Gravitational wave cosmology
Lecture 3

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Outline

- Lecture 1: introduction to gravitational waves
- Lecture 2: detecting gravitational waves
- Lecture 3: what we might learn from gravitational waves
GW science reach

![Gravitational Wave Amplitude vs Frequency Graph]

- Coalescence of Massive Black Holes
- Resolved Galactic Binaries
- Unresolved Galactic Binaries
- NS−NS and BH−BH Coalescence
- SN Core Collapse

Frequency [Hz]

Gravitational Wave Amplitude

- LISA
- LIGO
Listening to the Universe

GWs are like sound, not light:

- detectors are omnidirectional
- detectors don’t image
- GWs from bulk, not surface, processes
- phase coherent
- weak
- difficult to scatter/absorb
- frequencies of stellar mass events occur in human auditory band
What will we hear?
Lots of sources I won’t discuss

- Quasi-normal ringdown modes of black holes
- Extreme mass ratio inspiral
- Intermediate mass ratio inspirals
- Testing GR

Good review:
Sathyaprakash & Schutz 2009
Living Reviews in Relativity
GWs from inflation

- Quantum fluctuations (tensor in addition to scalar) in the early universe are amplified by inflation. Subsequent phase transitions also might generate GWs.
- Once emitted, GWs travel (almost) unimpeded.
- If detected, give pristine measurement of the very, very early universe: $10^{-24}$ seconds, not 400,000 years, after the Big Bang.
- Directly measure the expansion rate during inflation.
GWs from inflation

- Show up as Gaussian noise
- Noise is correlated for detectors separated by less than relevant wavelength
- Use correlations to detect background (assuming instrumental noise is uncorrelated)
- Energy density/strain noise in GWs in a frequency bin:

\[
\Omega_{GW} = \frac{1}{\rho_c} \frac{d \rho_{GW}}{d \ln f} \quad S_{GW}(f) = \frac{3H_0^2}{10\pi^2} f^{-3} \Omega_{GW}(f)
\]

\[
S_{GW}(f)^{1/2} = 5.6 \times 10^{-22} \Omega_{GW}^{1/2} \left( \frac{f}{100 \text{ Hz}} \right)^{-3/2} h_{100} \text{ Hz}^{-1/2}
\]
GWs from inflation
Supernovae

- Must be asymmetric!
- Large theoretical uncertainties
- Must be close (preferably in our galaxy)
Compact binary coalescence

- Most promising source
- Neutron star and/or black hole binary coalescence

- Stellar mass systems merge in LIGO band
- Systems are strong gravitational wave emitters
- Advanced LIGO will see such systems to cosmological distances (>300 Mpc)
How many binaries will LIGO detect?

- Population synthesis
  - Stellar Initial Mass Function
  - Cosmological star formation rate
  - Binary synthesis/individual stellar evolution
  - Natal kicks, Common envelope evolution, ...

- Prediction: 40 events per year in advanced LIGO
- Uncertainty: ~2 orders of magnitude!
Multi-messenger astronomy

- Combine GW & EM (and neutrinos? cosmic rays?)
- GW: teaches us about the physics
  - measure masses, spins, geometry
- EM: teaches us about the astrophysics
  - measure energy, baryonic timescale, beaming, environment
- Need both GW+EM to fully understand relativistic sources
- GW+EM help identify sources (in either direction)
Most promising GW+EM source: Short/hard Gamma-ray burst

- GRBs definitely exist
  - ~1/day in the Universe
- GRBs are very bright/relativistic
- GRBs can be detected “all sky” throughout the Universe
- GRBs have been observed “nearby”
- Some long, and some short (2 second divide)
Short GRBs are (almost certainly) binary systems

- Deep optical followup of GRBs does not show evidence for supernovae/stellar collapse
- GRBs are not associated with star formation
- GRBs are found far from the centers of their host galaxies
- Timescales are consistent
- Simulations produce GRBs

Koppitz & Rezzolla
Listening to a gamma-ray burst

sound courtesy of Sam Finn (PSU)
Short GRBs have happened at low redshift ($z < 0.2$)

Within range of advanced LIGO!
Short GRBs are beamed

- Detailed observations of the afterglows of short GRBs (Swift, XMM, Chandra, optical, radio)

