Testing GR with Cosmology

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The Large Scale Structure of the Universe

Planck

SDSS
Primordial Tilt

1992 (COBE): $n_s = 1 \pm 0.6$

2001 (Max+Boom): $n_s = 1.03 \pm 0.09$

2009 (WMAP5): $n_s = 0.963 \pm 0.014$

2013 (Planck+): $n_s = 0.9603 \pm 0.0073$
Outline

• the panorama of gravitation
• the cosmological arena
• cosmological linear perturbations
• what data to look at
• the future
Einstein Gravity

\[ \frac{1}{16\pi G} \int d^4 x \sqrt{-g} R(g) + \int d^4 x \sqrt{-g} \mathcal{L}(g, \text{matter}) \]

Lovelock’s theorem (1971): “The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant.”
“I think the best viewpoint is to pretend there are experiments and calculate. In this field we are not pushed by experiments- we must be pulled by imagination”

R. Feynman

GR1: Chapel Hill 1957
Modified Gravity

- New degrees of freedom
- Higher dimensions
- Higher-order
- Non-local

### Higher dimensions
- Strings & Branes
  - DGP
  - Horndeski theories
  - Cascading gravity
  - DGP
- Kaluza-Klein
- Higher-dimension gravity

### Non-local
- Horndeski theories
- 2T gravity
- Non-local
- General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.

### Higher-order
- Einstein-Cartan-Sciama-Kibble
- Chern-Simons
- Cuscuton
- Chaplygin gases
- Massive gravity
- TeVeS
- General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.

### Scalar
- Scalar-tensor & Brans-Dicke
- Ghost condensates
- Galileons
- the Fab Four
- KGB
- Coupled Quintessence
- Horndeski theories
- Scalar

### Vector
- Einstein-Aether
- Lorentz violation

### Tensor
- Bigravity
- Massive gravity
- Bimetric MOND

### Torsion theories
- Conformal gravity
- Einstein-Dilaton-Gauss-Bonnet
- Kaluza-Klein
- Generalisations of $S_{EH}$
- Strings & Branes
- Higher dimensions

### Einstein-Dilaton-Gauss-Bonnet
- Cascading gravity

### Generalisation of $S_{EH}$
- Strings & Branes
- Kaluza-Klein
- Higher dimensions

### New degrees of freedom
- TeVeS
- Scalar

### Lorentz violation
- Einstein-Aether
- Lorentz violation

### Lorentz violation
- Einstein-Aether
- Lorentz violation

### Tessa Baker 2013

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arXiv:
- 1310.1086
- 1209.2117
- 1107.0491
- 1110.3830

Tuesday, 14 January 14
Initial Conditions

Reionization ("EoR")

Dark ages

Recombination

---

Gran unification transition
- Electroweak and strong nuclear forces differentiate
- Inflation

Quantum gravity wall
- Spacetime description breaks down
Initial Conditions

Primordial Gravitational Waves

Primordial Tilt

Planck XXII

Planck + WP

Planck + WP + highL

Planck + WP + BAO

Natural Inflation

Power law inflation

Low Scale SSB SUSY

$R^2$ Inflation

$V \propto \phi^{2/3}$

$V \propto \phi$

$V \propto \phi^2$

$V \propto \phi^3$

$N_s = 50$

$N_s = 60$
Initial Conditions

Primordial Gravitational Waves

Primordial Tilt

Only 2 numbers

Planck XXII
Acceleration

Where strange things do happen...
Modified Gravity

New degrees of freedom

Higher dimensions

Non-local

Higher-order

Scalar\(\rightarrow\) Vector\(\rightarrow\) Tensor

Einstein-Dilaton-Gauss-Bonnet

Cascading gravity

Strings & Branes

Randall-Sundrum I & II

Kaluza-Klein

Generalisations of \(S_{EH}\)

Gauss-Bonnet

Lovelock gravity

DGP

2T gravity

Some degravitation scenarios

Higher-order

\(f(R)\)

\(f(G')\)

\(f(R_{\mu\nu}R^{\mu\nu}, \Box R, \text{etc.}\)

Lorentz violation

Einstein-Aether

Lorentz violation

Einstein-Aether

Massive gravity

Bigravity

EBI

Bimetric MOND

General R_{\mu\nu}R^{\mu\nu}, \Box R, \text{etc.}

Generalisations of \(S_{EH}\)

TeVeS

\(f(G)\)

