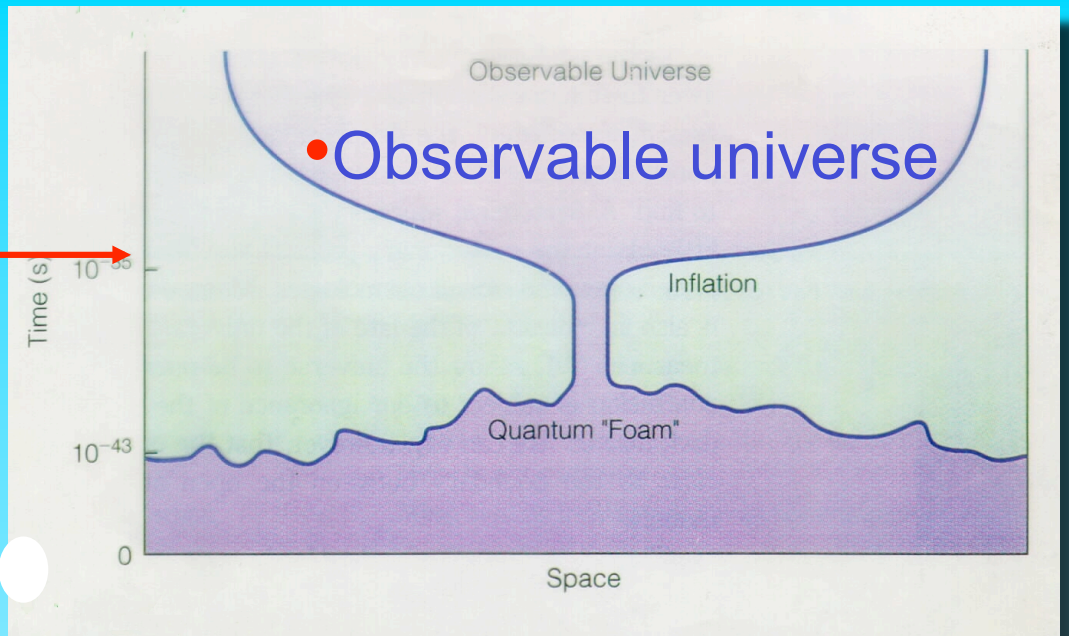
→

→ Most of the dark matter is NOT ordinary (baryonic) matter

→ **Weakly interacting massive particles (WIMPS)**

•Cosmic Inflation

•$t=10^{-35}$ s



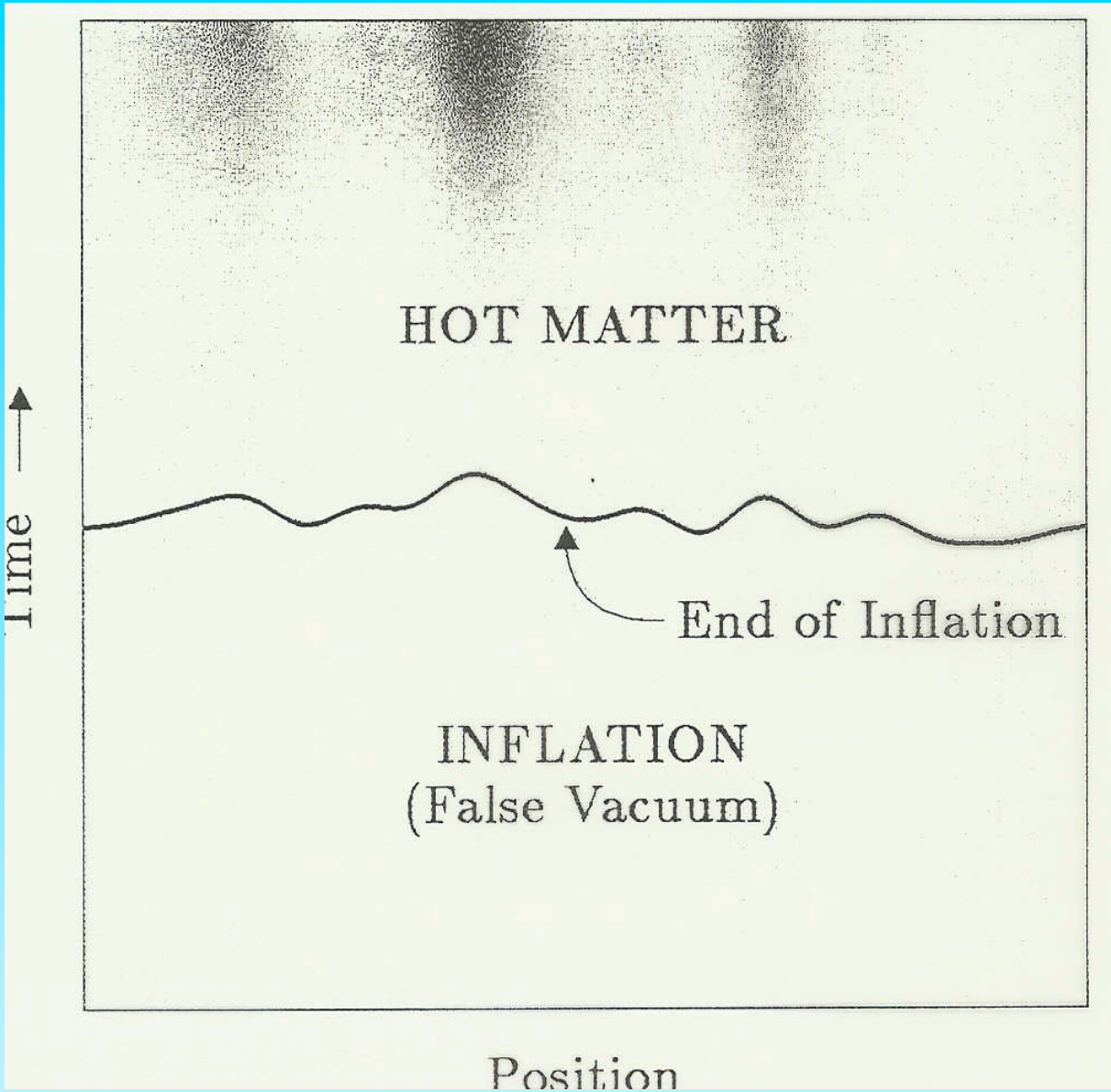
- Inflation theory predicts:

1. Flat geometry ($\Omega = 1$) •√
2. Small ripples in mass distribution

•Generation of primordial fluctuations

•Because of quantum fluctuations, different parts of the Universe finish inflating at slightly different times

•Quantum fluctuations are blown up to macroscopic scales during inflation



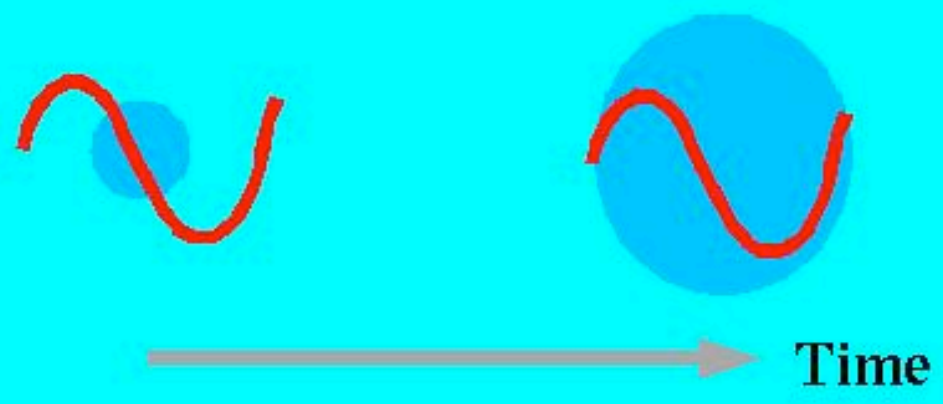
How fluctuations evolve

Key time: epoch of matter/radiation equality

Radiation dominated: $\delta \sim a^2$

Matter dominated: $\delta \sim a$

Key concept: is the fluctuation inside the horizon?