- Jet breaks are found, and imply beaming:
  - GRB 051221A: $\theta_j \approx 7^\circ$
  - GRB 111020A: $\theta_j \approx 3-8^\circ$
  - GRB 130603B: $\theta_j \approx 4-8^\circ$
Multi-messenger astronomy with GRBs

- Probe the inner engine of GRBs
  - binary progenitors? NS-NS vs NS-BH? masses?
  - beaming angles: measures total energy
- Properties of neutron star stuff
- Macronovae/r-process elements
- Event rates
  - predictions for LIGO
  - constraints on star formation and evolution
- Cosmological measurements
GWs and GRBs

- If GRBs are binary systems, can observe them in EM (as GRBs) and in GW (as binary inspirals)

- Observables:
  - Rate of short GRBs (in gamma-rays):
    \[ \mathcal{R}_{\text{GRB}} \gtrsim 10 \text{ yr}^{-1} \text{ Gpc}^{-3} \]
  - Beaming angle of short GRBs: \( \theta \)
  - Rate of GRB progenitors (in GWs):
    \[ \mathcal{R}_{\text{GW}} = \mathcal{R}_{\text{GRB}} / (1 - \cos \theta) \]
LIGO limits on GRB beaming

- LIGO S6/V2 didn’t see any binaries: constrains beaming
How long will LIGO have to wait?

- First binary within ~6 months for LIGO (HL; Hanford + Livingston), ~1 month for LIGO + Virgo (HLV)
- Best estimate for LIGO detection rate!

Chen & Holz 2013 PRL
To trigger or not to trigger?

- **Trigger**: (see GRB and GWs at the same time)
  - provides time and sky position: drastic improvement in GW search algorithm, and hence lower detection threshold
  - probably face-on, thus stronger signal

- **Untriggered**: (don’t see GRB in EM spectrum)
  - don’t need the gamma-rays to be pointing at us, so much higher space density
High rates. No trigger needed.

- Early sensitivity: ~10 events/year
- Mature sensitivity: ~50 events/year
- Will see untriggered before triggered
GW standard sirens

- Black holes are “simple”: they have no hair
- Binary black hole inspirals are well-modeled
- Binary black hole inspirals are understood from first principles

Schutz 1986, Nature
Dalal, DH, Hughes, & Jain 2006, PRD
Cutler and DH 2009, PRD
GWs from binary systems

- Strongest harmonic (widely separated):
  \[ h(t) = \frac{M_z^{5/3} f(t)^{2/3}}{D_L} F(\text{angles}) \cos(\Phi(t)) \]

- Dimensionless strain \( h(t) \)
- Luminosity distance \( D_L \)
- Accumulated GW phase \( \Phi(t) \)
- GW frequency \( f(t) = \frac{1}{2\pi} \frac{d\Phi}{dt} \)
- Position & orientation dependence \( F(\text{angles}) \)
- (Redshifted) chirp mass:
  \[ M_z = (1 + z) \left( m_1 m_2 \right)^{3/5} / \left( m_1 + m_2 \right)^{1/5} \]
Distance, but not redshift

- Gravitational waves provide a direct measure of luminosity distance, but they give no independent information about redshift.

- Gravitation is scale-free.
  - GWs from a local binary with masses $\left( m_1, m_2 \right)$ are indistinguishable from masses $\left( \frac{m_1}{1 + z'}, \frac{m_2}{1 + z} \right)$ at redshift $z$

- To measure cosmology, need independent measurement of redshift:
  - electromagnetic counterpart
What good is a counterpart?

- Determination of redshift
  - puts a point on the luminosity distance-redshift curve
- Precise location of GW source
  - drastic improvement in GW modeling, and hence distance determination
“Optical” counterpart?

- Roughly 5% of the system’s mass is being released in gravitational waves (~$10^{52/58}$ ergs)
- Even if only 1 part in $10^{10}$ of this available energy is converted into photons, the source would be easily visible at high redshift
- Need fantastic efficiency to remain “dark”
Potential standard sirens

- **LIGO**: stellar-mass binaries
- **LISA**: supermassive binary black holes
- **BBO**: stellar-mass binaries
LIGO standard sirens?:
LIGO standard sirens:

Short gamma-ray bursts!
Gamma-ray Burst Standard Sirens

- Short GRBs are known to occur at low redshift \( z < 0.2 \)
- Short GRBs are thought to be the result of binary mergers (NS or BH)
- Will be seen by aLIGO. Perfect standard siren!