Conformal gravity

Hoařava-Lifschitz

Horndeski theories

Torsion theories

Scalar-tensor & Brans-Dicke

Ghost condensates

Galileons

the Fab Four

KGB

Coupled Quintessence

Horndeski theories

Chern-Simons

Cuscuton

Chaplygin gases

Einstein-Cartan-Sciama-Kibble

f(T)

Tessa Baker 2013

arXiv:

1310.1086
1209.2117
1107.0491
1110.3830
An agnostic view: lessons from PPN

Expand around weak-field metric in a set of 10 parameters: \( \gamma, \beta, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4 \)

\[
-g_{00}(r) = 1 - \frac{2GM}{c^2r} + 2(\beta - \gamma) \left( \frac{2GM}{c^2r} \right)^2 \\
g_{rr}(r) = 1 + \gamma \frac{2GM}{c^2r}
\]

Perform similar expansion in non-GR theory. Map theory onto parameters.

E.g. in Brans-Dicke theory: \( \gamma = \frac{1 + \omega_{BD}}{2 + \omega_{BD}} \)
# Lessons from PPN

<table>
<thead>
<tr>
<th></th>
<th>( \gamma )</th>
<th>( \beta )</th>
<th>( \xi )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \alpha_3 )</th>
<th>( \zeta_1 )</th>
<th>( \zeta_2 )</th>
<th>( \zeta_3 )</th>
<th>( \zeta_4 )</th>
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<tbody>
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<td><strong>Einstein (1916) GR</strong></td>
<td>1</td>
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<td><strong>Bergmann (1968), Wagoner (1970)</strong></td>
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<td>( \beta )</td>
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<td>( c_0/c_1 - 1 )</td>
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<td><strong>Rastall (1979)</strong></td>
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<td>( a_2 )</td>
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<td><strong>Lee-Lightman-Ni (1974)</strong></td>
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<td><strong>Ni (1973)</strong></td>
<td>( a c_0/c_1 )</td>
<td>( b c_0 )</td>
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<td><strong>Rosen (1971)</strong></td>
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<td><strong>Yilmaz (1958, 1962)</strong></td>
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<td>-1</td>
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<td><strong>Page-Tupper (1968)</strong></td>
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<td>( \beta )</td>
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<td>-4 - 4( \gamma )</td>
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<td>-2 - 2( \gamma )</td>
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<td>( \zeta_4 )</td>
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<td><strong>Nordström (1913), Einstein-Fokker (1914)</strong></td>
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<tr>
<td><strong>Ni (1972) [flat]</strong></td>
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<td>1 - ( q )</td>
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<td>( \zeta_2 )</td>
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<td>( q )</td>
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<td><strong>Littlewood (1953), Bergman (1956)</strong></td>
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<td>-1</td>
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</table>
## Lessons from PPN

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bound</th>
<th>Effects</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma - 1$</td>
<td>$2.3 \times 10^{-5}$</td>
<td>Time delay, light deflection</td>
<td>Cassini tracking</td>
</tr>
<tr>
<td>$\beta - 1$</td>
<td>$2.3 \times 10^{-4}$</td>
<td>Nordtvedt effect, Perihelion shift</td>
<td>Nordtvedt effect</td>
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<tr>
<td>$\xi$</td>
<td>$0.001$</td>
<td>Earth tides</td>
<td>Gravimeter data</td>
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<tr>
<td>$\alpha_1$</td>
<td>$10^{-4}$</td>
<td>Orbit polarization</td>
<td>Lunar laser ranging</td>
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<tr>
<td>$\alpha_2$</td>
<td>$4 \times 10^{-7}$</td>
<td>Spin precession</td>
<td>Solar alignment with ecliptic</td>
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<tr>
<td>$\alpha_3$</td>
<td>$4 \times 10^{-20}$</td>
<td>Self-acceleration</td>
<td>Pulsar spin-down statistics</td>
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<tr>
<td>$\zeta_1$</td>
<td>$0.02$</td>
<td>-</td>
<td>Combined PPN bounds</td>
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<tr>
<td>$\zeta_2$</td>
<td>$4 \times 10^{-5}$</td>
<td>Binary pulsar acceleration</td>
<td>PSR 1913+16</td>
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<tr>
<td>$\zeta_3$</td>
<td>$10^{-8}$</td>
<td>Newton's 3rd law</td>
<td>Lunar acceleration</td>
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<tr>
<td>$\zeta_4$</td>
<td>$0.006$</td>
<td>-</td>
<td>Usually not independent</td>
</tr>
</tbody>
</table>
The Process