Inside horizon scale: DAMPING

3. THE GROWTH OF DENSITY FLUCTUATIONS – LINEAR THEORY

3.1 Linear Fluctuations

To study the evolution of linear perturbations, we write the density, pressure and velocity fields, $\rho(\overline{r}, t)$, $P(\overline{r}, t)$ and $\overline{u}(\overline{r}, t)$, as a mean value plus an initially small fluctuation:

$$\rho(\overline{r}, t) = \overline{\rho}(t) + \delta\rho(\overline{r}, t) = \overline{\rho}(t)(1 + \delta(\overline{r}, t)), \tag{3.1}$$

where $\delta = \delta\rho/\overline{\rho}$

$$P(\overline{r}, t) = \overline{P}(t) + \delta P(\overline{r}, t)$$

$$\overline{u} = H(t)\overline{r} + \overline{v}(\overline{r}, t) \tag{3.2}$$

where **v** is the peculiar velocity. For linear fluctuations, $\delta \ll 1$ and $v \ll Hr$.

3.2 The rate of growth equation

For an ideal, non-relativistic fluid, the equations of conservation of mass and momentum are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \overline{u}) = 0 \tag{3.3}$$

$$\frac{\partial \overline{u}}{\partial t} + \overline{u} \cdot \nabla \overline{u} = -\frac{\nabla P}{\rho} - \nabla \Phi \tag{3.4}$$

where Φ is the gravitational potential which, for a self-gravitating fluid, is given by Poisson's eqn:

$$\nabla^2 \Phi = 4\pi G(\rho_{tot} + \frac{3P_{tot}}{c^2}) - \Lambda c^2 \tag{3.5}$$

where the suffix *tot* indicates that all matter contributes to the gravitational potential whereas the conservation equations apply separately to the baryonic fluid and also to any component of non-baryonic dark matter. (Note that for radiation, $3P_{tot}/c^2 = \rho_\gamma$ so in the last equation, the P term is related to ρ_γ). (For a derivation of the fluid equations, see eg Binney & Tremaine *Galactic Dynamics*).

$$\frac{\partial^2 \delta}{\partial t^2}\Big|_{\overline{x}} + 2H\frac{\partial \delta}{\partial t}\Big|_{\overline{x}} = \frac{\nabla^2_{\overline{r}}\delta P}{\overline{\rho}} + \nabla^2_{\overline{r}}\phi \tag{3.20}$$

where

$$\nabla^2_{\overline{r}}\phi = 4\pi G(\delta\rho_{\rm tot} + \frac{3\delta P_{\rm tot}}{c^2}) \tag{3.21}$$

Note:

- Eqn (3.20) applies to any component of non-relativistic (cold) matter ($P \ll \frac{1}{3}\rho c^2$); δ refers to fluctuations in density and δP in pressure
- $\delta\rho_{\rm tot}$ and $\delta P_{\rm tot}$, on the other hand, are the sources of perturbation in the gravitational potential and so refer to the sum of all components of matter.
- The $2H\frac{\partial\delta}{\partial t}$ term is a cosmological drag term. It is the only term where the expansion of the universe comes in. Thus, the Hubble expansion tends to dampen fluctuations. Without this term, fluctuations would grow exponentially; with it, growth is a power-law at best.

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3.3 Solution for cold matter (dust)

We will now solve (3.20) for the case where pressure fluctuations are zero. This is known as the "dust solution" and corresponds to cold matter. Write:

$$\rho_{\rm tot} = \rho_m + \rho_r \tag{3.22}$$

and set $\delta P_{\rm tot} = 0$. Then, $\delta\rho_{\rm tot} = \delta\rho_m \equiv \rho_m\delta$. Eqn (3.20) becomes:

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_m\delta \tag{3.23}$$

where the dot denotes the time derivative at constant $\overline{x}$. Using (1.12), we can write this as

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\Omega_m\delta = 0 \tag{3.24}$$

Let us consider the solution in 3 different situations:

• Matter-dominated case, $\Omega \simeq \Omega_m \simeq 1$. This is the most important case since it describes the Universe since recombination until almost the present day. Setting $\Omega = 1$ in (2.11), we have $H^2 \propto \rho_m \propto a^{-3}$, from which it is easy to show that $H = \frac{2}{3t}$. Substituting in (3.24),

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0 \tag{3.25}$$

The general solution is:

$$\boxed{\delta = A(\overline{x})t^{2/3} + B(\overline{x})t^{-1}} \tag{3.26}$$

where A and B are constants. The first term is known as the "growing" mode and the second as the "decaying" mode.

• Λ-dominated; $\Omega_\Lambda \simeq 1\,(\Omega_m \simeq 0)$. Although the Universe has not yet reached this state, it is rapidly approaching it. Eqn (2.11) gives $H^2 = \frac{\Lambda c^2 \pi G}{3} = \text{const}$, so (3.24) gives

$$\ddot{\delta} + 2H\dot{\delta} = 0 \tag{3.28}$$

whose solution is

$$\boxed{\delta = A(\overline{x}) + B(\overline{x})e^{-2Ht}} \tag{3.29}$$

Thus, the growing mode is no longer growing, but has frozen in at a constant value.

• General case; $\Omega_m + \Omega_\Lambda = 1$. We will not attempt to solve this here, but a feel for the solution may be obtained by combining the two previous results. We write:

$$\delta = \delta_0(\overline{x})ag(a, \Omega_{m0}) \tag{3.30}$$

where $g(a, \Omega_{m0})$ is a correction factor that modifies the simple scaling of the $\Omega = 1$ case. The factor g is constant at early times ($\Omega \to 1$) and scales as $1/a$ at late times ($\Omega_\Lambda \to 1$). For $\Omega_m = 1/3$, one can show that $g \to g_i = 1.25$

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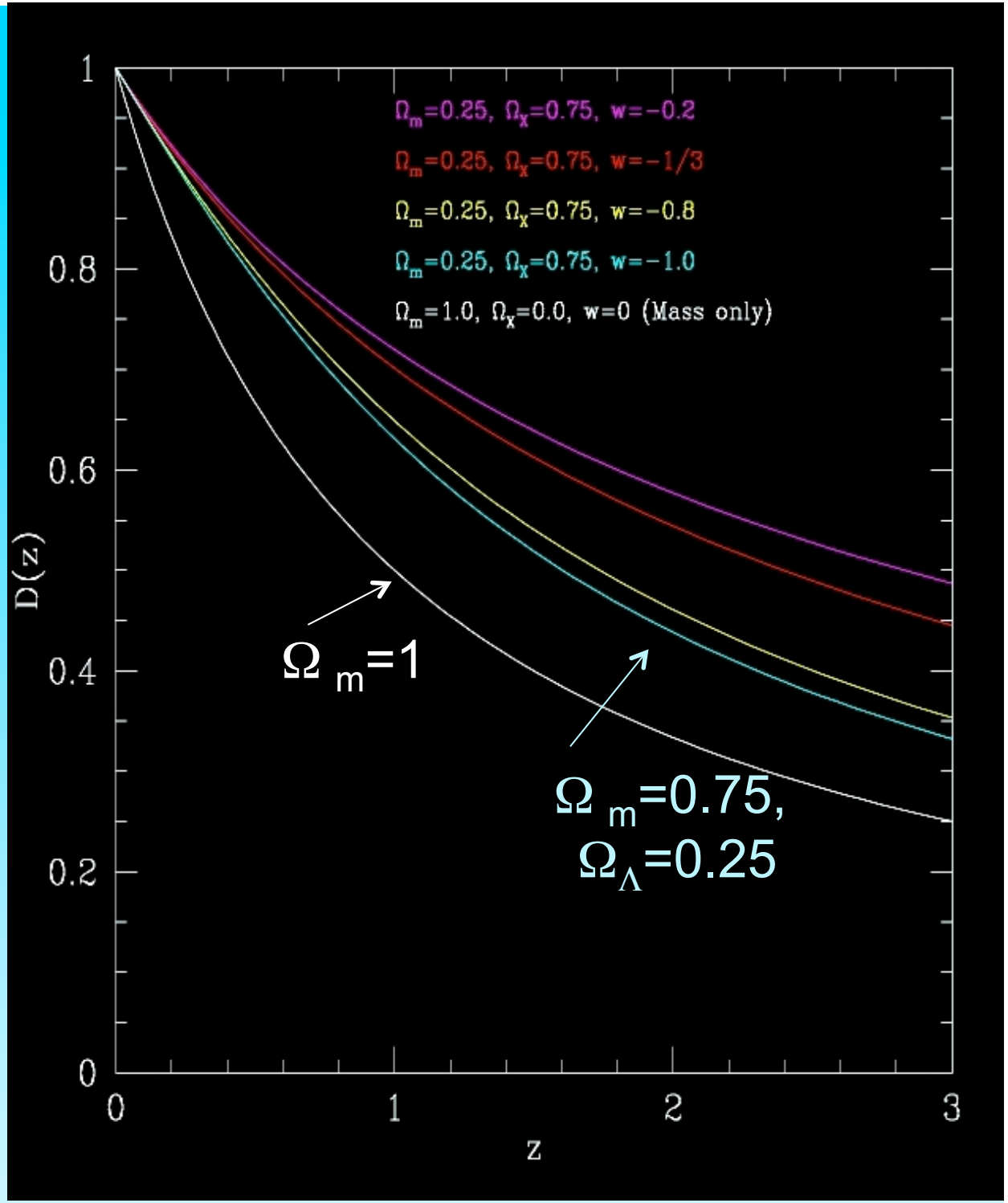
- Linear fluctuation growth rate in different cosmologies