Systematic “free” absolute distance

Dalal, DH, Hughes, & Jain 2006, PRD
Short gamma-ray bursts are perfect standard sirens

- Very bright in EM and GW
- Happen frequently
- Happen nearby
- Time of burst improves parameter estimation
- Can identify redshift from host galaxy

No distance ladder. Provides absolute distance
How well do GRB standard sirens measure distance?

- Markov-Chain Monte Carlo code
- Non-spinning restricted 3.5 post-Newtonian waveform
- Detection priors in population selection
- Independent interferometric noise realizations
- Advanced GW detector configurations

Measuring the Hubble constant

- Hubble: the overall scale of the Universe
- advanced LIGO/Virgo
- 15 isotropic NS-NS binaries
- assuming GW+EM: standard sirens
- distributions are non-Gaussian
- 3% measurement of $H_0$

Nissanke et al. 2013
Measuring the Hubble constant

add Japan+India: factor ~2
if GRBs are beamed: factor >2
NS-NS->NS-BH: factor ~4
Precision cosmology from GWs!

- Measure the Hubble constant to the few percent level

Nissanke et al. 2013
Golden binaries

- Most events are at detection threshold
- No extinction, obscuration, or evolution
- Can predict the full distribution of signal strengths
- Can predict the distribution of the strongest events

The loudest events provide the most physics!
GW strength in a LIGO detector

- **Signal-to-noise ratio** (restricted, first order, stationary phase):

\[
\text{SNR}^2 = 4 \frac{A^2}{D_L^2} \left[ F_+^2(\theta, \phi, \psi)(1 + \cos^2 \iota)^2 + 4F_\times^2(\theta, \phi, \psi) \cos^2 \iota \right] I_7
\]

with (this color text doesn’t show up?!!!):

\[
A = \sqrt{\frac{5}{96}} \frac{c}{\pi^{2/3}} \left( \frac{G M z}{c^3} \right)^{5/6}
I_7 = \int_{f_{\text{low}}}^{f_{\text{high}}} \frac{f^{-7/3}}{S_h(f)} \, df
\]

- sky position, orientation, inclination: \((\theta, \phi), \psi, \iota\)
- luminosity distance: \(D_L\)
- antenna power patterns (2 polarizations): \(F_+, F_\times\)
- LIGO noise spectral density: \(S_h(f)\)
- (redshifted) chirp mass:

\[
\mathcal{M} = (1 + z)(m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}
\]
Universal distribution of SNR

- Most events at detection threshold, with a tail to louder events
- Predictable, analytic, universal distribution
Universal distribution of loudest events


- For 4 events, loudest has SNR > 14.5 (for threshold = 12; 90% likelihood). SNR > 31 for 40 events. SNR > 42 for 100 events.
The louder, the better

- Larger SNR means better parameter estimation
- The best out of 4 events reduces the sky localization from \( \sim 30 \ \text{deg}^2 \) to \( \sim 15 \ \text{deg}^2 \). Out of 40 events, the best one is localized to better than \( \sim 2 \ \text{deg}^2 \)
- Probability of finding counterparts increases dramatically
GW science reach

- Coalescence of Massive Black Holes
- Resolved Galactic Binaries
- Unresolved Galactic Binaries
- NS-NS and BH-BH Coalescence
- SN Core Collapse

Graph showing gravitational wave amplitude vs. frequency for LISA and LIGO.
Big Bang Observer

- BBO sees $\sim 10^5$ NS-NS binaries to $z \sim 5$
- BBO sky localization uniquely identifies host
  - Extraordinary measurement of luminosity distance-redshift relation
  - Extraordinary measurement of gravitational lensing (and hence structure formation)

Ultra-precise cosmology

Cutler & DH 2009
BBO is a fantastic cosmological probe
Thus far we’ve only seen the Universe (and 95% of it is dark: dark matter and dark energy). In the next few years we will finally be able to listen to the Universe. This will be revolutionary!