Theory Space  Regime

Parametrization

Observables
The Universe: background cosmology

\[ ds^2 = a^2 \gamma_{\mu\nu} dx^\mu dx^\nu \]

**FRW equations**

\[ G_{\alpha\beta} = 8\pi GT_{\alpha\beta} \quad \rightarrow \quad \mathcal{H}^2 = \frac{8\pi G}{3} a^2 \rho \]

Any theory (modified gravity or otherwise)

\[ G_{\alpha\beta} = 8\pi GT_{\alpha\beta} + U_{\alpha\beta} \quad \rightarrow \quad \rho_X(\tau), P_X(\tau) \]

Tuesday, 14 January 14
Only measure $H(z)$ and $\Omega_K$
The Universe: large scale structure

![Graph showing linear, quasilinear, and nonlinear regions of the universe.](image)
Linear Perturbation Theory \((10 - 10,000 h^{-1} Mpc)\)

\[
ds^2 = a^2 (\gamma_{\mu \nu} + h_{\mu \nu}) dx^\mu dx^\nu
\]

Diffeomorphism invariance \(\rightarrow\) Gauge invariant

Newtonian potentials

\[
\rho \rightarrow \rho(\tau)[1 + \delta(\tau, \mathbf{r})]
\]

\[
\delta G_{\alpha \beta} = 8\pi G \delta T_{\alpha \beta}
\]

\[
\delta G^{(g_i)}_{00} : 2\nabla^2 \dot{\Phi} - 6\mathcal{H} k \dot{\Gamma} = 8\pi G a^2 \rho \delta^{(g_i)}
\]

\[
\delta G^{(g_i)}_{0i} : 2k \dot{\Gamma} = 8\pi G (\rho + P) \theta^{(g_i)}
\]

\[
\delta G^{(g_i)}_{ij} : \dot{\Phi} - \dot{\Psi} = 8\pi G a^2 (\rho + P) \Sigma^{(g_i)}
\]

\(+\ \delta G^{(g_i)}_{ii} \text{ equation}\)
Extending Einstein’s equations

\[\delta G_{\mu\nu} = 8\pi G \delta T^M_{\mu\nu} + \delta U_{\mu\nu}\]

Linear in \(\hat{\Phi}, \hat{\Gamma}, \hat{\chi}, \dot{\chi}\)

Skordis 2010
Baker, Ferreira, Skordis 2012
Bloomfield, Flanagan, Park, Watson 2012
Gleyzes, Gubitosi, Piazza, Vernizzi 2013
Pearson, Battye 2011
Extending Einstein’s equations

Key: Matter + Metric + New degree of freedom

\[-\alpha^2 \delta G_0^0(g^i) = \kappa \alpha^2 G \rho_M \delta_M^{(g^i)} + A_0 k^2 \hat{\Phi} + F_0 k^2 \hat{\Gamma} + \alpha_0 k^2 \hat{\chi} + \alpha_1 k \dot{\chi}\]
Extending Einstein’s equations

Key: Matter + Metric + New degree of freedom

\[-a^2 \delta G^0_0(g_i) = \kappa a^2 G \rho_M \delta^{(g_i)}_M + \alpha_0 k^2 \dot{\chi} + \alpha_1 k \ddot{\chi} + A_0 k^2 \hat{\Phi} + F_0 k^2 \hat{\Gamma}\]

\[-a^2 \delta G^0_i(g_i) = \nabla_i \left[ \kappa a^2 G \rho_M (1 + \omega_M) \theta^{(g_i)}_M + \beta_0 k \dot{\chi} + \beta_1 \ddot{\chi} \right] + B_0 k \hat{\Phi} + I_0 k \hat{\Gamma}\]

\[a^2 \delta G^i_i(g_i) = 3 \kappa a^2 G \rho_M \Pi^{(g_i)}_M + \gamma_0 k^2 \dot{\chi} + \gamma_1 k \ddot{\chi} + \gamma_2 \dddot{\chi} + C_0 k^2 \hat{\Phi} + C_1 k \hat{\Phi} + J_0 k^2 \hat{\Gamma} + J_1 k \hat{\Gamma}\]

\[a^2 \delta G^i_j = D^i_j \left[ \kappa a^2 G \rho_M (1 + \omega_M) \Sigma_M + \epsilon_0 \dot{\chi} + \frac{\epsilon_1}{k} \ddot{\chi} + \frac{\epsilon_2}{k^2} \dddot{\chi} \right] + D_0 \hat{\Phi} + \frac{D_1}{k} \hat{\Phi} + K_0 \hat{\Gamma} + \frac{K_1}{k} \hat{\Gamma}\]
Extending Einstein's equations