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3.4 The Jeans mass

Consider fluctuations in the baryon distribution which we will take to be a perfect fluid. Assume that the fluctuations are adiabatic, ie that the cooling time is long compared with the age of the Universe. Then, $\delta P = c_s^2 \delta\rho$, where c_s is the sound speed. The pressure is greater in overdense regions and this impedes the growth of the fluctuation. The Jeans mass is derived by comparing the relative strength of gravitational and pressure forces acting on a perturbation. If, for an adiabatic perturbation, we write:

$$\delta P \propto \delta\rho \propto e^{i\overline{k}\cdot\overline{x}} \tag{3.31}$$

then,

$$\nabla_r^2 \delta P = \frac{1}{a^2}\nabla_x^2 \delta P = \frac{c_s^2}{a^2}\nabla_x^2 \delta\rho = -\frac{k^2 c_s^2}{a^2}\delta\rho \tag{3.32}$$

Substitute in (3.20), setting $\delta\rho = \overline{\rho}\delta$:

$$\ddot{\delta} + 2H\dot{\delta} = -\frac{k^2 c_s^2}{a^2}\delta + 4\pi G\left[\overline{\rho}\delta + 3\left(\frac{c_s}{c}\right)^2 \overline{\rho}\delta\right]$$

$$\ddot{\delta} + 2H\dot{\delta} = \left(4\pi \mathrm{G}\overline{\rho} - \frac{k^2 c_s^2}{a^2}\right)\delta \tag{3.33}$$

Fluctuations can only grow if RHS is positive. This can be seen, for example, by trying a solution of the form $\delta \propto e^{iwt}$. The resulting algebraic equation is called the dispersion relation. For fluctuations to grow, w must be imaginary and this requires the RHS of the equation to be positive. Thus fluctuations grow only if their physical wavelength, $\lambda = \frac{2\pi a}{k}$, is greater than the Jeans wavelength:

$$\lambda_J = c_s\left(\frac{\pi}{\mathrm{G}\overline{\rho}}\right)^{1/2} \tag{3.34}$$

corresponding to masses greater than the Jeans mass:

$$\boxed{M_J = \frac{4\pi}{3}\overline{\rho}_m\left(\frac{\lambda_J}{2}\right)^3 = \frac{1}{6}\pi\overline{\rho}_m\left[c_s\left(\frac{\pi}{G\overline{\rho}}\right)^{1/2}\right]^3} \tag{3.35}$$

• At the epoch of matter-radiation equality, z_{eq}, $\rho_{nr} = \rho_{nr0}(a+z)^3$ is equal to $\rho_r = \rho_{r0}(a+z)^4$. This then gives $z_{eq} = 2.6 \times 10^4 \Omega_0 h^2$. At this epoch, the sounds speed is $\left(\frac{\partial P}{\partial \rho}\right) = \frac{c}{\sqrt{3}}$ and eqn (3.35) gives,

$$M_J = 3.5 \times 10^{15}(\Omega h^2)^{-2}\,\mathrm{M}_\odot \tag{3.36}$$

• Using values appropriate for the period after recombination (and recalling that $\overline{\rho} \propto \rho \propto a^{-3}$ and $T \propto c_s^2 \propto a^{-1}$) gives:

$$M_J \simeq 5a^{-3/2}\,\mathrm{M}_\odot \tag{3.37}$$

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3.5 The Meszaros effect

A perturbation in a *collisionless*, *non-relativistic* matter component (eg cold dark matter) experiences reduced growth during the period when the Universe is dominated by a relativistic component, eg radiation. To see this, let us consider eqn (3.20) for a non-relativistic matter component, which we will take to have $P = \delta P = 0$ (i.e. cold dark matter). The total mean density is $\overline{\rho}_{tot} = \overline{\rho}_{nr} + \overline{\rho}_r$ (with $\overline{\rho}_r > \overline{\rho}_{nr}$). The perturbed (non-relativistic) density is $\delta_{nr} = \delta\overline{\rho}_{nr}$. The relativistic (radiation) field is assumed to be uniform, $\delta_r = 0$, so $\delta\rho_{tot} = \delta_{nr} = \delta\overline{\rho}_{nr}$. Eqn.(3.20) then becomes (droping the overbars in $\overline{\rho}$, etc for clarity):

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi \mathrm{G}\rho_{nr}\delta = 0 \tag{3.38}$$

Changing variables to

$$y = \frac{\rho_{nr}}{\rho_r} = \frac{a}{a_{eq}} \tag{3.39}$$

where a_{eq} is the expansion factor at the epoch of matter - radiation equality. Using the Friedmann equation (2.6) with $k = 0$, $\Lambda = 0$ and $\rho = \rho_{nr} + \rho r$ and setting $\frac{d}{dt} = \dot{a}\frac{d}{da}$, eqn (3.37) becomes:

$$\frac{d^2\delta}{d^2y} + \frac{2+3y}{2y(1+y)}\frac{d\delta}{dy} - \frac{3y}{2y(1+y)} = 0. \tag{3.40}$$

which, as usual, has a growing and a decaying solution. The growing solution is

$$\delta_+ \propto 1 + \frac{3}{2}y \tag{3.41}$$

Before z_{eq} ($y < 1$), the growing mode is practically frozen. The total growth in the interval $t = 0 - t_{eq}$ is

$$\boxed{\frac{\delta_+(y=1)}{\delta_+(y=0)} = \frac{5}{2}} \tag{3.42}$$

After z_{eq}, the solution rapidly matches the growth rate in a matter-dominated Einstein-de-Sitter model,

$$\delta_+(y >> 1) \propto y \propto a \propto t^{2/3} \tag{3.43}$$

Physically, the explanation for the Meszaros effect is this. At early times, the dominant energy in radiation drives the Universe to expand so fast that the matter has no time to respond and δ is frozen. As the radiation becomes negligible, growth increases smoothly to the behaviour appropriate in an Einstein-de-Sitter universe.

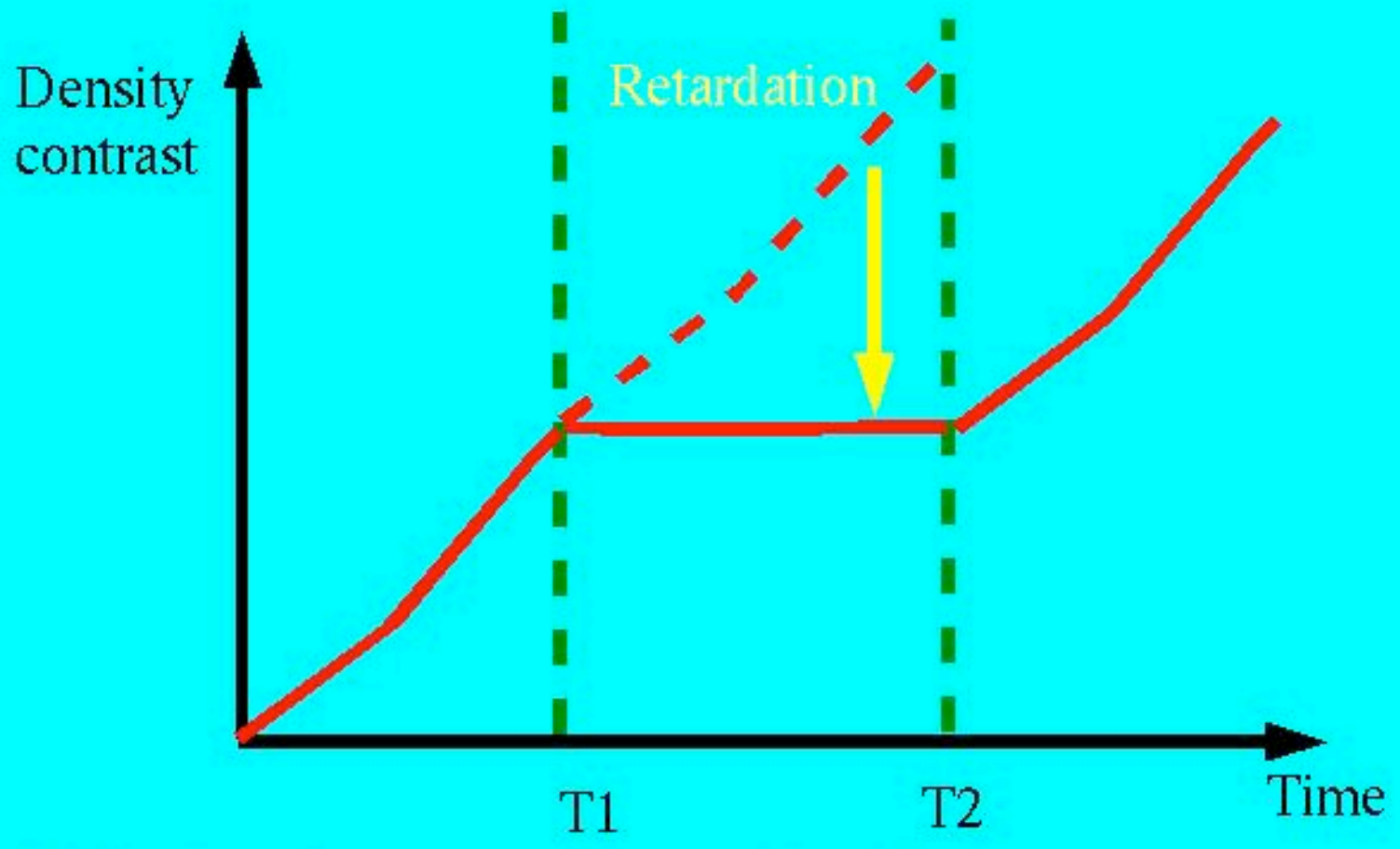
3.6 Free streaming

If a perturbation enters the horizon when the dark matter is still relativistic, then that perturbation is quickly damped because of *free streaming*: the particles which are moving at relativistic speeds in 3D simple move out of the high density part of the perturbation which is then erased. For thermal particles, the cut-off wavelength is,

$$\boxed{\lambda_{\mathrm{FS}} \propto m_X^{-1}} \tag{3.44}$$

where m_X is the dark matter particle mass

Evolution of a density perturbation (dark matter)

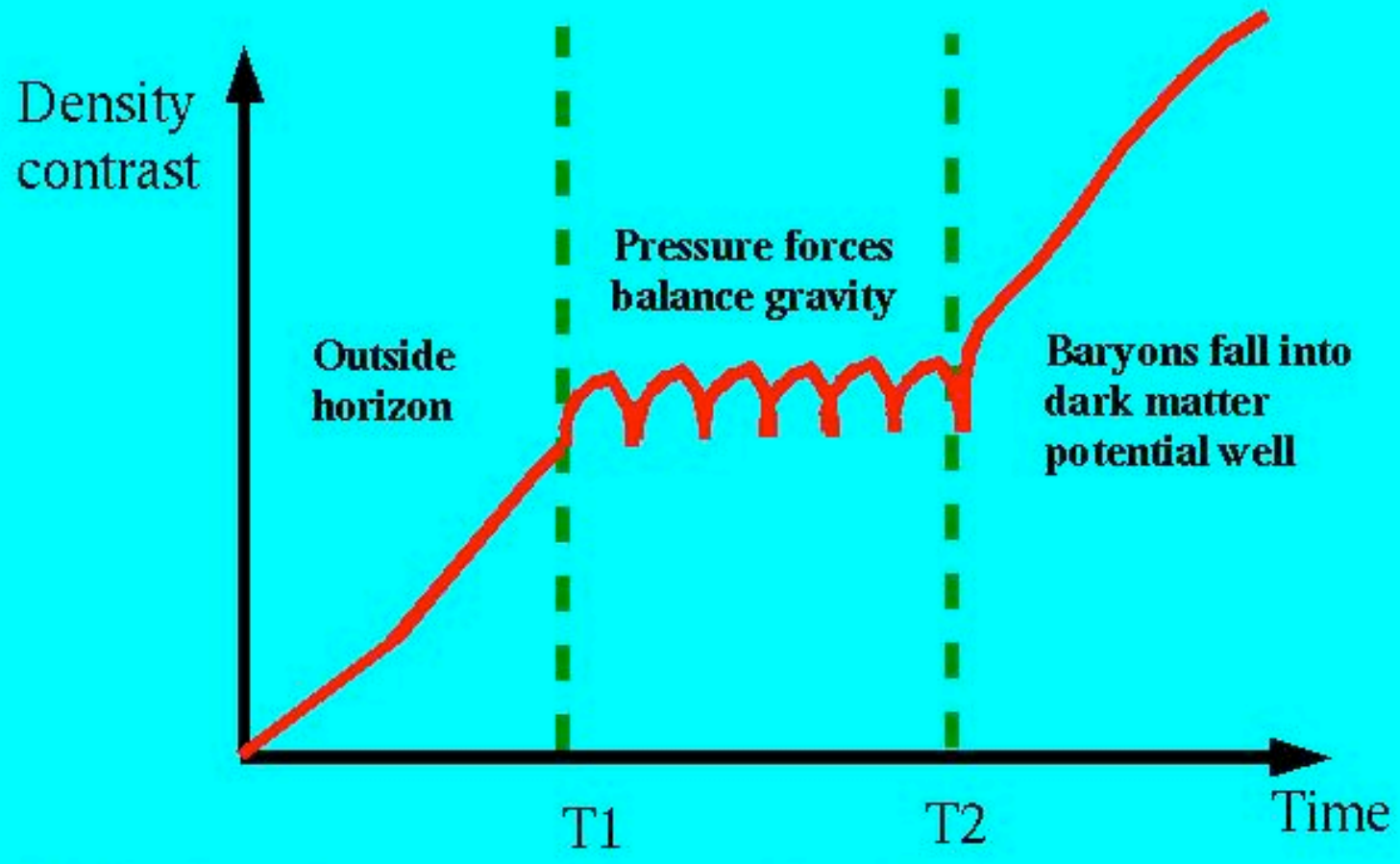


T1: horizon has expanded to enclose fluc'n

T2: epoch of matter-radiation equality

T1<t<T2 fluc'ns in radiation density and expansion of universe stop growth of density contrast

Evolution of a density perturbation (baryons)



T1: horizon has expanded to enclose fluc'n

T2: epoch of recombination: atoms form, photon pressure drops

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5. THE POWER SPECTRUM OF DENSITY FLUCTUATIONS

In this section, we collect the results of the preceeding section to arrive at a simple statistical description of the density and velocity fluctuation fields.