Integrability

7 to 9 free functions of time

ArXiv:1209.2117

Tuesday, 14 January 14
Integrability

Most general action with 1 d.o.f. (use unitary gauge)

\[ S = \int d^4x \sqrt{-g} \, L(N, K^\mu, K_{\mu\nu}, K^{\mu\nu}, (3)R, (3)R_{\mu\nu}, (3)R^{\mu\nu}, \ldots; t) \, . \]

Expand to 2nd order

\[ L(N, K, S, R, Z) = \bar{L} - \dot{\mathcal{F}} - 3H \mathcal{F} + (\dot{\mathcal{F}} + L_N) \delta N + L_R \delta R \]
\[ + \frac{A}{2} \delta K^2 + L_S \delta K^\mu \delta K^\nu + \left( \frac{1}{2} L_{NN} - \dot{\mathcal{F}} \right) \delta N^2 \]
\[ + \frac{1}{2} L_{RR} \delta R^2 + \mathcal{B} \delta K \delta N + \mathcal{C} \delta K \delta R + L_{NR} \delta N \delta R + L_{Z} \delta Z + O(3) \]

where:
\[ \mathcal{F} \equiv 2HL_S + L_K, \]
\[ A \equiv 4H^2L_{SS} + 4HL_{SK} + L_{KK}, \]
\[ B \equiv 2HL_{SN} + L_{KN}, \]
\[ C \equiv 2HL_{SR} + L_{KR} \, . \]

The \( L_X, L_{XY} \) 
are functions of time only.

Baker, Gleyzes, Ferreira, Vernizzi in prep
## Extending Einstein’s equations

<table>
<thead>
<tr>
<th>Scalar-Tensor</th>
<th>Galileons</th>
<th>K.G.B.</th>
<th>DGP</th>
<th>Einstein-Aether</th>
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<tbody>
<tr>
<td>f(R) gravity</td>
<td>The Fab Four</td>
<td>Quintessence</td>
<td>EBI</td>
<td>Horava-Lifschitz</td>
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<tr>
<td>f(G) theories</td>
<td>K-essence</td>
<td>Dark fluids</td>
<td>TeVeS</td>
<td>G-inflation</td>
</tr>
</tbody>
</table>

*ArXiv:1209.2117*
What about the non-linear regime?

Pros: Much better sampling of density field $N_{modes} \propto k^3$

$$F(R) = 1 + f(R)$$
What about the non-linear regime?

Baryon, feedback and bias
And now to what we observe: Light vs Matter

- For a perturbed line element of the form:

\[ ds^2 = a^2(\tau) [- (1 + 2\Phi) d\tau^2 + (1 - 2\Psi) \gamma_{ij} dx^i dx^j] \]

the equations of motion are:

\[ \frac{1}{a} \frac{d(a\mathbf{v})}{d\tau} = -\nabla \Phi \quad \text{(non-relativistic particles)} \]

\[ \frac{d\mathbf{v}}{d\tau} = -\nabla \perp (\Phi + \Psi) \quad \text{(relativistic particles)} \]
What we observe.

\[ \delta, \vec{\nu} \]

\[ \vec{\nu} \]

\[ \Phi, \Psi \]
Large Scales: the problem with cosmic variance

\[ C_\ell = G_{\text{eff}}(1 + \Omega_\Lambda) \]

\[ G_{\text{eff}} = G_0(1 + 0.1 \Omega_\Lambda) \]

\[ G_{\text{eff}} = G_0(1 - 0.1 \Omega_\Lambda) \]

\[ G_{\text{eff}} = G_0(1 + 0.2 \Omega_\Lambda) \]

\[ G_{\text{eff}} = G_0(1 - 0.2 \Omega_\Lambda) \]

\[ \text{ISW- late time effects on large scales} \]

\[ \propto \int (\dot{\Phi} + \dot{\Psi}) d\eta \]
Large scales: the problem with the Galaxy

Systematic effect due to stellar densities

Ross et al (BOSS) 2012
Large Scales: Tomography of Neutral Hydrogen

First attempts: the GBT
Not so large scale: “quasi-static” regime

A preferred length scale - the horizon

\[ \mathcal{H}^{-1} \equiv \left( \frac{\dot{a}}{a} \right)^{-1} \propto \tau \sim 3000h^{-1}\text{Mpc} \]

Focus on scales such that \( k\tau \gg 1 \)