5.1 Mathematical background

The density field, $\delta(\overline{x}, t)$, can be written as a sum of plan waves in Fourier space:

$$\delta(\overline{x}, t) = \frac{1}{(2\pi)^3} \int \delta_k(\overline{k}, t) e^{i\overline{k}\cdot\overline{x}} d^3k$$

$$\delta_k(\overline{k}, t) = \int \delta(\overline{x}, t) e^{-i\overline{k}\cdot\overline{x}} d^3x \tag{5.1}$$

The fact that $\delta(\overline{x}, t)$ averages to zero, $< \delta(\overline{x}, t) >= 0$, implies that $\delta_k(0) = 0$.

δ_k is usually assumed to be a *Gaussian random field* which means that the waves have random phases. In this case, the field can be specified entirely by its variance or power spectrum:

$$\boxed{P(\overline{k}) \equiv \left|\delta_k(\overline{k})\right|^2} \tag{5.6}$$

For an isotropic distribution, the power spectrum, averaged over all possible realizations, must be independent of direction, $< P(\overline{k}) >= P(k)$. Often, $P(k)$ is approximated as

$$P(k) \propto k^n \tag{5.7}$$

In practice, $P(k)$ is not a power-law, ie n varies with scale.

The density and mass fluctuation within a region of volume V is defined as:

$$\frac{\delta\rho}{\rho} = \frac{\delta M}{M} = \frac{1}{V} \int \delta(\overline{x}) d^3x \tag{5.8}$$

If δ_k is Gaussian, so is $\delta M/M$. The average density fluctuation (averaged over the whole of space) is zero:

$$\left\langle \frac{\delta M}{M} \right\rangle = 0 \tag{5.9}$$

It is (relatively) easy to find the mean squared value of $\delta M/M$:

$$\boxed{\sigma^2(M) = \left\langle \left(\frac{\delta M}{M}\right)^2 \right\rangle = \frac{1}{(2\pi)^3 V_u} \int P(k) W(\overline{k}; V) d^3k} \tag{5.10}$$

$$\sigma^2 \propto \int_0^{\frac{1}{x}} k^n d^3k \propto \left(\frac{1}{x}\right)^{3+n} \propto M^{-\left(\frac{3+n}{3}\right)} \tag{5.14}$$

since $M \propto x^3$, provided $n > -3$. We will express the present-day value of σ, assuming linear evolution, as:

$$\sigma_0 \equiv \left(\frac{M}{M_0}\right)^{-\alpha} \qquad \alpha = \frac{3+n}{6} \tag{5.15}$$

This gives the relative amplitude of fluctuations on different scales according to linear theory. Of course, σ scales with time in the same way as δ, ie as in (4.14):

$$\sigma = \sigma_0 g a \tag{5.16}$$

5.2 Dark matter power spectra

Density perturbations are thought to be generated by quantum fluctuations during inflation. These are adiabatic, Gaussian perturbations. As the universe inflates, the perturbation is stretched beyond the horizon. Once inflation ends, however, perturbations eventually re-enter the horizon. By self-similiarity arguments, when they do so, they all have the *same amplitude*, independently of wavelength (or mass). Smaller fluctuations re-enter the horizon first.

Let us compute the shape of the power spectrum at some time, say, t_{rec}. Consider a fluctuation that enters the horizon in the matter-dominated regime. Since $\sigma \propto \delta$, $\sigma_H/\sigma_{rec} = a_H/a_{rec}$, where the subscript H refers to horizon crossing. Thus,

$$\sigma_{rec} = \sigma_H \frac{a_{rec}}{a_H} = \sigma_H \left(\frac{t_{rec}}{t_H}\right)^{2/3} \tag{5.17}$$

where σ_H =const (ie independent of mass). During this time, the horizon mass scales as

$$M_H \propto \overline{\rho}_H (ct_H)^3 \propto t_H \tag{5.18}$$

because $\overline{\rho} \propto t^{-2}$ in the matter-dominated regime. Since at a_{enter}, $M \simeq M_H$,

$$\sigma_{rec} \propto t_H^{-2/3} \propto M^{-2/3} \tag{5.19}$$

Comparing with (5.15), $\frac{3+n}{6} = \frac{2}{3}$, so

$$\boxed{\alpha = \frac{2}{3} \qquad \text{or } n = 1} \tag{5.20}$$

This is called the Harrison-Zeldovich spectrum, $\boxed{P(k) \propto k.}$

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•Non-baryonic dark matter candidates

• Type	candidate	mass
hot	neutrino	a few eV
warm	Sterile neutrino	keV-MeV
cold	axion neutralino	10^{-5}eV- >100 GeV

It is usual to write

$$\boxed{P(k) = T^2(k)k} \tag{5.21}$$

where $T(k)$ is the *transfer function*.

Cold dark matter (CDM).

Density fluctuations cannot grow during the radiation-dominated era and so the power spectrum is maintained at the value at horizon crossing. Thus $\sigma \to \text{const}$ and, from (5.16), $\alpha \to 0$, so $n \to -3$.

For cold dark matter, a fitting formula is:

$$T_{\text{CDM}}(k) = \frac{\ln(1+2.34q)}{2.34q}[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{-1/4} \tag{5.22}$$

where

$$q = \frac{k}{\Gamma}h^{-1}\text{Mpc} \tag{5.23}$$

and the *shape parameter* Γ:

$$\Gamma = \Omega_{m0}h \tag{5.24}$$

Hot dark matter (HDM)

This corresponds to the case where the dark matter particles are still relativistic at matter-radiation equality. In this case, the dark matter particles free-stream out of the perturbations as soon as they come into the horizon and so fluctuations are erased until such time as the particles cease to be relativistic. The cut-off wavelength is,

$$\boxed{\lambda_{\text{FS}} \propto m_X^{-1}} \tag{5.25}$$

where m_X is the mass of the hot dark matter particle. A fitting formula for HDM is:

$$\Gamma_{\text{HDM}}(k) \simeq e^{-3/9q - 2.1q^2} \tag{5.26}$$

The power spectrum (or, more precisely, $k^3|\delta_k|^2$) for CDM and HDM is shown in Figure 1. Accurate transfer functions may be obtained using the programme on

http://www/physics.nyu.edu/matiasz/CMBFAST/cmbfast.html

The origin of cosmic structure

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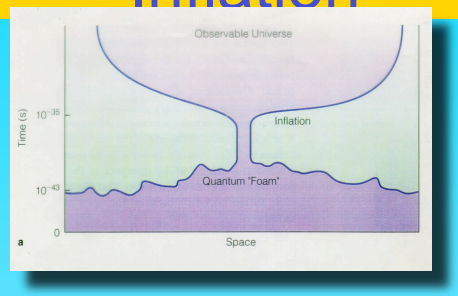
•Inflation



•→ QUANTUM FLUCTUATIONS:

$|\delta_k|^2 \propto k^n$ •n ≈1

Gaussian amplitudes

•

+

•Damping (nature of dark matter)

•→ FLAT UNIVERSE

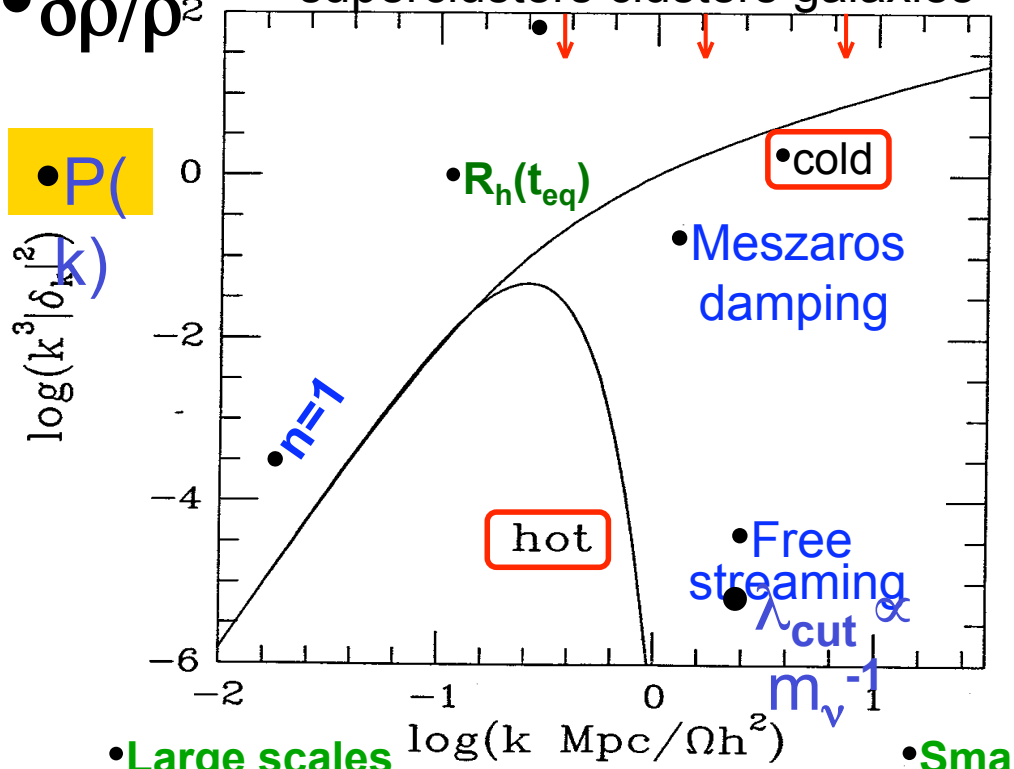
•$P(k)=Ak^n$

$T^2(k,t)$ → •Transfer function

•$\delta\rho/\rho^2$

•superclusters clusters galaxies

•P(k)



•Large scales

•Small scales

- Hot DM (eg ~30 ev neutrino)
  - Top-down formation
- Cold DM (eg ~neutralino)
  - Bottom-up (hierachical)

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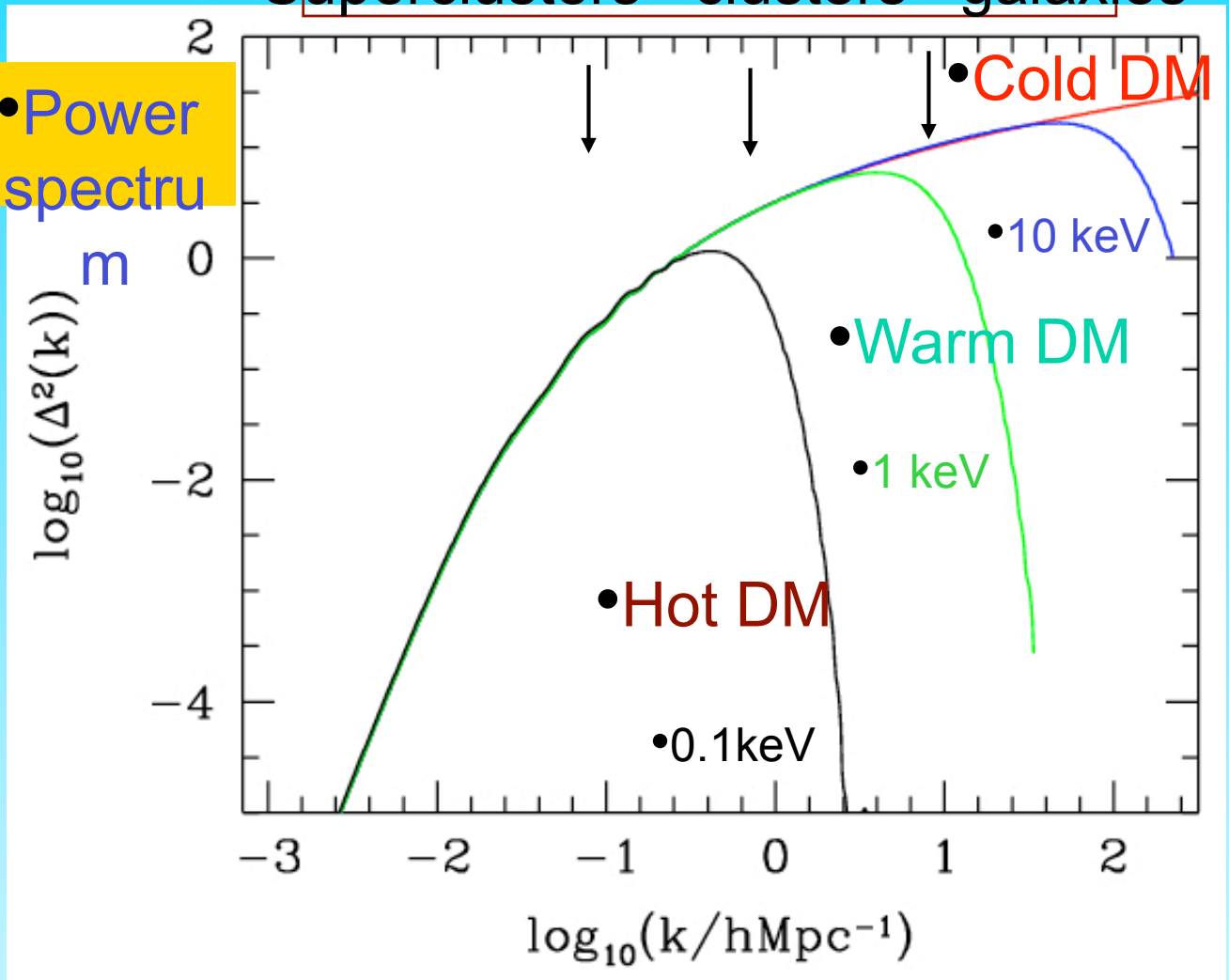
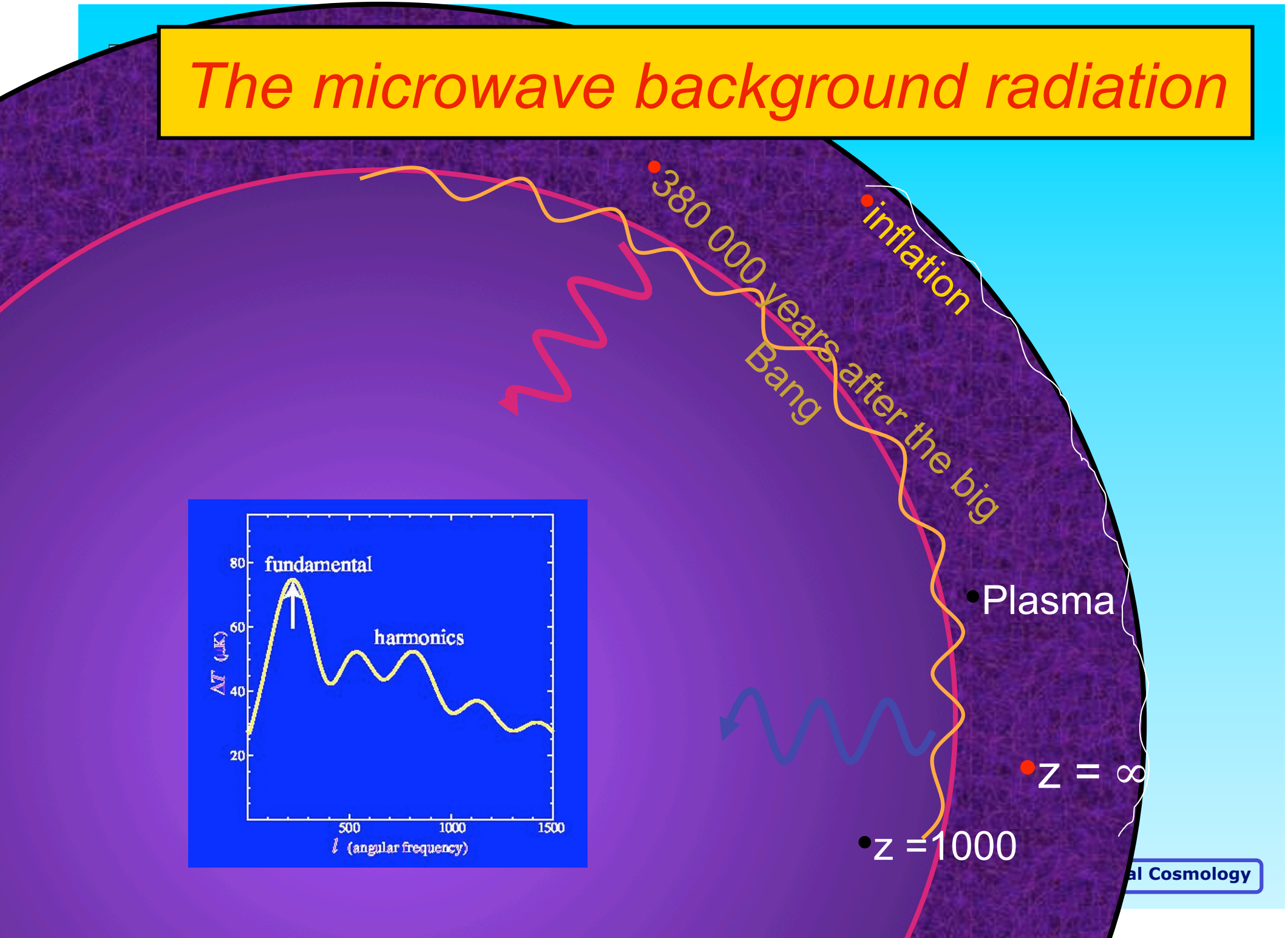
•Non-baryonic dark matter