Most surveys \( \leq 300h^{-1}\text{Mpc} \)

\[-k^2\Phi = 4\pi G\mu a^2 \rho \Delta \]

\[ \gamma \Psi = \Phi \]

Note: not applicable to CMB!
Not so large scale: “quasi-static” regime

The “quasi-static” functions reduce to a simple form

\[
\gamma = \frac{p_1(a) + p_2(a)k^2}{1 + p_3(a)k^2},
\]

\[
\mu = \frac{1 + p_3(a)k^2}{p_4(a) + p_5(a)k^2}.
\]

Baker et al 2012
Silvestri et al 2013

where \( p_i = p_i[L_K, L_{KK}, \cdots] \)

Goal: to use \( k \) and \( z \) dependent measurements of \((\gamma, \mu)\) to constrain PPF functions
Growth of Structure

Growth rate

\[ f(k, a) = \frac{d \ln \delta_M(k, a)}{d \ln a} \]

\( f \) satisfies a simple ODE

\[ \frac{df}{d \ln a} + qf + f^2 = \frac{3}{2} \Omega_M \xi \]

with

\[ q = \frac{1}{2} \left[ 1 - 3w(1 - \Omega_M) \right] \]

\( \xi = \frac{\mu}{\gamma} \)
Growth of structure: Redshift Space Distortions

Real space:  Redshift space:

Linear regime  Squashing effect

Turnaround  Collapsed

Collapsing  Finger-of-god

Guzzo et al 2008
\[ f(k, a) = \frac{d \ln \delta_M(k, a)}{d \ln a} \]
Weak Lensing
Galaxy Weak Lensing

Simpson et al 2012
(CFHTLens)

Reyes et al 2010

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State of Play in 2014

no constraints on GR

however...
The Future

Euclid
Mapping the Geometry of the Dark Universe

The Square Kilometre Array
The International Radio Telescope for the 21st Century

Large Synoptic Survey Telescope
# The Future is now

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Now</th>
<th>Soon</th>
<th>Future</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photo-z:LSS (weak lensing)</td>
<td>DES, RCS, KIDS</td>
<td>HSC</td>
<td>LSST, Euclid, SKA</td>
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<tr>
<td>Spectro-z (BAO, RSD, ...)</td>
<td>BOSS</td>
<td>MS-DESI, PFS, HETDEX, Weave</td>
<td>Euclid, SKA</td>
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<tr>
<td>SN Ia</td>
<td>HST, Pan-STARRS, SCP, SDSS, SNLS</td>
<td>DES, J-PAS</td>
<td>JWST, LSST</td>
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<td>CMB/ISW</td>
<td>WMAP</td>
<td>Planck</td>
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<tr>
<td>sub-mm, small scale lensing, SZ</td>
<td>ACT, SPT</td>
<td>ACTPol, SPTPol, Planck, Spider, Vista</td>
<td>CCAT, SKA</td>
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<tr>
<td>X-Ray clusters</td>
<td>ROSAT, XMM, Chandra</td>
<td>XMM, XCS, eRosita</td>
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<tr>
<td>HI Tomography</td>
<td>GBT</td>
<td>Meerkat, Baobab, Chime, Kat 7</td>
<td>SKA</td>
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</tbody>
</table>
The Future: Redshift Space Distortions

Percival 2013

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# Model Dependent Constraints

<table>
<thead>
<tr>
<th>Theory</th>
<th>parameter</th>
<th>now</th>
<th>future</th>
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</thead>
<tbody>
<tr>
<td>Brans-Dicke</td>
<td>$1/\omega$</td>
<td>0.006</td>
<td>$4.19 \times 10^{-4}$</td>
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<td>Einstein-Aether</td>
<td>$c_1$</td>
<td>few</td>
<td>0.222</td>
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<tr>
<td></td>
<td>$c_3$</td>
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<td></td>
<td>$\alpha$</td>
<td>few</td>
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<tr>
<td>DGP</td>
<td>$1/(r_c H_0)$</td>
<td>0.075</td>
<td>0.004</td>
</tr>
</tbody>
</table>
Summary

• The large scale structure of the Universe can be used to test gravity (different eras probe different scales).

• There is an immense landscape of gravitational theories (how credible or natural is open for debate).

• We need a unified framework to test gravity (“PPF” modelled on PPN).

• Focus on linear scales at late times (for now).

• Non-linear scales can be incredibly powerful but much more complicated.

• Need new methods and observations to access the really large scales (is HI tomography the future?).

• Current measurements are not constraining.

• There are a plethora of new experiments to look forward to.