•Superclusters clusters galaxies

•Power spectrum



The microwave background radiation

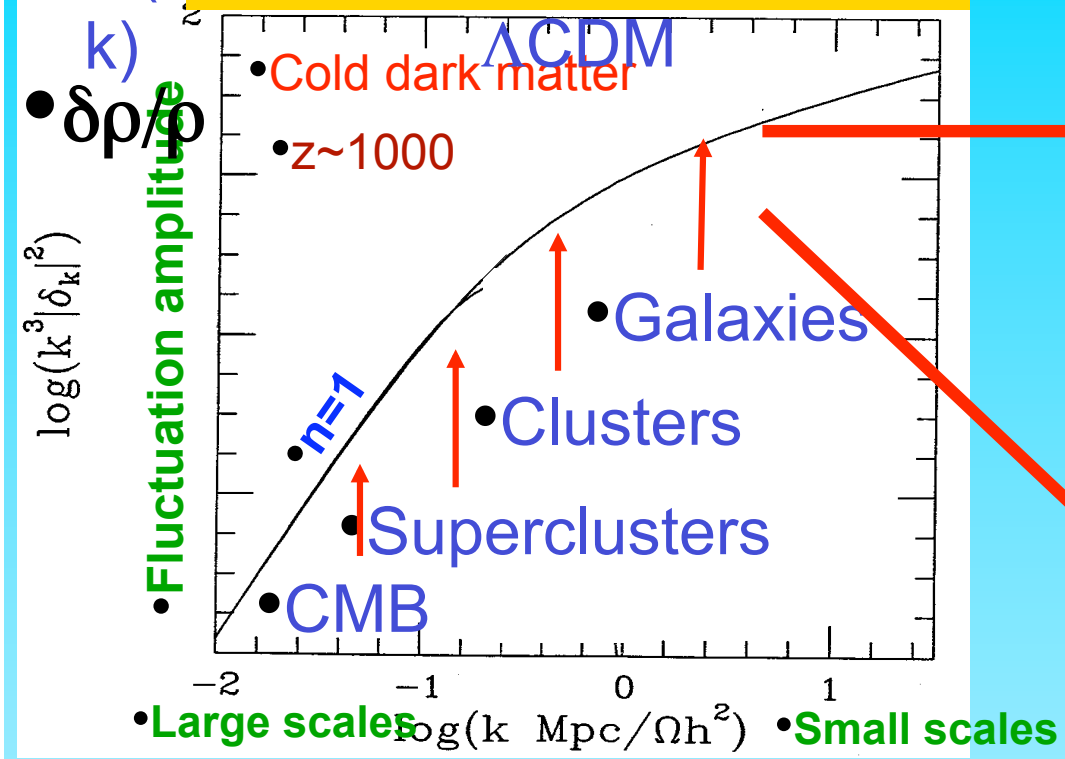


•Testing the CDM paradigm

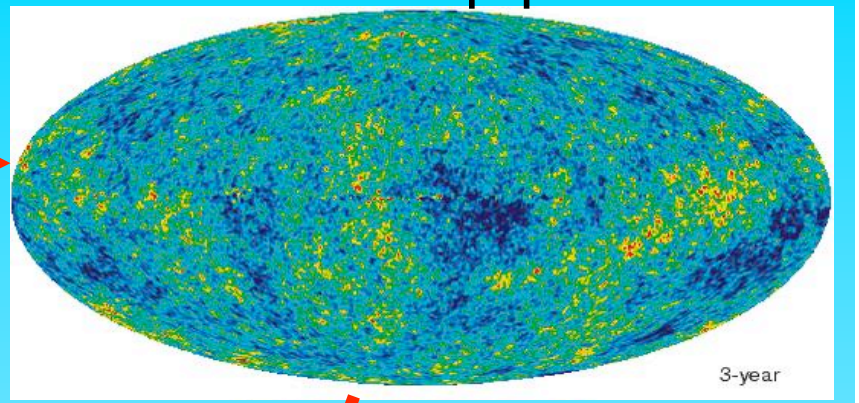
•P(•Initial conditions :

k)

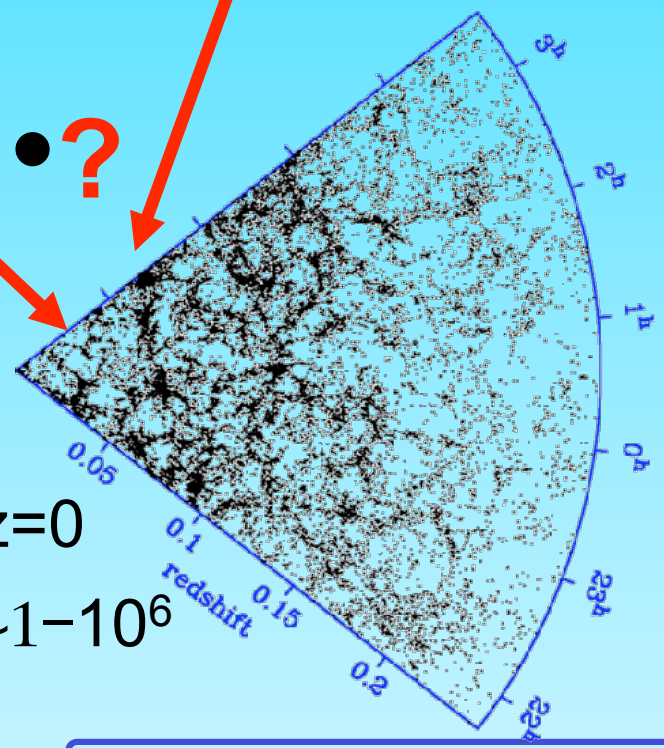
•δρ/ρ



•z=1000 •$\delta\rho/\rho \sim 10^{-5}$



•?



•z=0

•$\delta\rho/\rho \sim 1-10^6$

• The galaxy distribution evolves from fluctuations seen in CMB by gravitational amplification.

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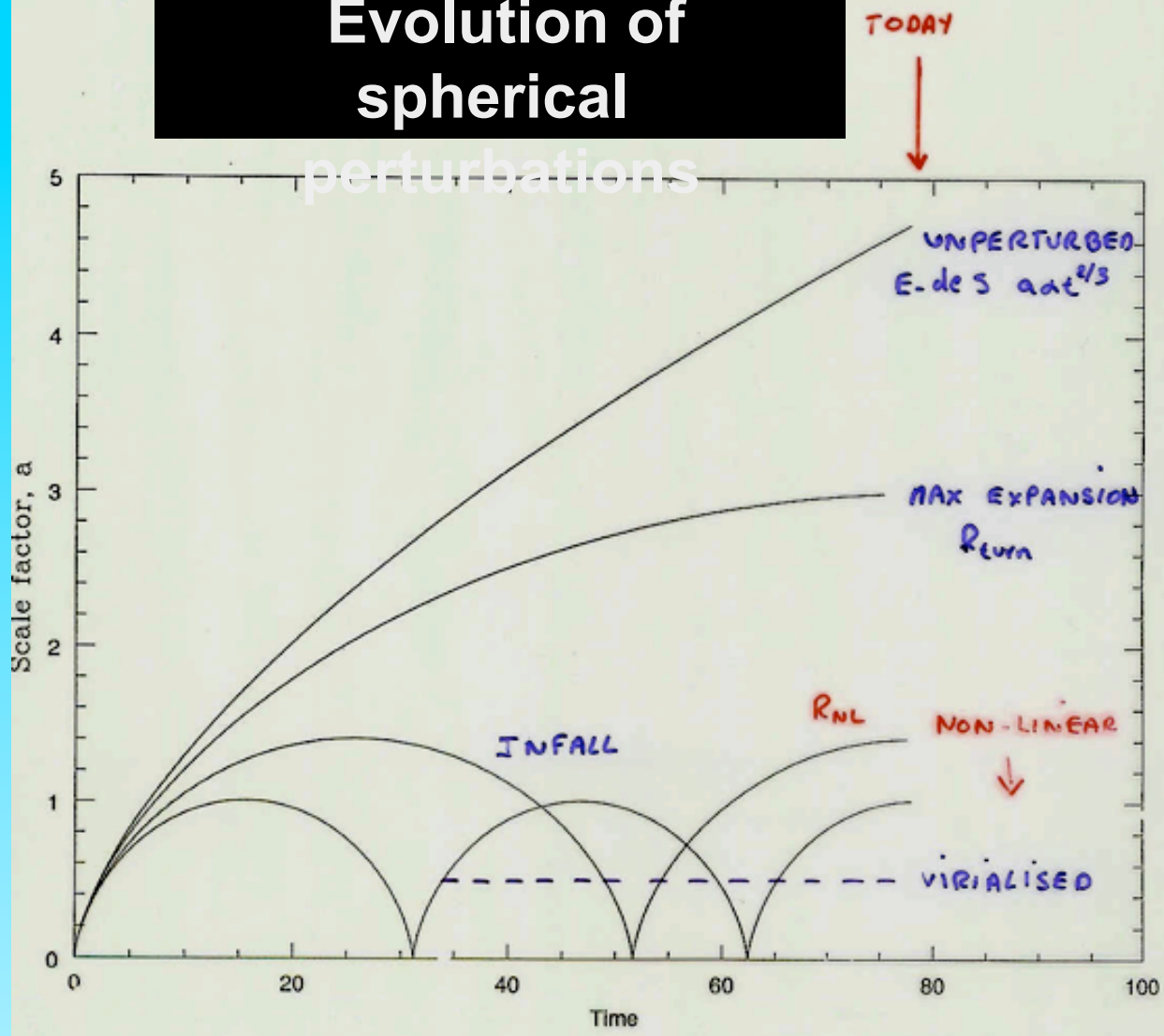
Evolution of spherical perturbations

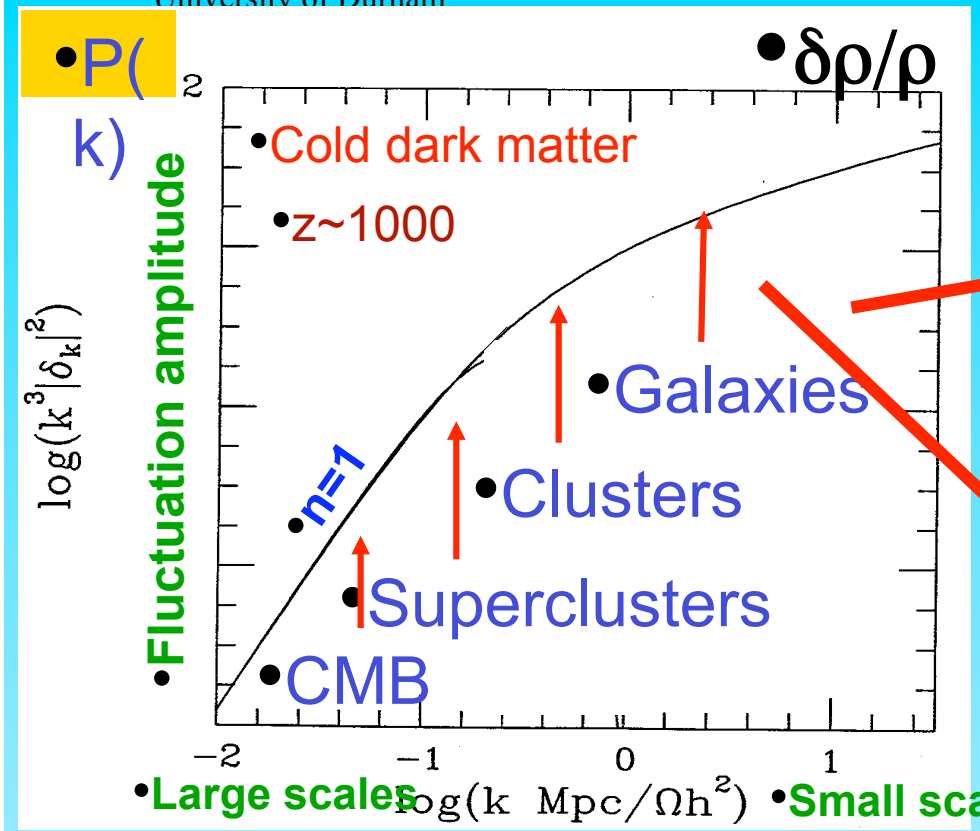


- R_{NL} SEPARATES UNMIXED, UNSHOCKED (PRIMORDIAL) MATERIAL FROM MIXED, SHOCKED, NON-LINEAR MATERIAL
- IN E-d-S, R_{NL} CORRESPONDS TO $\delta\rho/\rho \sim 100$

•Testing the CDM paradigm

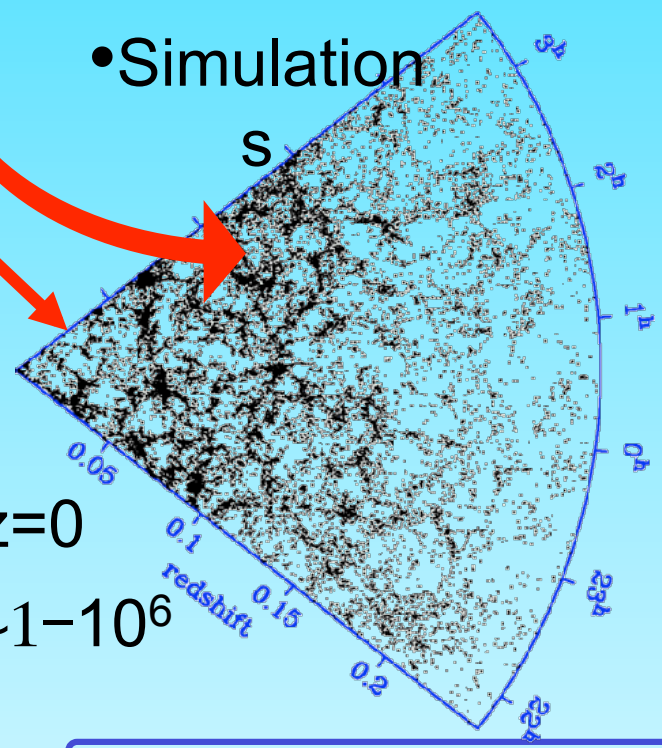
•P(k)

•δρ/ρ



•"Cosmology machine"

•Simulations



•Initial conditions : ΛCDM

•Basic DM physics simple: structure grows primarily by gravity

•z=0

•$\delta\rho/\rho \sim 1-10^6$

→ •N-body simulations

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